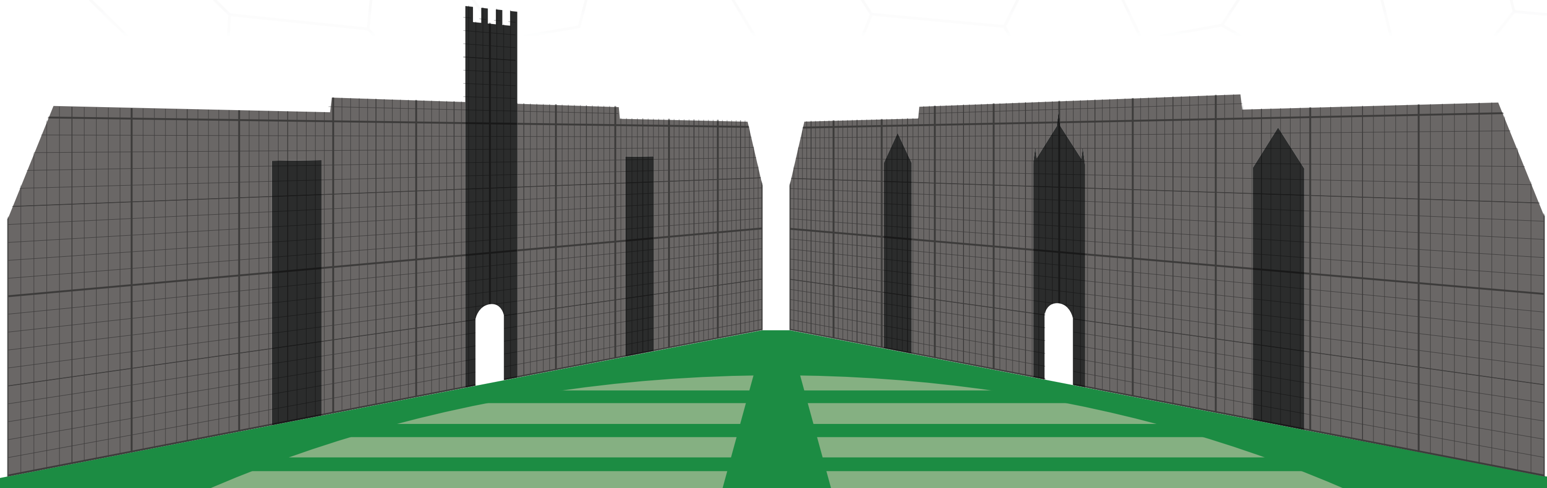
Conference Proceedings



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Building mathematics: how artefacts can be used to engage students with their learning of mathematics

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Abstract

There has been much discussion about what techniques can be employed successfully to engage students with their learning of mathematics. At university level these include problem based approaches and enrichment activities. Work also identifies the importance of engaging students in meaningful mathematical discussion as an essential component of students' mathematics learning.

At pre-undergraduate level there is a range of evidence supporting the use of mathematical objects or artefacts to introduce new concepts. It is argued that this type of activity gives a visual and physical perspective to the learning and reinforces newly introduced concepts. However this notion is not widely used to support the learning of undergraduate level mathematics.

The Mathematics team at Middlesex utilised and built on ideas and techniques within these areas to develop a series of tasks and activities to engage students with mathematics and promote understanding of advanced concepts. These included the construction of mathematical artefacts that supported concepts and ideas that the students were learning within the mathematics curriculum. The series of activities developed are referred to as 'Building Mathematics' activities. The process of constructing, use of these artefacts, and the discussion it promoted is shown to help develop a deep understanding of mathematical concepts.

In this article we will discuss the types of activities used, the artefacts produced and the impact on students learning of mathematics. We will reflect on the successes and challenges of the venture and discuss plans for future development and enhancement of the initiative.

1. Background

The step-up from pre-university mathematics learning to undergraduate mathematics has been studied for a number of years. It has been argued that pre-university mathematics fails to prepare students sufficiently for many of the more abstract concepts studied, (Smith, 2004; Hawkes & Savage, 2000). One of the resulting issues is disengagement. Indeed the QAA benchmark statement, (QAA, 2015), highlights the importance of engagement in the learning process. In (Borovik & Gardiner, 2006) the authors indicate that "*mathematics requires a high level of motivation and emotional involvement on the part of the learner*", a sentiment that most mathematicians and mathematics educators will agree with. Engagement is thus seen as core to developing a deep understanding of the subject. Providing an engaging and intellectually stimulating learning experience is a key component to this. Techniques used to do this include problem and activity based learning and enrichment activities.

One method often utilised to develop engagement with mathematics at pre-university level is the use of mathematical artefacts and instruments. In the current article we define an artefact as a physical object. This can be generalised, and has been in the literature, to any object produced by humans including sounds, physical gestures, technology and so on, although there is some discussion on the precise definition of an artefact.

This follows a Vygotskian approach, (Vygotsky, 1986). Vygotsky put forward the theory that meaningful activity plays a role and is a generator of understanding. According to Vygotsky higher mental functions should be viewed as the products of mediated activity. In the case of mathematical learning we see the artefact as the mediator between the learner and the concept. Vygotsky also indicated that knowledge and understanding could be facilitated through social activity. The social aspect of activities is also, then, a vital part of the learning process.

In Crawford et al. (1993), and as cited in Crawford (1996), the authors claim that traditional approaches to teaching mathematics at universities have limited mediated activity. The authors found that 80% of students felt that mathematics was simply a set of techniques designed to solve particular problems. Furthermore it was claimed that students merely learnt mathematics in order to perform well on assessment and did not seek a deeper understanding of the subject. More recently, Bartolini and Mariotti (2008) discuss the role, use and contribution of artefacts and instruments as tools to develop a deeper understanding. Despite this research being done at the pre-university level we expect the use of artefacts to have a similar impact at more advanced levels.

Drawing on the research described here the programme team at Middlesex University designed a number of tasks and workshops designed to engage students with their learning of mathematics. A weekly timetabled session was introduced called 'Engaging with Mathematics' and embedded activities were used in modules throughout the year. The current article will discuss details of and the general aim of some of the activities where artefacts were used. Some of these activities have previously been reported by the second author in (Megeney, 2015) in the context of employability skills.

2. Engaging students using artefacts

In this paper we will discuss two of the engagement activities the team developed although other activities were designed.

2.1. Visualising higher dimensions

Students often find the move from two and three-dimensional geometry to higher dimensions difficult. Indeed the fact that one cannot visualise these dimensions requires an abstract understanding of the links between geometric objects and their mathematical descriptions. In order to allay these concerns in students the programme team designed a visualisation activity to demonstrate objects in four dimensions by considering their shadows in three dimensions. This, of course, is a familiar technique in lower dimensions where, for example, one might draw a cube on a piece of paper.

Using the construction kit Zometool™ students were asked to build various three dimensional objects and study their shadows using a torch or a projector. Students learned that familiar objects like cubes, tetrahedrons, dodecahedrons and so on produced shadows that often bared a resemblance to the original shape, but also occasionally did not. In particular producing a hexagonal shadow from a cube was facilitated by the team and demonstrated by the students.

Using the students' familiarity with these shapes allowed them to develop a deeper understanding of four dimensional objects. For example students found that the two-dimensional square shadow of a cube allowed for a generalisation of the cube as a three-dimensional shadow of the four-dimensional hypercube. This led students to the notion of projections from higher dimensions.

It was found that these activities helped students overcome their initial fear of working in higher dimensions. The programme team found that, by the end of the sessions, students were not only able to work in higher dimensions but were comfortable doing so. In fact the final activity includes working together to build a shadow of the four-dimensional equivalent of a dodecahedron, the 120-cell. And this part of the activity promoted communication and group discussion.

These activities fed directly into the content of their first year courses where Zometool™ and the students' familiarity with it facilitated their understanding of the abstract concepts including finite-dimensional vector spaces and linear algebra where higher dimensions are frequently discussed and, importantly, treated no differently from two and three dimensions.

2.2. Sierpinski Tetrahedron

The second activity was designed as an enrichment activity. This activity reinforced the notions of recursion and induction that the students encounter in their modules. These concepts are introduced pre-university and are often taught as a series of steps with little emphasis on the understanding of what can be a challenging idea. Examples encountered of the principle of induction in particular often follow a similar familiar style. The programme team felt that working with these concepts constructively using the self-similarity of fractals would improve students' confidence in applying these ideas to less familiar situations. Furthermore it offered the opportunity for the team to challenge the ideas of measurement in space in preparation for more advanced concepts like measure and Hausdorff dimension that they may encounter later in their degrees.

Students worked together to produce a model of a Sierpinksi tetrahedron. Some groups worked on smaller tetrahedrons, others on fitting these together. Each group required coordination and communication in order to correctly construct their parts.

Towards the end of the first year, students take part in an external, public facing event designed to encourage children and young people to consider studying STEM subjects. At this event the students ran the Sierpinski tetrahedron activity themselves, communicating the ideas to children and young people. This required students to tailor their communication of more advanced ideas to the audience: some of the participants were happy to discuss concepts at length, some of the younger children simply wanted to play.

3. Discussion

Anecdotal evidence from staff and students confirm the influence of interaction with artefacts on the learning of more advanced topics. Reflections of students further solidify these links and suggest an increased confidence in working on advanced mathematics. One student said "I found the ideas behind symmetrical objects, especially higher dimension ones, extremely fascinating. … I find it hard to imagine anything higher than 3D, so I really appreciated having an idea of how things can be viewed". Another student told us "[the activities] helped each of us grow our knowledge [sic] and ability, but at the same time, have fun".

Other students confirmed that the communication of ideas reinforced their understanding of the ideas, one saying that "explaining what a Sierpinski tetrahedron is helped deepen my understanding".

The opportunity for social interaction between students and staff was also noted positively by students. One student said "we got to work closely with the staff during all these activities. This was very good to build up confidence and self-esteem". Yet another said that the "merits of interacting with staff, students and the public was that we were able to work together, learn together, and literally build together. This helped each of us grow our knowledge and ability, but at the same time, have fun; there were also a lot of successful teamwork and communication between each other". Another student said that "these activities created an environment of active, involved and explanatory learning".

The improvement in student engagement has been noted by staff teaching on the programme. This extends work done in (Megeney, 2015) on promoting the development of employability skills using mathematics engagement activities.

The project reported in this article is moving into its third year. Feedback received to date is very positive, both from students and staff. There is evidence that motivation and engagement of students in the subject and their course is improved. Students are more confident in expressing their opinions and questioning theory. There was, in particular, a notable improvement in student engagement following their participation in the external event described in section 2.2.

One consequence of this is that a number of students have indicated a wish to become involved with these and similar activities in the future. Furthermore there is an increase in involvement with peer assisted learning schemes at the university as a result of the activities.

The innovations discussed in this article are not without their challenges. The time required to design a meaningful mediated activity can be substantial and should not be underestimated. Ensuring the mathematical content of activities is sufficiently clear and communicated can be a challenge in itself. Furthermore balancing the fun element of an activity and the learning experience can be difficult, this should be considered carefully in the design of the activities.

It was clear that on occasion staff or students involved in the activities were not as receptive to the learning experience as we would have liked. Catering for different learning styles is therefore an important aspect of the design. Indeed including the student voice in the design of activities is an important aspect.

In the future we expect to continue and extend the range and type of activities following the success of the Engaging with Mathematics series and other activities. In particular the mathematics undergraduate programmes will continue to take an active part in external outreach events. Furthermore the team are planning on involving final year students in developing and delivering activity workshops akin to those described in this article.

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