

Time-based Resource Allocation for Downlink in Heterogeneous Wireless Cellular Networks

David O. Denedo · Quoc-Tuan Vien ·
Ca V. Phan

Received: date / Accepted: date

Abstract Heterogeneous Wireless Cellular Networks (HWCNs) are an essential part of current and future cellular networks as a result of several benefits they offer regarding the ever-increasing user traffic requirement. Network resources are nevertheless limited, and as such, an efficient allocation of resources is vital for the development of the HWCNs. An improvement in the coverage area leads to increased mobile user satisfaction which in turn yields higher revenue for network operators. Minimising power consumption helps reduce CO_2 emissions for economically and environmentally efficient HWCNs. In this paper, by exploiting stochastic geometry, we first analyse the Coverage Probability (CP) of a typical HWCN and evaluate the impacts of propagation model/building types, cell allocation and per-tier power allocation. It is shown that a higher allocation of resources in a more lossy environment generally leads to a higher CP up to a certain threshold. Also, previous research shows that large-scale user behaviour varies over time. To this end, this paper proposes a hybrid Resource Allocation (RA) scheme, namely Time-based RA (TRA), to solve a hybrid optimisation problem of improving coverage area during periods of peak user traffic while minimising total power consumption during off-peak periods. Numerical results show that the proposed scheme achieves up to 25% higher CP during the peak period subject to limited total available power and 57% savings in total power consumption during the period of minimal user traffic given a target coverage probability, when compared to the schemes with equally allocated resources.

D. O. Denedo
Faculty of Science and Technology, Middlesex University, United Kingdom
E-mail: dd731@live.mdx.ac.uk, daveden22@gmail.com

Q.-T. Vien
Faculty of Science and Technology, Middlesex University, United Kingdom
E-mail: q.vien@mdx.ac.uk

C. V. Phan
Faculty of Electrical and Electronics Engineering, Ho Chi Minh City University of Technology and Education, Vietnam.
E-mail: capv@hcmute.edu.vn

Keywords Heterogeneous networks · Resource allocation optimisation · Coverage probability · Power consumption · Time-based Resource Allocation

1 Introduction

The proliferation of mobile devices was projected to reach well over 50 billion connected devices by 2020 [1] arising from increased demand for services like video streaming, social media and teleconferencing, which are bandwidth-intensive [2]. Despite the ever-increasing traffic demand, the available allocatable resources are limited [3]. The need to adequately satisfy this enormous number of mobile devices coupled with their high bandwidth requirement have facilitated the development of Heterogeneous Wireless Cellular Networks (HWCNs) [4–8].

In HWCNs, the existing tower-mounted macrocells are overlaid with smaller cells, such as microcells, picocells and femtocells [9, 10], to form a multi-tier system. Different tiers of Base Stations (BSs) are characterised by their transmit power, propagation characteristics, backhaul and physical form factor [11]. The tower-mounted macrocells provide an extensive coverage area, whereas the network operator-managed pico-cells help enhance the coverage area by targeting cell edges and blind-spots. The network could also comprise user-operated femtocells to improve throughput and coverage area within small indoor areas, such as home and office. Potential benefits derived from adopting the HWCN as compared to homogeneous macrocell deployment include higher spectral efficiency through frequency reuse [12, 13], improved Quality of Service (QoS) [1] and better coverage area, especially for indoor environments and cell edge users [14].

Among various aspects of cellular networks, two essential aspects that should be considered are Coverage Probability (CP) and Energy Efficiency (EE). With regards to the CP, mobile users are significant stakeholders and providing better coverage would improve user satisfaction, which in turn generates higher revenue for the network providers. Similarly, the EE is an essential factor affecting a wide variety of cellular network stakeholders because an increase in the EE lowers cost for the network providers. It is especially beneficial to the environment and complies with policies on green technology.

Considering both CP and EE in the network design, this paper aims to address the following concerns:

1. How do resource allocation (RA) and propagation affect the coverage area of downlink in HWCN, in which cells may experience different environment and are supplied with different power despite existing within the same tier?
2. Ideally, increasing the transmit power of BSs should lead to an enhancement of coverage. However, there exists inter-cell interference from neighbouring cells. So, how can the per-tier power be allocated to improve the overall network coverage?
3. Dense deployment of cells results in overlapping cell regions with increased inter-cell interference. How can the cells in the tiers be distributed to improve the system performance?

4. Subject to the dynamic nature of user traffic across time as well as constraints in network resources, how can the available resources be allocated effectively to satisfy the time-varied user traffic?

To this extent, we first investigate the impacts of various parameters on the coverage area of an HWCN, based on which optimisation problems (OPs) are proposed to find the optimal RA in terms of both power allocation and cell density for time-varied user traffic. The main contributions of this paper can be summarised as follows:

1. Exploiting stochastic geometry for modelling the distribution of cells in different tiers, CP is analysed for the downlink in a typical HWCN.
2. The impacts of propagation model/building types, heterogeneity through tier splitting, cell intensity allocation and per-tier power allocation on the CP of the HWCN are evaluated, putting the effect of inter-cell interference into consideration.
3. Two OPs are developed, including: i) maximise CP during the peak period of user traffic subject to limited resources in terms of total available power, and ii) minimise the total power consumption during the off-peak period subject to QoS requirement of the coverage.
4. A heuristic algorithm, namely Time-based Resource Allocation (TRA), is proposed to solve the above two OPs so as to either maximise the CP or minimise the total power consumption. The proposed TRA finds the optimal cell allocation to maximise CP during the peak period of user traffic and optimal power allocation to minimise the total power consumption subject to a CP threshold during the period of minimal user traffic.
5. Numerical results are provided for the proposed TRA algorithm to validate its effectiveness compared to the equal RA scheme.

2 Related Work

Anticipated to reach over 50 billion connected devices by 2020 [1], there exists a disparity between the growth in mobile traffic compared to the available resources provided by network operators [3]. Integrating the current and the future cellular technology, HWCN can bridge the gap between mobile traffic and network resources. An overview of HWCNs was provided in [11] showing the coexistence between macro-cells and picocells. A baseline model for HWCNs was also established in [15], which included several essential parameters such as outage probability and received Signal-to-Interference-plus-Noise-Ratio (SINR).

Stochastic geometry has been identified as a potential tool to capture the random spatial placement of BSs for accurate modelling and analysis of HWCNs [1]. In [9], the impacts of propagation model and per-tier power allocation were evaluated using the stochastic geometry, and a heuristic algorithm was proposed to solve an OP of minimising the total power consumption subject to constraints of CP and limited per-tier power resources.

EE has been investigated as a measure of system throughput over total power consumed by the entire network [16]. Although HWCNs are shown to help reduce the energy consumption as a result of the deployment of low-powered small cells,

the cumulative power expended by those small cells themselves still add up to a substantial amount in a dense network. The authors in [14] investigated the energy-efficient allocation of resources in the downlink of the HWCNs with CP constraints, focusing on maximising the overall EE of the femtocells within the network.

The deployment of advanced technologies comes up with vital questions about their impacts on the environment. Specifically, the drive to achieve ubiquitous network coverage gave rise to the evolution of LTE, but it also led to a substantial increase in carbon dioxide emission [14]. A significant amount of the global energy has been consumed by the ICT in which radio access nodes in cellular networks would consume a significant portion of the energy. About 15% of the total power in cellular networks is expended on transmission, while the cooling system consumes about 33%. Therefore, an effective way to tackle the power issue would be to reduce the number of deployed access nodes [3, 17, 18].

In [19], it was shown that dense deployment of small cells leads to higher throughput and user QoS at the expense of an increased deployment and operational cost as well as an increased energy consumption. Stochastic geometry tools were also employed to optimise the BS cell densities to make a compromise between network performance and deployment costs. The impacts of the transmit power parameters and the intensity of small cell distribution on the EE of the network were studied in [20] taking the spectral efficiency and CP into consideration. It was verified that as the number of small cells is large enough, the EE gain diminishes up to the point of even vanishing since the inter-cell interference becomes prevailing. A focusing-searching algorithm was accordingly proposed to optimise the cell intensity configuration for the small cells to achieve the maximum EE. The authors in [21] applied Lyapunov optimisation framework to strike a balance between the average throughput and latency while guaranteeing the EE.

In [22], it was identified that the energy saved by controlling the transmit power alone is limited since the required power for actual transmission is quite small in comparison with the total power consumed in macro BSs. Here, the total power consumption comprises of cooling, signal processing, and computation as well as RF signal amplification. A method to reduce energy cost by joint spectrum allocation, user association and cell activation was developed in [22] for a small network having up to 20 BSs.

Dealing with EE issues, a framework was introduced in [3] which involves the centralisation of baseband processing to significantly reduce energy consumption, processing power aggregation and dynamic RA. In [14], the authors claimed that the communication among macrocells and femtocells is in a coarse timescale, and thus the centralised solution would not be feasible.

Traditionally, a high traffic demand was assumed with continuously active macro BSs. However, it was revealed that there are high fluctuations in traffic demand, both spatially and temporally in cellular networks. In fact, the traffic during the day is generally higher than that at night within commercial areas, and hence switching some BSs off during off-peak period, while concurrently increasing the coverage area of other BSs to prevent dead zones, could potentially lead to higher energy savings. Stochastic geometry tools were also employed to analyse the CP and EE of HWCNs through the deployment of sleep mode in [23]. Specifically, the idle mode capability

(IMC) of the BSs allows them to switch off their transmission modules when there is no active user equipment (UE) discovered within the coverage area. Thanks to the IMC, energy wastage can be reduced and the CP can be enhanced by reducing the interference caused by the unused BSs which have been switched off.

In [24], it was suggested that the application of sleep mode technology may negatively impact the Quality of Experience (QoE). The authors considered the QoE as the desired level of buffer starvation probability which arises from video streaming when the buffer of the mobile device gets depleted. The media player has to be delayed allowing for more packets to fill up the buffer. In order to reflect that fact, the authors in [24] analysed the starvation probability of a buffer where bursty on-off arrival occurs, taking into account the presence of different tiers of BSs and sought to optimise the on-off switching framework subject to the QoE requirements.

One major challenge in HWCN is RA and interference management arising because the resources of all tiers are provided by the same service provider [21]. Mobile network operators who own multiple frequency spectrum bands can allocate higher frequency bands to small cells to achieve a higher throughput while lower frequency bands can be allocated to macro cells to achieve better coverage area. According to [17], an enhanced EE can be achieved by tackling one or a combination of total energy consumption at the BS, cell deployment strategies, or user association, along with on-off scheme. The authors in [25] further compared the performance of different approaches for user association and RA. In [26], three use cases in HWCNs were investigated, including mobile broadband, machine-type communication and ultra-reliable low-latency communication, with the deployment of scheduling and RA optimisation.

Several research works on HWCNs have focused on improving various aspects such as minimising the impact of interference from neighbouring cells, user association, load balancing, EE, and improving CP. While a few works considered location-based RA for downlink in the HWCNs, this research focuses on the RA based on the variation in user traffic over time.

3 System Model

This paper investigates the downlink of a typical HWCN with K tiers of BSs. Each tier is characterised by their transmit power, cell intensity and physical form factor of macrocells, microcells and picocells (see Fig. 1).

It is assumed that there exist M_k propagation models in the k -th tier, $k = 1, 2, \dots, K$. The spatial distribution of the BSs within the m_k -th propagation model of the k -th tier is modelled as homogeneous Poisson Point Process (PPP) Φ_{k,m_k} with intensity λ_{k,m_k} . The attenuation coefficient for the propagation model is denoted by α_{k,m_k} ($\alpha_{k,m_k} > 2$) and the transmit power of the serving BS is denoted by Δ_{k,m_k} . The communication channel between the transmitting BS and a UE located at a point x_{k,m_k} from the BS, is assumed to experience Rayleigh fading having power of $h_{k,m_k} \sim Exp(1)$ and additive white Gaussian noise n_{k,m_k} having power of N_0 . The received signal at a UE within

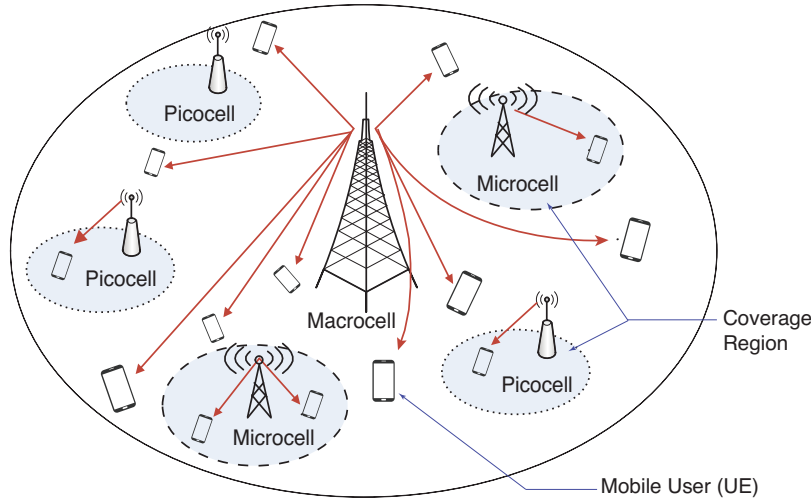


Fig. 1 System Model for a Typical HWCN

the k -th tier can be written by

$$y_{k,m_k} = \frac{\sqrt{\Delta_{k,m_k}} \sqrt{h_{k,m_k}}}{\sqrt{\|x_{k,m_k}\|^{\alpha_{k,m_k}}}} s_{k,m_k} + \sum_{k=1}^K \sum_{m=1}^M \sum_{x \in \Phi_{k,m} \setminus x_{k,m}} \frac{\sqrt{\Delta_{k,m}} \sqrt{h_x}}{\sqrt{\|x\|^{\alpha_{k,m}}}} s_{k,m} + n_{k,m_k}. \quad (1)$$

The received SINR at the UE can be therefore expressed as:

$$\gamma_{k,m_k} = \frac{\Delta_{k,m_k} h_{k,m_k} \|x_{k,m_k}\|^{-\alpha_{k,m_k}}}{\sum_{k=1}^K \sum_{m=1}^M \sum_{x \in \Phi_{k,m} \setminus x_{k,m}} \Delta_{k,m} h_x \|x\|^{-\alpha_{k,m}} + N_0} \quad (2)$$

For reliable communication, a UE located at x_{k,m_k} in the m -th propagation model should receive the desired signal power above a threshold value from the serving BS. This paper assumes an open access model of connectivity whereby the UE has unrestricted access to any tier in an HWCN, provided that its received SINR (i.e. γ_{k,m_k}) exceeds a given threshold SINR denoted by τ_{k,m_k} . The CP¹, denoted by \hat{p} , of the downlink in the HWCN can be thus expressed as:

$$\hat{p} = \Pr \left\{ \bigcup_{\substack{1 \leq k \leq K, 1 \leq m \leq M \\ x_{k,m_k} \in \Phi_{k,m_k}}} \gamma_{k,m_k} > \tau_{k,m_k} \right\} \quad (3)$$

4 Coverage Probability of Downlink in an HWCN

In this section, we investigate CP of downlink in a typical HWCN, taking the impact of the propagation model and cell intensity into account. Assuming a maximum SINR connectivity model, the CP of the downlink in the HWCN can be calculated by [9]:

¹ The CP is defined as the likelihood that a UE receives the desired signal from the transmitting BS.

$$\begin{aligned}
\hat{p} &= \sum_{k=1}^K \sum_{m=1}^M \lambda_{k,m_k} \int_{\mathcal{R}^E} \exp\left(-\frac{\tau_{k,m_k} N_0}{\Delta_{k,m_k}} \|x_{k,m_k}\|^{\alpha_{k,m_k}}\right) \\
&\times \exp\left[-2\pi \sum_{t=1}^K \sum_{n=1}^{|M_k|} \frac{\lambda_{t,n}}{\alpha_{t,n}} \left(\frac{\Delta_{t,n}}{\Delta_{k,m_k}}\right)^{\frac{2}{\alpha_{t,n}}}\right. \\
&\left. \times \csc\left(\frac{2\pi}{\alpha_{t,n}}\right) (\tau_{k,m_k})^{\frac{2}{\alpha_{t,n}}} \|x\|^{\frac{2\alpha_{k,m_k}}{\alpha_{t,n}}}\right]
\end{aligned} \tag{4}$$

From the analysis of (4) as in [9], the following remarks were observed:

1. The propagation environment having a higher path loss exponent achieves a higher CP. It is worth noting that \hat{p} increases as $\alpha_{t,n}$ increases, where $t = 1, 2, \dots, K$ and $n = 1, 2, \dots, M_k$. Also, from (2), it can be observed that a more lossy environment leads to a lower interference, consequently resulting in a higher CP.
2. Allocating a higher cell intensity to the more lossy propagation model achieves an enhanced CP. It is evident that \hat{p} increases over λ_{k,m_k} for propagation models having the higher path loss exponent (i.e. α_{k,m_k}). The resulting implication is that assigning the cell intensities in increasing degree of path loss exponent for the propagation models in a tier achieves a better coverage region.
3. The effect of background noise (i.e. N_0) on the coverage region is negligible. Deductions from (2) show that the effect of interference is more dominant than that of the background noise. Also, from (4), \hat{p} is shown to be mostly dependent on the interference parameters. Therefore, in an HWCN consisting of several different BSs, the effect of interference would overshadow that of background noise, and as such, the background noise parameter can be ignored.

5 Resource Allocation for an HWCN

It is revealed that the assigned cell intensity and power allocation of individual tiers have significant impacts on the coverage area of HWCNs. However, increasing either the cell intensity or the transmit power of BSs does not necessarily lead to an improvement in the coverage region due to the increase in cross-tier interference. Furthermore, user traffic in the HWCNs is not static over time. There exists a temporal variation in the user traffic [27]. This section investigates the impact of RA on downlink of a typical HWCN, taking the dynamic nature of the user traffic into account.

5.1 Cell Intensity Allocation for an HWCN

The distribution of BSs within tiers is shown to have a considerable impact on CP of HWCNs, especially when there exist different propagation environment [9]. Also, the power constraint needs to be considered given limited resource available for the entire network. Such power constraint as well as cross-tier interference motivate the

need to optimise the cell intensity allocation to achieve maximum CP. An OP can be developed as follows:

$$\arg \max_{\{\lambda_{k,m_k}\}} \hat{p} \quad (5)$$

$$s.t. \quad \sum_{k=1}^K \sum_{m_k=1}^{M_k} \lambda_{k,m_k} \Delta_{k,m_k} \leq \Delta_{avail}, \quad (6)$$

where $k = 1, 2, \dots, K$, $m_k = 1, 2, \dots, M_k$, Δ_{avail} is the total available power within the network and Δ_{k,m_k} is the pre-defined power level for the cell in the m -th propagation model of the k -th tier. As mentioned in the second remark given in Section 4, allocating a higher cell intensity to the propagation model with the highest path loss leads to a higher CP. Without any loss of generality, it can be assumed that $\alpha_{k,m_k} \leq \alpha_{k,m_k+1}$ where $m_k = 1, 2, \dots, M_k - 1$. Consequently, an auxiliary constraint can be introduced for achieving maximum coverage:

$$0 \leq \lambda_{k,m_k} \leq \lambda_{k,m_k+1} \leq \lambda_{max}, \quad (7)$$

where λ_{max} denotes the maximum assignable cell intensity in the HWCN.

As stated in [28], the CP can be numerically determined using (4), and the OP in (5) subject to constraints (6) and (7) can be solved by using heuristic approach.

5.2 Power Allocation for an HWCN

Additionally, the per-tier power allocation has a significant impact on the coverage region of HWCNs, especially in cases where the cell intensities in different propagation models vary. The OP for achieving maximum CP subject to the per-tier power allocation can be similarly expressed as in (5). However, given limited total power (i.e. Δ_{avail}), various assignments of power levels in the tiers lead to different interference effects. During temporal periods of high user traffic, the goal is to minimise the total power consumption in the network to achieve the CP requirement. As a result, a second OP can be formulated as:

$$\arg \min_{\{\Delta_{k,m_k}\}} \sum_{k=1}^K \sum_{m_k=1}^{M_k} \lambda_{k,m_k} \Delta_{k,m_k} \quad (8)$$

$$s.t. \quad \hat{p} \geq \hat{p}_0, \quad (9)$$

$$0 < \Delta_{k,m_k} \leq \Delta_{k,m_k}^{(max)}, \quad (10)$$

where $k = 1, 2, \dots, K$, $m_k = 1, 2, \dots, M_k$. \hat{p}_0 denotes the CP threshold and $\Delta_{k,m_k}^{(max)}$ denotes the highest power that can be allocated to the BS in the m_k -th propagation model of the k -th tier.

In a similar way for cell intensity allocation, the OP in (8) given the constraints in (9) and (10) can be solved via a heuristic search.

5.3 Proposed Time-based Resource Allocation Scheme for HWCN

In this section, a heuristic algorithm is proposed to solve two OPs put forward in sections 5.1 and 5.2 considering time-varied user traffic. Let $t_s = 1, 2, \dots, T$, denote the total time spectrum upon which the UE traffic varies. Assuming there exists a point t_n on time T for which $0 \leq t_s \leq t_n$ represents the period with high UE traffic. The period of low UE traffic is thus in the range $t_n + 1 \leq t_s \leq T$. A hybrid OP can be formulated as follows:

$$\begin{cases} \arg \max_{\{\lambda_{k,m_k}\}} \hat{p}, & \text{if } 0 \leq t_s \leq t_n; \\ \arg \min_{\{\Delta_{k,m_k}\}} \sum_{k=1}^K \sum_{m_k=1}^M \lambda_{k,m_k} \Delta_{k,m_k}, & \text{otherwise.} \end{cases} \quad (11)$$

For the period $0 \leq t_s \leq t_n$, let us define ξ as a set of all possible permutations of the per-tier cell intensity in the range $[0, \lambda_{max}]$ which satisfies the constraint in (7). The elements in the set ξ can either follow a uniform distribution or a specific configuration of cells in the HWCN.

For the time interval $t_n + 1 \leq t_s \leq T$, let β denote a set of all available power levels for the cells in the range $[0, \Delta_{k,m_k}]$, $k = 1, 2, \dots, K$ and $m_k = 1, 2, \dots, M_k$.

The proposed TRA algorithm is summarised in Algorithm 1 and is further described by a flowchart shown in Fig. 2.

6 Numerical Results

In this section, we present numerical results to investigate the impact of propagation model/building types, the distribution of the propagation models, heterogeneity via tier splitting, cell intensity allocation and power allocation on the CP of downlink in an HWCN. The simulation is carried out in MATLAB for different parameters of the HWCN to validate the TRA optimisation scheme.

6.1 Impact of Propagation Models

Figure 3 plots the CP for downlink of a three-tier HWCN against a threshold SINR in the third tier (i.e. τ_3) with respect to various propagation models. The threshold SINR for the first and second tiers are set as 1 dB (i.e. $\tau_1 = \tau_2 = 1$ dB). Also, all three propagation models ($M = 3$) are assumed to exist in each tier (i.e. $\mathbb{B} = [[1, 1, 1]^T, [1, 1, 1]^T, [1, 1, 1]^T]^T$). All propagation models in the first tier are allocated a cell power of 25 W, representing the macrocell. On the other hand, the cells in three propagation models in the second and third tier have pre-set power levels of 1 W, 4 W and 8 W, respectively.

All three propagation models in the first tier are assumed to have a cell intensity of 1/100. In the second and third tiers, however, the cell intensities are set to 5/200, 25/200 and 40/200 respectively, corresponding to the three propagation models. The assignment of the cell intensities follows the constraint given in (7). Three scenarios

Algorithm 1 TRA Algorithm for Downlink in an HWCN

```

1:  $t_s \leftarrow$  all time intervals
2:  $\Delta_{avail} \leftarrow$  total available power in the network
3: if  $t_s = 1$  to  $t_n$  {During peak periods} then
4:    $(\hat{p}_{max}, j_{max}) \leftarrow (0, 0)$ 
5:   for  $k = 1$  to  $K$  do
6:     Find  $\xi_k \leftarrow$  all permutations of set  $\xi$  satisfying  $0 < \lambda_{k,m_k} \leq \lambda_{k,m_{k+1}} \leq \lambda_{max}$ 
7:   end for
8:    $\xi \leftarrow \{\xi_k\}$ 
9:   for  $j = 1$  to  $|\xi|$  do
10:     $\Delta_{sum}[j] \leftarrow \varepsilon_{k=1}^K$ 
11:    if  $\Delta_{sum}[j] \leq \Delta_{avail}$  then
12:      Find  $\hat{p}$  using (5)
13:      if  $\hat{p} \geq \hat{p}_{max}$  then
14:         $\hat{p}_{max} \leftarrow \hat{p}$ 
15:         $j_{max} \leftarrow j$ 
16:      end if
17:    end if
18:  end for
19: else
20:    $\Delta_{max} \leftarrow \infty$ 
21:    $(\Delta_{min}, i_{min}) \leftarrow (0, 0)$ 
22:   Find  $\beta_A \leftarrow$  all possible permutations of set  $\beta$  satisfying the set  $\beta$ 
23:   for  $i = 1$  to  $|\beta_A|$  do
24:     Find  $\hat{p}$  using (5)
25:     if  $\hat{p} \geq \hat{p}_{max}$  then
26:        $\Delta_{sum} \leftarrow \sum_{k=1}^K \sum_{m_k=1}^M [\Delta_{k,m_k}]_i \lambda_{k,m_k}$ 
27:       if  $\Delta_{sum} \leq \Delta_{max}$  then
28:          $\Delta_{min} \leftarrow \Delta_{sum}$ 
29:          $i_{min} \leftarrow i$ 
30:          $\Delta_{max} \leftarrow \Delta_{min}$ 
31:          $\hat{p}_{opt} \leftarrow \hat{p}$ 
32:       end if
33:     end if
34:   end for
35: end if

```

Outputs 1:
Optimal cell allocation: $\{[\lambda_{k,m_k}]_{j_{max}}\}$
Maximum CP: \hat{p}_{max}
Total Power Consumption: $\Delta_{sum}[j_{max}]$

Outputs 2:
Optimal power allocation: $\{[\Delta_{k,m_k}]_{i_{min}}\}$
Total power consumption: Δ_{min}
CP: \hat{p}_{opt}

of propagation models are considered, having different sets of path loss exponents as follows: [2.1, 2.5, 2.9], [2.3, 2.7, 3.1] and [2.5, 2.9, 3.3].

As stated in the third remark in Section 4, the effect of noise on the system is minimal and is thus ignored, leaving only an interference-influenced HWCN. In addition, as shown in Fig. 3, a more lossy propagation model achieves a higher CP, which is in agreement with the first remark stated in Section 4.

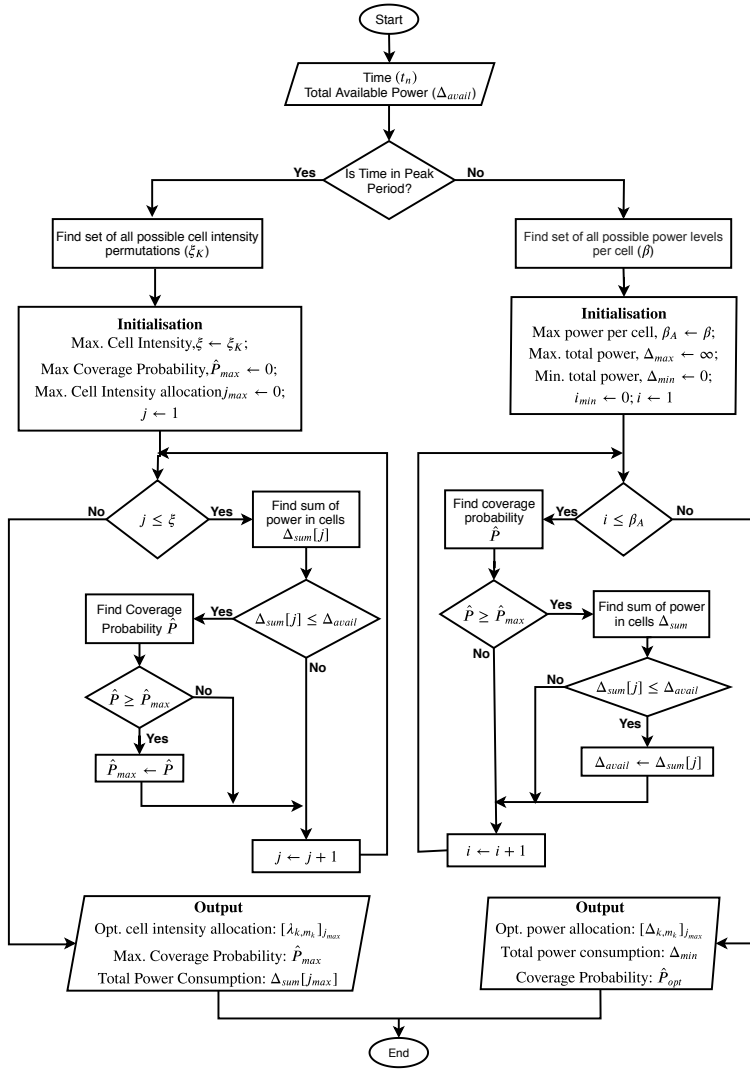


Fig. 2 Flow Chart of Hybrid Optimisation Scheme

6.2 Impact of Propagation Model Distribution

In Fig. 4, the achieved CP is plotted against threshold SINR in the third tier (i.e. τ_3) considering the impact of propagation model distribution. Similar to Fig. 3, the threshold SINR for the first and second tier are set as 1 dB. A total of six different propagation model distributions are considered, all having the path loss exponent of [2.1, 2.5, 2.9] in each tier with respect to the three types of propagation models. For each distribution, the power of the cells in the first tier is fixed as 25 W, while those in the second and third tiers are allocated 1 W, 4 W and 8 W with respect to the

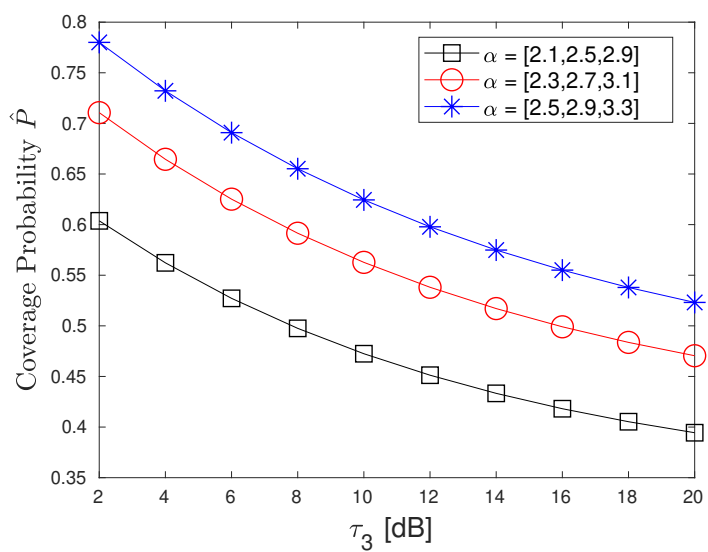


Fig. 3 CP vs τ_3 considering different propagation models

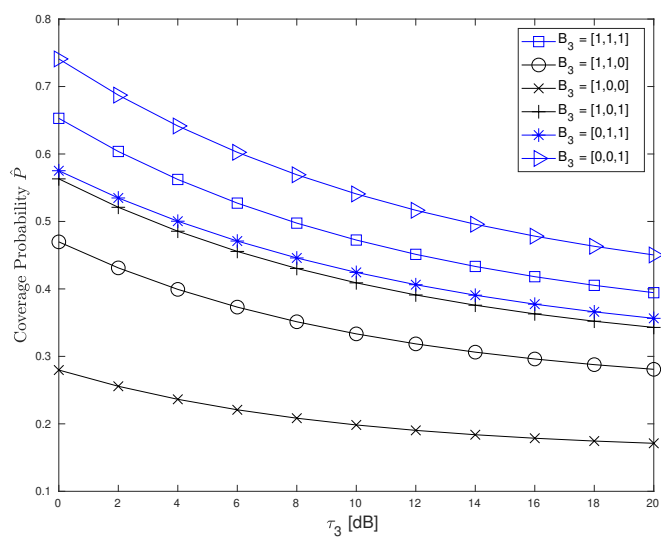


Fig. 4 CP vs τ_3 considering different propagation model distributions

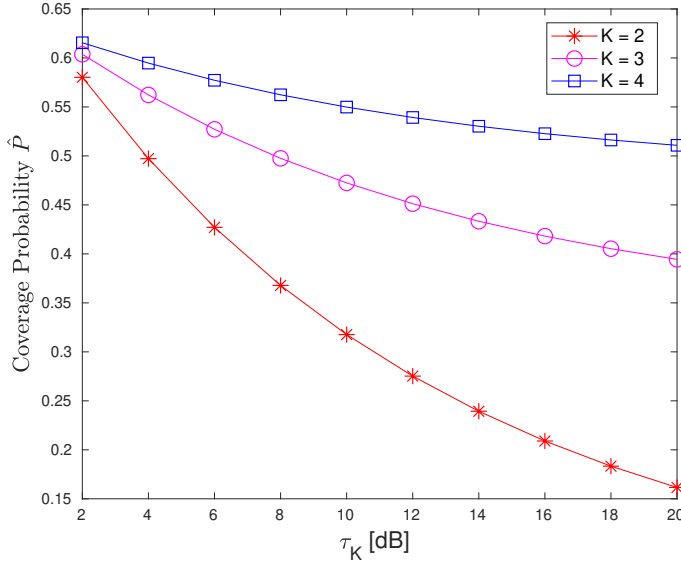


Fig. 5 CP vs τ_3 considering tier multiples via splitting

corresponding propagation models. The propagation matrix can be thus expressed as $\mathbb{B} = [[1, 1, 1]^T, \mathbb{B}_2^T, \mathbb{B}_3^T]$, where $\mathbb{B}_2 = \mathbb{B}_3$ and they are varied as shown in Fig. 4.

Similarly, for each scenario of propagation model distribution, the cell intensity is set as 1/100 for all three propagation models; whereas, in the second and third tiers, the cell intensities are [5/200, 25/200, 40/200], [50/400, 90/400, 0], [70/200, 0, 0], [60/400, 0, 80/400], [0, 50/400, 90/400] and [0, 0, 70/200], corresponding to the variations in \mathbb{B}_2 and \mathbb{B}_3 .

The analysis of the results in Fig. 4 leads to the following observations:

1. The propagation model $\mathbb{B}_3 = [1, 0, 0]$ having a higher cell intensity allocated to the propagation environment with the least path loss exponent yields the lowest CP, whereas the propagation model $\mathbb{B}_3 = [0, 0, 1]$ in which a higher cell intensity is allocated to the more lossy propagation model yields the highest CP.
2. Allocation of cell intensities in increasing order of path loss exponent in the propagation model yields a better performance as seen in $\mathbb{B}_3 = [1, 1, 1]$, $\mathbb{B}_3 = [0, 1, 1]$ and $\mathbb{B}_3 = [0, 0, 1]$. This verifies the second remark given in Section 3.2.

6.3 Impact of Tier Heterogeneity

To investigate the effect of tier heterogeneity on the coverage area of HWCNs via tier splitting, Fig. 5 plots CP against the threshold SINR in the last tier (i.e. τ_K), considering several multiples of tiers (i.e. K). The threshold SINR in the K -th tier is varied in each case while keeping the threshold SINR of the remaining tiers fixed at $\tau_k = 1$ dB, $k = 1, 2, \dots, K-1$.

Three scenarios of $K = \{2, 3, 4\}$ are considered, each of which experience three propagation models with path loss exponent of [2.1, 2.5, 2.9]. The cells in the first tier have a power and cell intensity allocation of 25 W and [1/100, 1/100, 1/100], respectively. For $K = 2$, the cells in the second tier are allocated power of [1 W, 4 W, 8 W] with intensity of [5/100, 25/100, 40/100] corresponding to the three propagation models. For simplicity, the third tier in the scenario of $K = 3$ is derived by splitting the second tier of the first scenario (i.e. $K = 2$) into two halves (i.e. $\lambda_2^{(K_3)} = \lambda_3^{(K_3)} = [5/200, 25/200, 40/200]$). Similarly, the third scenario of $K = 4$ is a derivative of the second scenario of $K = 3$ by keeping the first two tiers the same and splitting the third tier into two halves (i.e. $\lambda_3^{(K_4)} = \lambda_4^{(K_4)} = [5/400, 25/400, 40/400]$).

It can be observed in Fig. 5 that an increase in the number of tiers leads to an overall improvement in the network performance. For example, when a threshold SINR of 12 dB is required in the lowest tier, a CP of 54% is achieved in a four-tier HWCN, whereas a two-tier HWCN provides a CP of only 27%. This result verifies the benefit derived from using small cells in the HWCNs.

6.4 Impact of Cell Intensity Allocation

Investigating the impact of cell intensity allocation on the CP of a three-tier HWCN, Fig. 6 plots the achieved CP against threshold SINR in the third tier (τ_3), considering various cell intensity allocations. The threshold SINR in the first two tiers are set as 1 dB. For the investigation, five different cell intensity distributions are considered as shown in Fig. 6. Similar to the simulation parameters used in Section 6.1, all three propagation models (i.e. $M = 3$) are assumed to exist in each tier (i.e. $\mathbb{B} = [[1, 1, 1]^T, [1, 1, 1]^T, [1, 1, 1]^T]^T$). The per-tier cell power of all propagation models in the first tier is set as 25 W. The second and third tiers are pre-set with power levels of 1 W, 4 W and 8 W, respectively, corresponding to the three propagation models for all cell intensity distributions.

The results in Fig. 6 show that a higher CP is achieved for a cell configuration having higher cell intensity allocated to the propagation model with the higher path loss. For instance, when a threshold SINR of 10 dB is required in the third tier, the cell configuration with higher cell intensity allocated to the propagation model with the lowest path loss achieves a CP of 30%, while the cell configuration which allocates higher cell intensity to the propagation model with the highest path loss achieves the highest CP of 47%. This again verifies the second remark observed in Section 4.

6.5 Impact of Per-Tier Power Allocation

The impact of per-tier power allocation on CP is illustrated in Fig. 7 where the CP is plotted versus threshold SINR in the third tier considering different per-tier power allocations and simulation parameters as outlined in Table 1.

For a threshold SINR of 10 dB in the third tier (i.e. $\tau_3 = 10$ dB) with a fixed cell intensity of [5/200, 25/200, 40/200] and a propagation model with path loss exponent of [2.1, 2.5, 2.9], allocating a higher power in the propagation model with the lowest

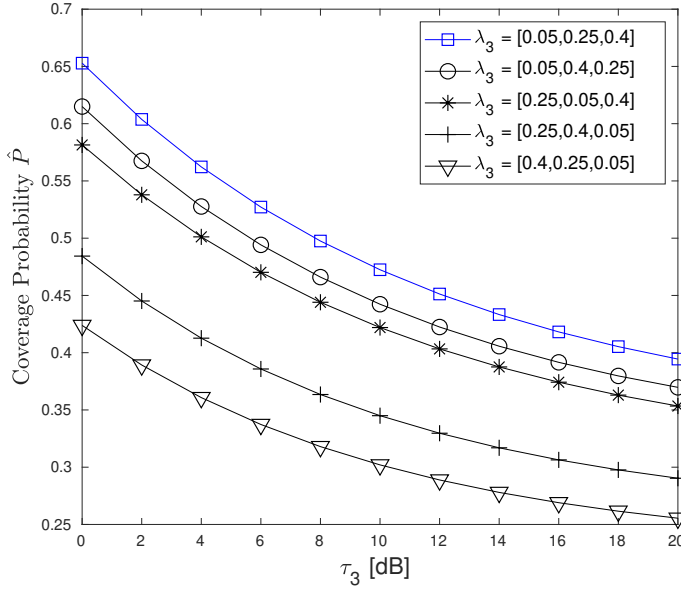


Fig. 6 CP vs τ_3 considering different cell intensities

Table 1 Simulation parameters for impact of power allocation

Case	First Tier		Second & Third Tier	
	Δ_1	λ_1	$\Delta_{2,m}$	$\Delta_{2,m}$
$\Delta_{k,m_k}^{(1)}$	25	1/100	[1, 4, 8]	[5/200, 25/200, 40/200]
$\Delta_{k,m_k}^{(2)}$	25	1/100	[1, 8, 4]	[5/200, 40/200, 25/200]
$\Delta_{k,m_k}^{(3)}$	25	1/100	[4, 1, 8]	[25/200, 5/200, 40/200]
$\Delta_{k,m_k}^{(4)}$	25	1/100	[4, 8, 1]	[25/200, 40/200, 5/200]
$\Delta_{k,m_k}^{(5)}$	25	1/100	[8, 1, 4]	[40/200, 5/200, 25/200]
$\Delta_{k,m_k}^{(6)}$	25	1/100	[8, 4, 1]	[40/200, 25/200, 5/200]

path loss (i.e. $\Delta_{3,m_3} = [8, 4, 1]$) yields a CP of 32%. When higher power is assigned to the second propagation model, (i.e. $\Delta_{3,m_3} = [4, 8, 1]$), the CP increases to 34%. Finally, when higher power is allocated to the propagation model with the highest path loss (i.e. $\Delta_{3,m_3} = [1, 4, 8]$), a CP of 43% is achieved. The result shows that the assignment of power in the various cells has a significant impact on the CP due to the differences in path loss and cell intensity in the various tiers.

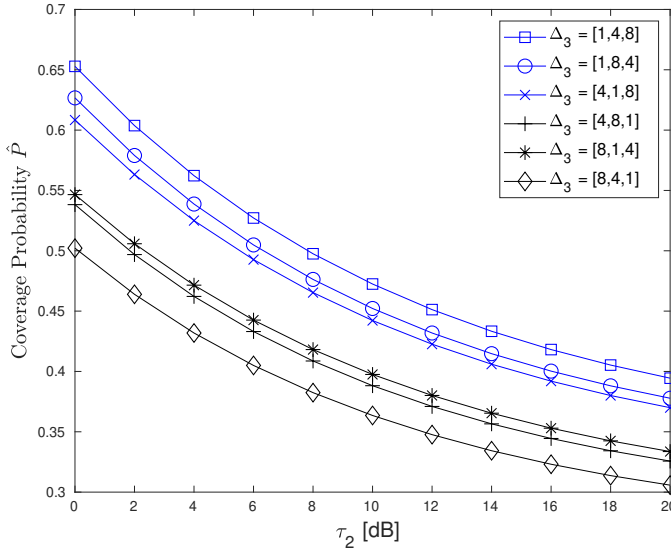


Fig. 7 CP vs τ_3 considering different power allocations

6.6 Resource Allocation Optimisation

This section presents the results and analysis of the proposed TRA algorithm. Cell intensity optimisation and power consumption minimisation are first analysed in Sections 6.6.1 and 6.6.2, respectively. The analysis of the hybrid optimisation scheme with the TRA is then presented in Section 6.6.3.

6.6.1 Simulation Results for Cell Allocation Optimisation Scheme

Considering the impacts of cell intensity allocation as shown in Fig. 6 and taking the power constraint into account, Figs. 8 and 9 show the relationship between the maximum CP (i.e. \hat{p}), total available power per unit area (i.e. Δ_{avail}) and the sum of power required by the tiers per unit area (i.e. Δ_{req}) in an HWCN. The simulation compares two schemes: the heuristic cell optimisation scheme and the equal cell intensity allocation scheme. The threshold SINR and per-tier power for the first and second tier are set as $\{1 \text{ dB}, 30 \text{ W}\}$ and $\{10 \text{ dB}, 1 \text{ W}\}$, respectively. The first tier is allocated with a cell intensity of $[1/100, 1/100, 1/100]$, while the second tier (i.e. λ_2) is set in the range $[0.1, 1]$ in a bid to determine the optimal cell allocation which achieves a maximum CP.

Figures 8 and 9 indicate that at specific values of the total available power constraint, a higher CP is achieved while also utilising a lower total required power in the cells. In particular, from Fig. 8, given a power constraint of 3.2 W m^{-2} , a maximum CP of 42% can be achieved using the proposed optimisation scheme, whereas the fixed cell allocation scheme only achieves a CP of only 35%. Moreover, Figure 9

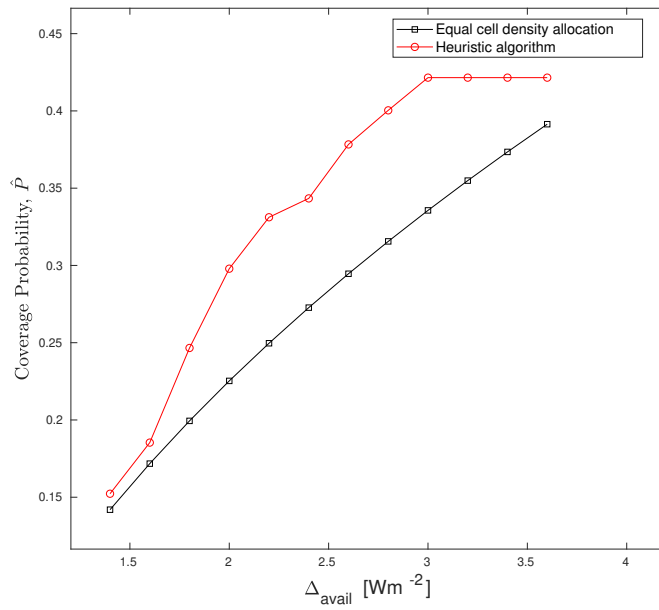


Fig. 8 Maximum CP vs Total Available Power per area

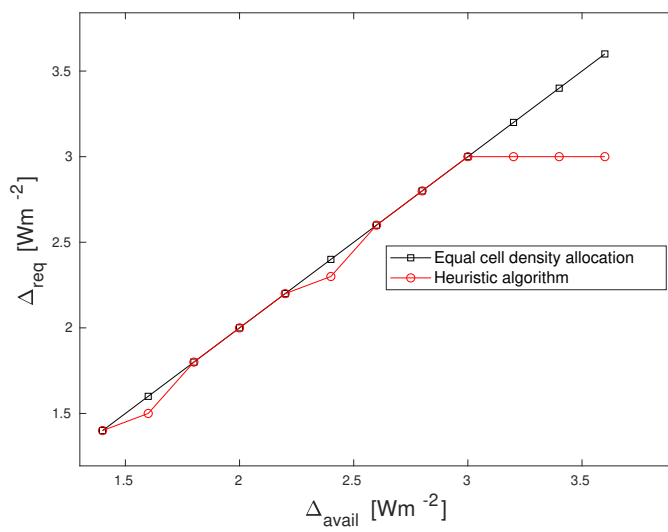


Fig. 9 Total required power in cells per area vs total available power per area

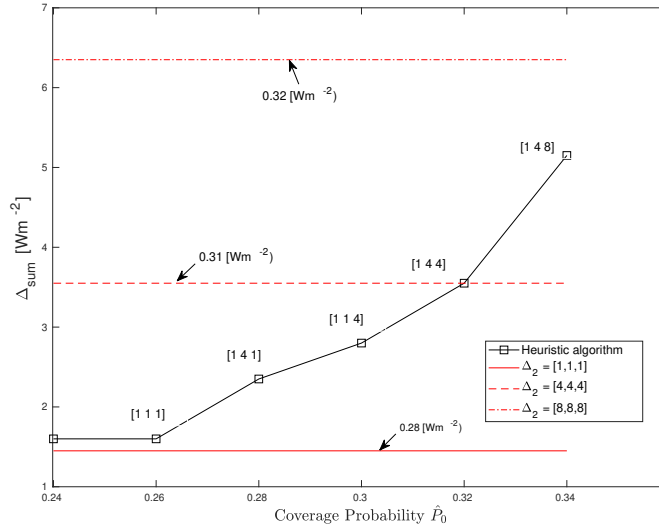


Fig. 10 Total Per-Tier Required Power vs Target CP in a Two-Tier HWCN

shows that the proposed optimisation scheme only requires a power of $3Wm^{-2}$ to achieve the maximum CP, while the equal cell allocation scheme could reach as high as $3.6Wm^{-2}$. The result obtained accordingly shows that the proposed optimisation scheme can achieve up to a 20% increase in CP with a lower energy when compared with the equal cell allocation scheme.

6.6.2 Simulation Results for Power Allocation Optimisation Scheme

Following the analysis of the impact of power allocation on CP in Section 5.2, it is observed that the CP of a typical HWCN is affected by the allocated per-tier power. One of the objectives of this paper is the minimisation of the total power to achieve a threshold CP during periods of minimal user traffic. In this section, the proposed heuristic power allocation algorithm is compared with three equal power allocation schemes. For all the schemes, the cells in the first tier are assigned a power of 25 W with an intensity of $\lambda_1 = [1/100, 1/100, 1/100]$. For the second tier, the three equal power allocation schemes are allocated a cell intensity of $\lambda_2 = [5/100, 25/100, 40/100]$ and a power allocation of $\Delta_2^{(equal_1)} = [1, 1, 1]$, $\Delta_2^{(equal_2)} = [4, 4, 4]$ and $\Delta_2^{(equal_3)} = [8, 8, 8]$, whereas for the proposed optimisation scheme, the power of the cells in the second tier is chosen from a permutation of the set $[1, 4, 8]$ to obtain the optimal power settings to exceed a threshold CP.

Figure 10 shows that when an appropriate power allocation scheme is applied, the required threshold CP can be achieved while saving energy. For example, to achieve a threshold CP (i.e. \hat{p}) of 0.32 and 0.34, the equal power allocation scheme $\Delta_{k,m_k}^{(equal_2)} = [25; 4, 4, 4]$ and $\Delta_{k,m_k}^{(equal_3)} = [25; 8, 8, 8]$ require a power allocation of $3.55Wm^{-2}$ and

$6.35 W m^{-2}$ respectively, whereas using the proposed heuristic allocation scheme which assigns a power level of $\Delta_{k,m_k}^{(opt)} = [25; 1, 4, 1]$ and $\Delta_{k,m_k}^{(opt)} = [25; 1, 1, 4]$ only require a total power per area of $2.35 W m^{-2}$ and $2.8 W m^{-2}$, respectively. The results obtained show that over 33% of the total power consumption can be saved using the proposed heuristic algorithm as compared to an equal power allocation scheme. Minimising power consumption not only saves cost for the network operators but is also environmentally friendly.

6.6.3 Proposed Hybrid Resource Optimisation Scheme

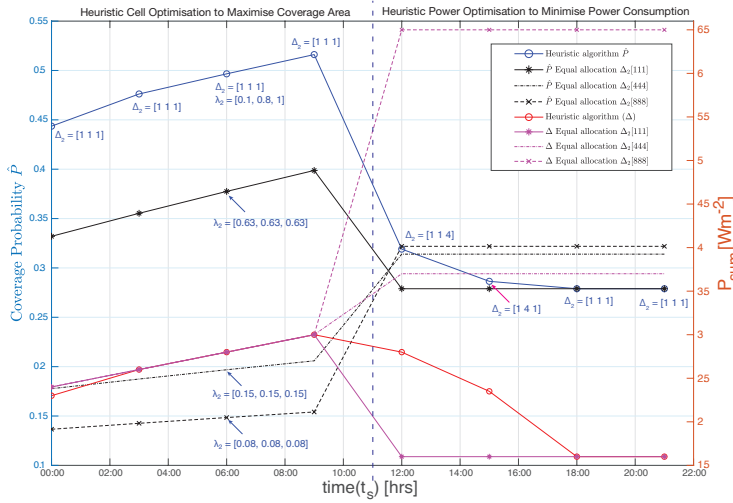


Fig. 11 CP and Power Consumption vs Time considering different RAs

This section presents the numerical results and analysis to justify the effectiveness of the proposed TRA optimisation scheme. Fig. 11 presents the result of the comparison between the TRA scheme and equal RA scheme in a two-tier HWCN. Considering time-based variations in user traffic, the simulation divides a 24-hour timescale into eight different time intervals, each of which has a different resource constraint as shown in Table 2. The goal in the first four timeslots is to maximise CP given a constraint in the total power, which simulates the peak period of user traffic. On the other hand, the goal in the last four timeslots is to optimise the allocation of per-tier power levels to minimise the total power consumption, which represents the off-peak period.

The threshold SINRs for the first and second-tier are set to be 1 dB and 10 dB, respectively, with a cell intensity of $[1/100, 1/100, 1/100]$ for all the schemes under comparison. The per-tier power allocation for the equal RA scheme is set as 30 W in

Table 2 Time vs Resource Constraint for two-tier HWCN

Time	Resource Constraint
$t_1 = 0h$	$\Delta = 2.4Wm^{-2}$
$t_2 = 3h$	$\Delta = 2.6Wm^{-2}$
$t_3 = 6h$	$\Delta = 2.8Wm^{-2}$
$t_4 = 9h$	$\Delta = 3.0Wm^{-2}$
$t_5 = 12h$	$\hat{p} = 30\%$
$t_6 = 15h$	$\hat{p} = 28\%$
$t_7 = 18h$	$\hat{p} = 26\%$
$t_8 = 21h$	$\hat{p} = 24\%$

Table 3 Simulation Parameters for Cell Intensity Optimisation

Parameter	First Tier	Second Tier
Pathloss, α_{k,m_k}	[2.1, 2.5, 3.1]	[2.1, 2.5, 3.1]
Threshold SINR, τ_{k,m_k}	1 dB	10 dB
Power in tiers, $\Delta_{k,m_k}(W)$	30	[1, 1, 1]
Cell intensity, λ_{k,m_k}	1/100	$[\lambda_{opt_1}, \lambda_{opt_2}, \lambda_{opt_3}]$

the first tier and the second tier is allocated [1, 1, 1], [4, 4, 4] and [8, 8, 8], respectively, corresponding to the three equal RA schemes with three propagation models.

For the first OP of maximising CP, the cell intensities for the second tier of the equal RA schemes are calculated based on the total available power, the product of the cell intensity and power in the first tier as well as the given power in the second tier. The simulation parameters used for the cell optimisation scheme are outlined in Table 3.

For the second OP of minimising total power consumption, the cell intensities for the second tier of the equal RA schemes are set as [5/100, 25/100, 40/100] for all schemes corresponding to the three propagation models. The simulation parameters for the power optimisation scheme are outlined in Table 4.

From Fig. 11 and the constraint given in Table 2, during the time slot $t_3 = 6h$, the intended goal is to achieve the highest CP given a total available power constraint of $\Delta = 2.8Wm^{-2}$. The equal RA scheme with $\Delta_2^{(equal_3)} = [8, 8, 8]$ can only allocate a cell intensity of $\lambda_2^{(equal_3)} = [8/100, 8/100, 8/100]$ due to the power constraint and also achieved a low CP of 14.84%. The equal RA scheme with $\Delta_2^{(equal_2)} = [4, 4, 4]$ is able to allocate more cells ($\lambda_2^{(equal_2)} = [63/100, 63/100, 63/100]$) following on the total available power constraint. A higher CP of 37.7% is achieved using this scheme. However, the proposed scheme achieved the highest CP of 47.6%.

Table 4 Simulation Parameters for Power Allocation Optimisation

Parameter	First Tier	Second Tier	
Pathloss, α_{k,m_k}	[2.1, 2.5, 3.1]	[2.1, 2.5, 3.1]	
Threshold SINR, τ_{k,m_k}	1 dB	10 dB	
Cell intensity, λ_{k,m_k}	1/100	$[\lambda_{opt_1}, \lambda_{opt_2}, \lambda_{opt_3}]$	
Power in tiers, $\Delta_{k,m_k} (W)$	$\Delta_{k,m_k}^{(opt)}$	30	$[\Delta_{opt_1}, \Delta_{opt_2}, \Delta_{opt_3}]$
	$\Delta_{k,m_k}^{(opt)}$	30	[1, 1, 1]
	$\Delta_{k,m_k}^{(opt)}$	30	[4, 4, 4]
	$\Delta_{k,m_k}^{(opt)}$	30	[8, 8, 8]

Also, for the time slot $t_6 = 15h$, the goal is to minimise total power consumption while achieving a threshold CP $\hat{p} = 28\%$. The first equal RA scheme $\Delta_2^{(equal_1)} = [1, 1, 1]$ has the lowest power consumption but could not achieve the required threshold CP. The second and third equal RA schemes can exceed the target CP but they require a total power of $3.7Wm^{-2}$ and $6.5Wm^{-2}$, respectively. Finally, using the proposed optimisation scheme, the threshold CP is achieved and it requires only a total power of $2.35Wm^{-2}$ by allocating a power allocation of $\Delta_2^{(opt)} = [1, 4, 1]$ in the second tier.

7 Conclusion

Heterogeneous Wireless Cellular Network (HWCN) is a research area of both current and future wireless cellular networks because of the various advantages it offers. A lot of research have been carried out to tackle the various issues identified in HWCNs such as management of the inter-cell interference, load balancing across tiers, spectral efficiency, EE, and maximising CP. This research focuses more on minimising power consumption and maximising coverage area based on the identified variations in user traffic across time.

Energy consumption and coverage area are two major areas which are beneficial to a wide range of cellular network stakeholders. Minimising the power consumption not only saves cost for the network operators, but also benefit the environment, especially with the growing concerns regarding global warming. On the other hand, improving CP is beneficial for mobile users, which in turn improves business for the network operators.

In this research, the impact of various factors, such as the propagation model, cell intensity as well as power allocation, on the CP of a typical HWCN have been analysed. Heuristic algorithms have been proposed to solve the two identified OPs of maximising CP during periods of peak user traffic and minimising power consumption during off-peak periods. The simulation results have shown that the proposed scheme achieves a higher coverage of up to 25% during the peak periods and a power saving of up to 57% during the off-peak period compared to the scheme in which

the resources are equally allocated. The future work would be the investigation of employing the proposed TRA for the downlink in practical HWCNs.

References

1. ElSawy, H., Hossain, E., Haenggi, M.: Stochastic geometry for modeling, analysis, and design of multi-tier and cognitive cellular wireless networks: A survey. *IEEE Communications Surveys & Tutorials* **15**(3), 996–1019 (2013)
2. Peng, M., Wang, C., Li, J., Xiang, H., Lau, V.: Recent advances in underlay heterogeneous networks: Interference control, resource allocation, and self-organization. *IEEE Communications Surveys & Tutorials* **17**(2), 700–729 (2015)
3. Hu, R.Q., Qian, Y.: An energy efficient and spectrum efficient wireless heterogeneous network framework for 5g systems. *IEEE Communications Magazine* **52**(5), 94–101 (2014)
4. Swami, P., Bhatia, V., Vuppala, S., Ratnarajah, T.: User fairness in NOMA-HetNet using optimized power allocation and time slotting. *IEEE Systems Journal* **15**(1), 1005–1014 (2021)
5. Xu, C., Zheng, G., Zhao, X.: Energy-minimization task offloading and resource allocation for mobile edge computing in NOMA heterogeneous networks. *IEEE Transactions on Vehicular Technology* **69**(12), 16001–16016 (2020)
6. Ali, Z.J., Noordin, N.K., Sali, A., Hashim, F.: Fair energy-efficient resource allocation for downlink NOMA heterogeneous networks. *IEEE Access* **8**, 200129–200145 (2020)
7. Gopalam, S., Hanly, S.V., Whiting, P.: Distributed user association and resource allocation algorithms for three tier HetNets. *IEEE Transactions on Wireless Communications* **19**(12), 7913–7926 (2020)
8. Lai, J.Y., Wu, W.H., Su, Y.T.: Resource allocation and node placement in multi-hop heterogeneous integrated-access-and-backhaul networks. *IEEE Access* **8**, 122937–122958 (2020)
9. Vien, Q.T., Akinbote, T., Nguyen, H.X., Trestian, R., Gemikonakli, O.: On the coverage and power allocation for downlink in heterogeneous wireless cellular networks. In: 2015 IEEE international conference on Communications (ICC), pp. 4641–4646. IEEE (2015)
10. Phan, C.V., Vien, Q.T.: Optimising coverage efficiency in heterogeneous wireless cellular networks. *IET communications* **14**(17), 3022–3029 (2020)
11. Ghosh, A., Mangalvedhe, N., Ratasuk, R., Mondal, B., Cudak, M., Visotsky, E., Thomas, T.A., Andrews, J.G., Xia, P., Jo, H.S., et al.: Heterogeneous cellular networks: From theory to practice. *IEEE communications magazine* **50**(6), 54–64 (2012)
12. Xie, B., Zhang, Z., Hu, R.Q., Qian, Y.: Spectral efficiency analysis in wireless heterogeneous networks. In: 2016 IEEE International Conference on Communications (ICC), pp. 1–6. IEEE (2016)
13. Coskun, C.C., Davaslioglu, K., Ayanoglu, E.: Three-stage resource allocation algorithm for energy-efficient heterogeneous networks. *IEEE Transactions on vehicular technology* **66**(8), 6942–6957 (2017)
14. Xu, X., Kutrolli, G., Mathar, R.: Energy efficient power management for 4g heterogeneous cellular networks. In: 2013 IEEE 9th International Conference on Wireless and Mobile Computing, Networking and Communications (WiMob), pp. 231–238. IEEE (2013)
15. Dhillon, H.S., Ganti, R.K., Baccelli, F., Andrews, J.G.: Modeling and analysis of k-tier downlink heterogeneous cellular networks. *IEEE Journal on Selected Areas in Communications* **30**(3), 550–560 (2012)
16. Xu, Y., Hu, Y.: Robust energy-efficient downlink resource allocation in heterogeneous networks with outage probability constraint. *Wireless Personal Communications* **104**(1), 441–458 (2019)
17. Chavarria-Reyes, E., Akyildiz, I.F., Fadel, E.: Energy consumption analysis and minimization in multi-layer heterogeneous wireless systems. *IEEE Transactions on Mobile Computing* **14**(12), 2474–2487 (2015)
18. Soh, Y.S., Quek, T.Q., Kountouris, M., Shin, H.: Energy efficient heterogeneous cellular networks. *IEEE Journal on selected areas in communications* **31**(5), 840–850 (2013)
19. Cai, R., Zhang, W., Ching, P.C.: Cost-efficient optimization of base station densities for multitier heterogeneous cellular networks. *IEEE Transactions on Wireless Communications* **15**(3), 2381–2393 (2015)
20. Ren, Q., Fan, J., Luo, X., Xu, Z., Chen, Y.: Energy efficient base station deployment scheme in heterogeneous cellular network. In: 2015 IEEE 81st Vehicular Technology Conference (VTC Spring), pp. 1–5. IEEE (2015)

21. Liu, J.S., Lin, C.H., Huang, H.C.: Joint congestion control and resource allocation for energy-efficient transmission in 5g heterogeneous networks. *EURASIP Journal on Wireless Communications and Networking* **2019**(1), 1–16 (2019)
22. Zhuang, B., Guo, D., Honig, M.L.: Energy-efficient cell activation, user association, and spectrum allocation in heterogeneous networks. *IEEE Journal on Selected Areas in Communications* **34**(4), 823–831 (2016)
23. Ma, C., Ding, M., López-Pérez, D., Lin, Z., Li, J., Mao, G.: Performance analysis of the idle mode capability in a dense heterogeneous cellular network. *IEEE Transactions on Communications* **66**(9), 3959–3973 (2018)
24. Farrokhi, A., Ercetin, O.: Qoe based random sleep-awake scheduling in heterogeneous cellular networks. In: 2016 IEEE Wireless Communications and Networking Conference, pp. 1–6. IEEE (2016)
25. Fooladivanda, D., Rosenberg, C.: Joint user association and resource allocation in heterogeneous cellular networks: Comparison of two modeling approaches. In: 2019 31st International Teletraffic Congress (ITC 31), pp. 66–74. IEEE (2019)
26. Vora, A., Kang, K.D.: Effective 5g wireless downlink scheduling and resource allocation in cyber-physical systems. *Technologies* **6**(4), 105 (2018)
27. Wang, X., Turgut, E., Gursoy, M.C.: Coverage in downlink heterogeneous mmwave cellular networks with user-centric small cell deployment. *IEEE Transactions on Vehicular Technology* **68**(4), 3513–3533 (2019)
28. Vien, Q.T., Le, T.A., Nguyen, H.X., Karamanoglu, M.: An energy-efficient resource allocation for optimal downlink coverage in heterogeneous wireless cellular networks. In: 2015 International Symposium on Wireless Communication Systems (ISWCS), pp. 156–160. IEEE (2015)