# Cognitive Beamforming Design for Dual-Function Radar-Communications

Tuan Anh Le[1], Ivan Ku[2], Xin-She Yang[1], Christos Masouros[3], and Tho Le-Ngoc[4]
[1]Faculty of Science and Technology, Middlesex University, London, NW4 4BT, UK
[2]Faculty of Engineering, Multimedia University, Persiaran Multimedia, 63100 Cyberjaya, Selangor, Malaysia
[3]Department of Electronic and Electrical Engineering, University College London, London, WC1E 7JE, UK
[4]Department of Electrical and Computer Engineering, McGill University, Montreal, Quebec, H3A 0G4, Canada
Email: {t.le; x.yang}@mdx.ac.uk, kccivan@mmu.edu.my, c.masouros@ucl.ac.uk, tho.le-ngoc@mcgill.ca

***Abstract*—This paper introduces a dual-function radar-communication (DFRC) system with cognitive radio capability to tackle the spectral scarcity problem in wireless communications. Particularly, a cognitive DFRC system operates on a spectrum owned by a primary system to simultaneously perform data communication and target tracking while maintaining its interference to the primary users (PUs) below a certain threshold. To achieve this, an optimization problem is formulated to jointly design the beamforming vectors for both the radar and communication functions in minimizing the mean square error (MSE) of the beam patterns between the designed and desired waveforms under three constraints: i) the signal-to-interference-plus-noise ratio (SINR) at each data communication user; ii) the per-antenna transmit power; and iii) the interference imposed on each PU. The semidefinite relaxation technique is utilized to search for the optimal solution to the optimization problem. The simulation results indicate that our proposed cognitive DFRC approach can effectively protect the PUs while simultaneously perform its communication and radar functions.**

***Index Terms*—Dual-function radar-communications, cognitive radio.**

## I. Introduction

Dual-function radar-communications (DFRC) has recently attracted increasing interests as a means of sharing spectrum, hardware, and signalling between radar and communication systems. DFRC has been recognized as a promising technology in beyond 5G and 6G networks [1]–[3]. In order to balance the radar's and communication's performances in the DFRC system, the radar and communication functions are dedicated separate waveforms that are jointly designed [2] by means of the beamforming design approach. In this approach, the metric for the radar function is normally the mean square error (MSE) between the beam patterns of the designed and desired waveforms [2], [4]. Alternatively, it can be the minimum weighted beam pattern gain in the desired directions of the targets [5] or the cross correlation beam pattern [2]. On the other hand, the metrics for the communication function are commonly signal-to-interference-plus-noise ratio (SINR) [2], [5], [6] or the achievable secrecy rate [4]. The beamforming design problems for DFRC are then cast as optimization problems. For example, the weighted sum of the MSE and cross correlation beam patterns is minimized subject to the communication users' SINR and transmit power constraints as proposed in [2], while, in [5], the MSE beam pattern is minimized under the constraints on the communication users' SINR and the transmit power. In [6], the objective is to maximize the worst SINR among the users subject to the power constraint and the covariance of the transmit waveform being equal to a given optimal covariance of the multiple-input multiple-output (MIMO) radar.

Cognitive radio was introduced and developed for effective spectrum sharing and efficient spectrum utilization in wireless communications systems, see e.g., [7]–[9] and references therein. The combination of DFRC with cognitive radio offers additional improvement to the spectrum utilization. To that end, cognitive radio was firstly adopted in DFRC in [10] where opportunistic spectrum sharing between a primary rotating radar system and a secondary communication system was allowed. In fact, this approach can be considered as a radar-centric method where the performance of the communication function can be potentially compromised. Hence, to improve the performance of the communication function while maintaining the performance of the radar function in a cognitive DFRC system, the beamforming design approach is desirable. To the best of the authors' knowledge, such approach does not exist in the literature.

Motivated by the above-mentioned fact, this paper proposes a jointly design beamforming approach for a cognitive DFRC system. An optimization problem for a cognitive DFRC system is introduced whereby the MSE between the beam patterns of the designed and desired waveforms is minimized subject to i) the SINR level of each communication user is above a required level; ii) the per-antenna transmit power is at a fixed level; and iii) the interference level imposed on each primary user is below a predefined threshold. Adopting the semidefinite relaxation (SDR) technique, the non-convex optimization is then transformed into a convex form which then can be solved by interior point methods. Simulations are finally carried out to evaluate the performance of the proposed cognitive DFRC approach.

***Notation:*** Lower or upper case letter $a$ or $A$: a scalar; bold lower case letter $\mathbf{a}$: a column vector; bold upper case letter $\mathbf{A}$: a matrix; $(\cdot)^T$: the transpose operator; $(\cdot)^\star$: the complex conjugate operator; $(\cdot)^H$: the complex conjugate transpose operator; $\mathbb{E}[\cdot]$: the expected value operator; $\mathrm{Tr}(\cdot)$: the trace operator; $\mathbf{A} \succeq \mathbf{0}$: $\mathbf{A}$ is a positive semidefinite matrix; $\mathbb{H}^{M\times M}$: the set of $M\times M$ Hermitian matrices; $\mathbb{C}^{M\times 1}$: the set of $M\times 1$ complex element vectors; $a \sim \mathcal{CN}(0,\sigma^2)$: $a$ is a zero mean circularly symmetric complex Gaussian random variable with variance $\sigma^2$.

## II. System Model

Consider a DFRC system consisting of an integrated MIMO base station (BS) and MIMO radar subsystem for data communication and target tracking, respectively. An $M$-antenna array is shared between the MIMO BS and MIMO radar subsystem. The DFRC system does not own any radio spectrum but operates cognitively on the spectrum owned by the primary system. Under the primary system permission, the DFRC system communicates with $U$ single-antenna secondary users (SUs) while simultaneously tracks $K$ targets, as long as its interference levels imposed on $L$ primary users (PUs) are kept at below predefined thresholds.

Let $\mathbf{w}_i \in \mathbb{C}^{M\times 1}$ and $x_i^c$ be the transmit beamforming vector and the intended data symbol, respectively, for the communication function of the $i$-th SU, where $\mathbb{E}\left[|x_i^c|^2\right] = 1$ and $i \in \{1,\cdots,U\}$. While, $\mathbf{v}_t \in \mathbb{C}^{M\times 1}$ and $x_t^r$ represent the radar beamforming vector and radar waveform, respectively, for the radar function of tracking targets, where $\mathbb{E}\left[|x_t^r|^2\right] = 1$ and $t \in \{1,\cdots,M\}$, that is, we have $M$ radar beams tracking $K$ targets. It is also noted that $U + K + L \leq M$ so that there is enough spatial degree of freedom to accommodate the beamforming requirements of all the PUs, SUs and targets that are under consideration. Furthermore, let $\mathbf{W} = \left[\mathbf{w}_1, \mathbf{w}_2, \cdots, \mathbf{w}_U\right]$, $\mathbf{x}^c = \left[x_1^c, x_2^c, \cdots, x_U^c\right]^T$, $\mathbf{V} = \left[\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_M\right]$ and $\mathbf{x}^r = \left[x_1^r, x_2^r, \cdots, x_M^r\right]^T$. As a result, the $M\times 1$ signal output vector of the $M$-antenna array can be written as:

$$\mathbf{x} = \mathbf{W}\mathbf{x}^c + \mathbf{V}\mathbf{x}^r. \tag{1}$$

Finally, the transmit power, $p_i$, allocated to the $i$-th SU is given by:

$$p_i = \left[\sum_{i=1}^{U} \mathbf{w}_i\mathbf{w}_i^H + \sum_{t=1}^{M} \mathbf{v}_t\mathbf{v}_t^H\right]_{i,i} = \left[\mathbf{W}\mathbf{W}^H + \mathbf{V}\mathbf{V}^H\right]_{i,i}, \tag{2}$$

where $\left[\cdot\right]_{i,i}$ denotes the $i$-th entry on the diagonal of the matrix.

### A. Radar Metric

Assuming the communication data symbols and radar waveforms are uncorrelated, that is, $\mathbb{E}\left[x_i^c x_t^r\right] = 0$, we define the covariance matrix of the output vector as

$$\mathbf{R} = \mathbb{E}\left(\mathbf{x}\mathbf{x}^H\right) = \mathbf{W}\mathbf{W}^H + \mathbf{V}\mathbf{V}^H = \sum_{i=1}^{U} \mathbf{w}_i\mathbf{w}_i^H + \sum_{t=1}^{M} \mathbf{v}_t\mathbf{v}_t^H. \tag{3}$$

The major task of a radar function is to steer its radar beams towards predefined directions so that the signal bounced back from the targets can be analyzed [2]. The beamforming pattern of the signal output from the DFRC system at the direction $\theta \in [-180°, 180°]$ can be expressed as:

$$\mathcal{P}(\mathbf{R},\theta) = \mathbb{E}\left[\left|\mathbf{a}^H(\theta)\,\mathbf{x}\right|^2\right] = \mathbf{a}^H(\theta)\,\mathbf{R}\mathbf{a}(\theta), \tag{4}$$

where the steering vector at angle $\theta$ is $\mathbf{a}^H(\theta) = \left[1, \exp\left(j2\pi\frac{d}{\lambda}\sin\theta\right), \cdots, \exp\left(j2\pi\frac{d(M-1)}{\lambda}\sin\theta\right)\right]$, the antenna spacing is $d$ and the carrier wavelength is $\lambda$.

One of the important criteria in radar beamforming is to match the designed and desired radar beam patterns [2], [11]. The MSE between the designed and desired radar beam patterns for all targets is the performance metric of the radar function in the DFRC system [2], [11] and is given by:

$$\mathcal{L}(\mathbf{R},\omega) = \frac{1}{G}\sum_{t=1}^{G}\left|\omega\mathcal{D}(\theta_t) - \mathcal{P}(\mathbf{R},\theta_t)\right|^2, \tag{5}$$

where $\mathcal{D}(\theta_t)$ is the desired radar beam pattern for the $t$-th target, $\{\theta_t\}_{t=1}^G$ are the sampled angle grids and $\omega$ is the scaling factor which is a variable to be optimized. The role of $\omega$ is to properly scale $\mathcal{D}(\theta_t)$ as the desired radar beam patterns are usually given in normalized forms [11]. The designed and desired radar beam patterns are said to be matched when the MSE defined in (5) is minimized.

### B. Communication Metric

The received signal at the $i$-th SU is:

$$y_i = \mathbf{h}_{s,i}^H\mathbf{x} + n_i = \mathbf{h}_{s,i}^H\left(\sum_{i=1}^{U}\mathbf{w}_i x_i^c + \sum_{t=1}^{M}\mathbf{v}_t x_t^r\right) + n_i, \tag{6}$$

where $\mathbf{h}_{s,i} \in \mathbb{C}^{M\times 1}$ denotes the $M$ channel coefficients between the BS and the $i$-th SU and $n_i \sim \mathcal{CN}(0,\sigma^2)$ is the additive noise at the $i$-th SU. Let $\{\mathbf{w}_i\} = \{\mathbf{w}_1, \mathbf{w}_2, \cdots, \mathbf{w}_U\}$ and $\{\mathbf{v}_t\} = \{\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_M\}$ represent two sets of beamforming vectors. Besides that, let $\mathbf{H}_{s,i} = \mathbf{h}_{s,i}\mathbf{h}_{s,i}^H$ be the instantaneous channel state information (CSI) or $\mathbf{H}_{s,i} = \mathbb{E}[\mathbf{h}_{s,i}\mathbf{h}_{s,i}^H]$ be the statistical CSI. Therefore, the SINR at the $i$-th SU can be written as:

$$\frac{\mathbf{w}_i^H\mathbf{H}_{s,i}^H\mathbf{w}_i}{\sum_{j=1,j\neq i}^{U}\mathbf{w}_j^H\mathbf{H}_{s,i}^H\mathbf{w}_j + \sum_{t=1}^{M}\mathbf{v}_t^H\mathbf{H}_{s,i}^H\mathbf{v}_t + \sigma_i^2}.$$

Similarly, let $\mathbf{h}_{p,q} \in \mathbb{C}^{M\times 1}$ denote the channel coefficients between the BS and the $q$-th PU and $\mathbf{H}_{p,q} = \mathbf{h}_{p,q}\mathbf{h}_{p,q}^H$ be the instantaneous CSI or $\mathbf{H}_{p,q} = \mathbb{E}[\mathbf{h}_{p,q}\mathbf{h}_{p,q}^H]$ be the statistical CSI. The interference imposed by the DFRC transmission on the $q$-th PU can, thus, be expressed as:

$$\sum_{j=1}^{U}\mathbf{w}_j^H\mathbf{H}_{p,q}^H\mathbf{w}_j + \sum_{t=1}^{M}\mathbf{v}_t^H\mathbf{H}_{p,q}^H\mathbf{v}_t.$$

## III. Proposed Optimization Problem

Let $P_m$, $I_t$ and $\eta_i$ be the available transmit power at the BS, the interference tolerant threshold at PUs and the required SINR level of the $i$-th SU, respectively. We aim to jointly design the two sets of beamforming vectors $\{\mathbf{w}_i\}$ and $\{\mathbf{v}_t\}$ so that the MSE in (5) is minimized subject to the following three constraints: i) the SINR level at each SU is guaranteed to be above $\eta_i$; ii) the transmit power at each BS antenna does not exceed the BS's budget $\frac{P_m}{M}$; and iii) the interference level inflicted at each PU is kept below $I_t$. To that end, we introduce the optimization problem as follows:

$$\begin{aligned}
\min_{\{\mathbf{w}_i\},\ \{\mathbf{v}_t\},\ \omega} \quad & \mathcal{L}\left(\{\mathbf{w}_i\},\{\mathbf{v}_t\},\omega\right) \\
\text{s. t.} \quad & \text{SINR}_i \geq \eta_i, \forall i, \\
& \left[\mathbf{W}\mathbf{W}^H + \mathbf{V}\mathbf{V}^H\right]_{i,i} = \frac{P_m}{M},\ \forall i, \\
& \sum_{j=1}^{U} \mathbf{w}_j^H \mathbf{H}_{p,q}^H \mathbf{w}_j + \sum_{t=1}^{M} \mathbf{v}_t^H \mathbf{H}_{p,q}^H \mathbf{v}_t \leq I_t,\ \forall q.
\end{aligned} \tag{7}$$

It can be observed that the optimization problem shown in (7) is non-convex due to the SINR constraint. By letting $\mathbf{F}_i = \mathbf{w}_i\mathbf{w}_i^H$ and $\mathbf{V}_t = \mathbf{v}_t\mathbf{v}_t^H$, one can rewrite (3) as $\mathbf{R} = \sum_{i=1}^{U}\mathbf{F}_i + \sum_{t=1}^{M}\mathbf{V}_t$. In the sequel, we will cast the objective function in a quadratic form with respect to the optimization variables $\mathbf{R}$ and $\omega$. We start by rewriting (4) as follows:

$$\begin{aligned}
\mathcal{P}(\mathbf{R},\theta) &= \mathbf{a}^H(\theta)\,\mathbf{R}\mathbf{a}(\theta) = \text{vec}\left(\mathbf{a}^H(\theta)\,\mathbf{R}\mathbf{a}(\theta)\right) \\
&= \left(\mathbf{a}^T(\theta)\otimes\mathbf{a}^H(\theta)\right)\text{vec}(\mathbf{R}).
\end{aligned} \tag{8}$$

Therefore,

$$\omega\mathcal{D}(\theta_t) - \mathcal{P}(\mathbf{R},\theta_t) = \left[\mathcal{D}(\theta_t),\ -\mathbf{a}^T(\theta_t)\otimes\mathbf{a}^H(\theta_t)\right]\begin{bmatrix}\omega \\ \text{vec}(\mathbf{R})\end{bmatrix}. \tag{9}$$

Hence, one can equivalently cast (5) as $\mathcal{L}(\mathbf{R},\omega) = \mathbf{r}^H\mathbf{\Omega}\mathbf{r}$ where $\mathbf{r} = \left[\omega, \text{vec}(\mathbf{R})\right]^T \in \mathbb{C}^{(M^2+1)\times 1}$ stacks all optimization variables in a vector and $\mathbf{\Omega} \in \mathbb{C}^{(M^2+1)\times(M^2+1)}$ is given as:

$$\mathbf{\Omega} = \frac{1}{G}\sum_{t=1}^{G}\begin{bmatrix}\mathcal{D}(\theta_t) \\ -\mathbf{a}^{\star}(\theta_t)\otimes\mathbf{a}(\theta_t)\end{bmatrix}\left[\mathcal{D}(\theta_t),\ -\mathbf{a}^T(\theta_t)\otimes\mathbf{a}^H(\theta_t)\right]. \tag{10}$$

With some manipulations, one can equivalently rewrite (7) as:

$$\begin{aligned}
\min_{\mathbf{F}_i,\ \mathbf{V}_t} \quad & \mathbf{r}^H\mathbf{\Omega}\mathbf{r} \\
\text{s. t.} \quad & \left(1+\frac{1}{\eta_i}\right)\text{Tr}\left(\mathbf{H}_{s,i}^H\mathbf{F}_i\right) - \sum_{j=1}^{U}\text{Tr}\left(\mathbf{H}_{s,i}^H\mathbf{F}_j\right) \\
& - \sum_{t=1}^{M}\text{Tr}\left(\mathbf{H}_{s,i}^H\mathbf{V}_t\right) - \sigma_i^2 \geq 0,\ \forall i, \\
& \mathbf{R} = \sum_{i=1}^{U}\mathbf{F}_i + \sum_{t=1}^{M}\mathbf{V}_t, \\
& \mathbf{R}_{i,i} = \frac{P_m}{M},\ \forall i, \\
& \sum_{j=1}^{U}\text{Tr}\left(\mathbf{H}_{p,q}\mathbf{F}_j\right) + \sum_{t=1}^{M}\text{Tr}\left(\mathbf{H}_{p,q}\mathbf{V}_t\right) \leq I_t,\ \forall q, \\
& \mathbf{F}_i \succeq \mathbf{0},\ \forall i \in \{1,\cdots,U\}, \\
& \mathbf{V}_t \succeq \mathbf{0},\ \forall t \in \{1,\cdots,M\}.
\end{aligned} \tag{11}$$

To arrive at (11), we have relaxed rank-one constraints on $\mathbf{F}_i$ and $\mathbf{V}_t$, i.e., rank($\mathbf{F}_i$) = 1 and rank($\mathbf{V}_t$) = 1. The optimization problem in (11) is convex. Hence, interior-point methods can be adopted to solve the problem.

If the optimal solutions obtained by solving (11) are rank-one matrices $\mathbf{F}_i$ and $\mathbf{V}_t$, then they are also the optimal solutions to the original optimization problem in (7) where the optimal solutions $\mathbf{w}_i$ and $\mathbf{v}_t$ are attained from the product of the square root of the eigenvalue and the eigenvector of the corresponding $\mathbf{F}_i$ and $\mathbf{V}_t$ [12]. On the other hand, if the optimal solution $\mathbf{F}_i$ and $\mathbf{V}_t$ are not rank-one matrices, then the randomized technique introduced in [13] can be utilized to obtain approximated/sub-optimal solution to the original optimization problem in (7).

## IV. Numerical Results

### A. Simulation Setup

In this section, we present the numerical results to evaluate the performance of the proposed cognitive DFRC approach. The cognitive DFRC system tracks three targets, i.e., $K = 3$, located at $-60°$, $0°$, and $40°$ with respect to the broadside of the antenna array. Hence, the desired beams for the MIMO radar include three main beams at $\overline{\theta}_1 = -60°$, $\overline{\theta}_2 = 0°$, and $\overline{\theta}_3 = 40°$. Each ideal beam has a beamwidth of $10°$. The desired beam patterns $\mathcal{D}$ in (5) for each beam is set as

$$\mathcal{D}(\theta) = \begin{cases} 1, & \overline{\theta}_t - 5 \leq \theta \leq \overline{\theta}_t + 5,\ t = \{1,\cdots,G\} \\ 0, & \text{otherwise.}\end{cases} \tag{12}$$

The sampled angle grids $\{\theta_t\}_{t=1}^{G}$ for each beam pattern are attained using uniform sampling with a step of $1°$ between $-90°$ and $90°$, i.e., $G = 181$.

The BS serves two SUs, i.e., $U = 2$, located at $-30°$ and $20°$ with respect to the broadside of the antenna array. There is one PU, i.e., $L = 1$, located at $-40°$ with respect to the broadside of the antenna array. The noise variance is 0.1 while the PU's interference tolerance threshold is set to 0.01, 0.05, and 0.1.

We adopt the statistical CSI model where the channel covariance matrix from the BS to the $i$-th SU, $\mathbf{H}_{s,i} = \mathbf{H}(\zeta_{s,i},\delta_a)$, and to the $q$-th PU, $\mathbf{H}_{p,q} = \mathbf{H}\left(\zeta_{p,q},\delta_a\right)$, are the function of the angle of departure, $\zeta_{s,i}$ or $\zeta_{p,q}$, and the standard deviation of the angular spread, $\delta_a$. The $(m,n)$th element of $\mathbf{H}(\zeta,\delta_a)$ is given in [14] as:

$$\exp\left(\frac{j2\pi\Delta}{\psi}\left[(n-m)\sin\zeta\right]\right)\exp\left(-2\left[\frac{\pi\Delta\delta_a}{\psi}\left\{(n-m)\cos\zeta\right\}\right]^2\right), \tag{13}$$

where $\psi$ is the carrier wavelength, $\delta_a = 2°$, and the antenna spacing is set as $\Delta = \psi/2$. The CVX package [15] is adopted to obtain the solution for problem (11).

### B. Performance Evaluation

Fig. 1 compares the transmit beam patterns of the proposed cognitive DFRC approach, i.e., problem (11), against those of the non-cognitive DFRC approach in problem (31) of [2], the radar-only approach in problem (19) of [11], and the comm-only approach in problem (12) of [16].[1] It can be seen from the figure that the proposed cognitive DFRC approach steers the same beams towards the three targets, i.e.,

[1] For a fair comparison, problems (31) of [2] and (19) of [11] are considered in this experiment with the same objective function as that of the proposed problem (7).

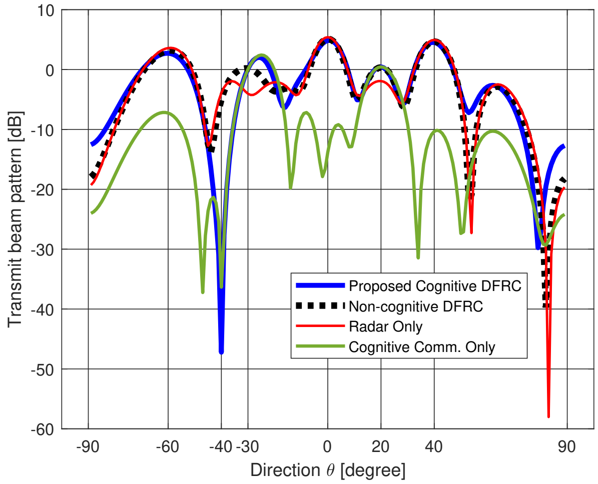


Fig. 1: The radiation beam patterns of the $M = 10$ antenna array for different approaches. The interference tolerant level at the PU is 0.01. The required SINR at each SU is 10 dB.

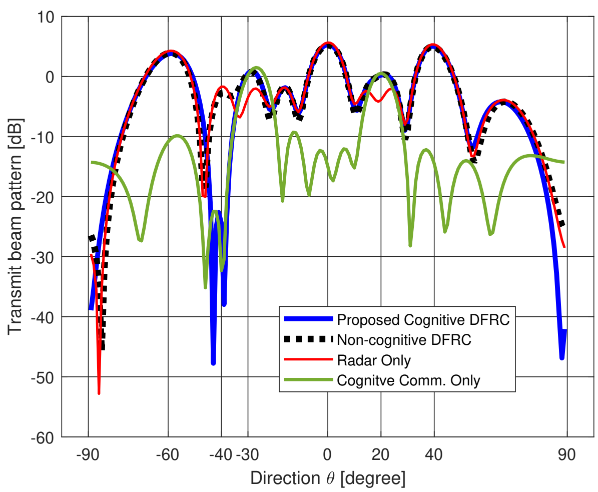


Fig. 2: The radiation beam patterns of the $M = 12$ antenna array for different approaches. The interference tolerance level at the PU is 0.01. The required SINR at each SU is 10 dB.

at $-60°$, $0°$, and $40°$, as those of the non-cognitive DFRC and the radar-only approaches. The proposed cognitive DFRC, cognitive-communication-only, and non-cognitive DFRC approaches provide 0 dB beam levels at the directions of the communication users, i.e., at $-30°$ and $20°$, while the radar-only approach has much lower beam levels at these directions as expected. The proposed cognitive DFRC and cognitive-communication-only approaches effectively form a deep null to protect the PU at $-40°$ whereas the non-cognitive DFRC and radar-only approaches fail to maintain an acceptable interference level, i.e., $-20$ dB, at the PU.

Trends similar to Fig. 1 can also be observed in Fig. 2 for $M = 12$. As the number of antennas increases, the resolution of the MIMO antenna array is improved, resulting in sharper

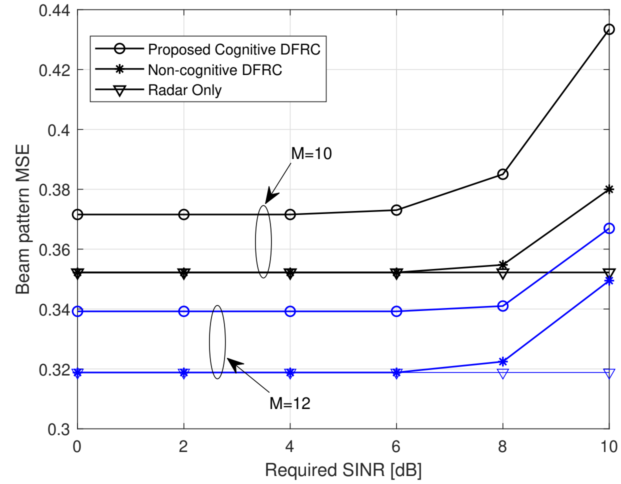


Fig. 3: The beam pattern MSE versus the target SINR level at SUs with different number of antennas. The interference tolerance level at the PU is 0.01.

and more effective beams. This, in turn, improves the system performance which will be further discussed next.

Fig. 3 plots the beam pattern MSE of the proposed cognitive DFRC, non-cognitive DFRC, and radar-only approaches versus the required SINR level with different numbers of antennas, i.e., $M$, at the BS. It can be seen that the radar-only approach attains the lowest MSE in the observed SINR range. The non-cognitive DFRC approach achieves the same MSE performance as the radar-only approach in the SINR range from 0 dB to 8 dB. At SINR of 10 dB, the non-cognitive approach has higher MSE than the radar-only approach. This is due to the fact that the radar function has been traded off for the communication function in order to provide a relatively high SINR level whereas in lower SINR range, the communication function can be met without any performance loss of the radar function.

Fig. 3 indicates that maintaining the required null at the PU while transmitting to the SUs results in the highest MSE level. For instance, the MSE offered by the proposed cognitive DFRC is 0.2 higher than that offered by the non-cognitive DFRC in the SINR range from 0 dB to 6 dB for $M = 10$ and $M = 12$. This performance gap occurs because having more constraints on the optimization problem narrows down the feasibility set which, in turn, affects the optimal solution. The results shown in Fig. 3 also reveal that the MSE can be reduced when more transmit antennas are available. For example, the MSE attained by the proposed cognitive DFRC is reduced from 0.373 to 0.239 at the SINR level of 6 dB when the number of antennas increases from 10 to 12. The improvement of the MSE is due to having more degrees of freedom and greater antenna resolution from the increased number of antennas.

In Fig. 4, the beam pattern MSE is plotted against the interference tolerance level at the PU for the proposed cognitive

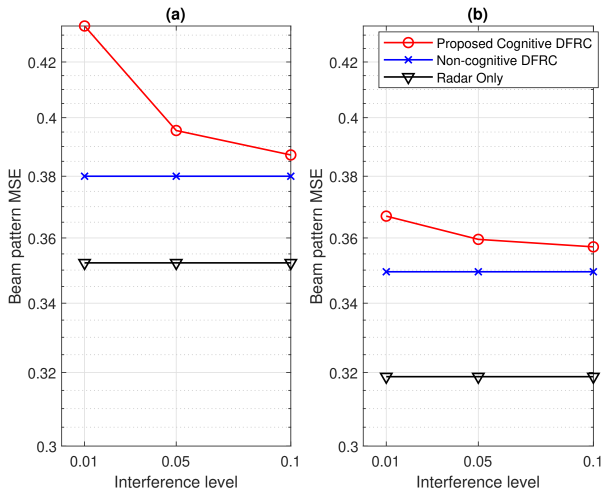


Fig. 4: The beam pattern MSE versus the interference tolerance level for: a) $M = 10$ antennas, and b) $M = 12$ antennas. The required SINR at each SU is 10 dB.

DFRC, non-cognitive DFRC and the radar-only approaches. The required SINR level at each SU is 10 dB. As expected, the non-cognitive and radar-only approaches have a constant level of MSE as they do not take into account the existence of the PU. It can be observed from the figure that adding more functions, i.e., the communication and cognitive functions, onto the system results in a higher beam pattern MSE level. This is the cost of using frequency resource from the primary system. The beam pattern MSE of the proposed cognitive DFRC approach converges to that of the non-cognitive DFRC approach as the interference tolerance level increases. Therefore, when a low level of interference is tolerated at the PU, the proposed cognitive DFRC approach is desirable. On the other hand, when the primary system tolerates high level of interference, the non-cognitive DFRC approach can be chosen. Furthermore, a similar effect of the number of antenna has on the MSE shown in Fig. 3 can also be observed in Fig. 4.

## V. Conclusion

This paper proposes an approach to jointly design the radar and communication beamforming vectors for a cognitive DFRC system. Simulation results indicate that the proposed cognitive DFRC approach can effectively form a null towards the PU while simultaneously communicate with its SUs and track several targets. Trade-offs between the radar function, i.e., the beam pattern MSE between the designed and desired waveforms, and the communication function, i.e., the PU's interference tolerance level and the SUs' SINR level, have been observed. The performance of the proposed cognitive DFRC approach can be improved when higher degree of freedom is available by increasing the numbers of the BS's antennas.

## trtrReferences

tr

[1] T. Zhang, G. Li, S. Wang, G. Zhu, G. Chen, and R. Wang, "ISAC-accelerated edge intelligence: Framework, optimization, and analysis," *IEEE Trans. Green Commun. and Netw.*, vol. 7, no. 1, pp. 455–468, Mar. 2023.

[2] X. Liu, T. Huang, N. Shlezinger, Y. Liu, J. Zhou, and Y. C. Eldar, "Joint transmit beamforming for multiuser MIMO communications and MIMO radar," *IEEE Trans. Signal Process.*, vol. 68, pp. 3929–3944, Jul. 2020.

[3] F. Liu, Y. Cui, C. Masouros, J. Xu, T. X. Han, Y. C. Eldar, and S. Buzzi, "Integrated sensing and communications: Toward dual-functional wireless networks for 6G and Beyond," *IEEE J. Sel. Areas in Commun.*, vol. 40, no. 6, pp. 1728–1767, Jun. 2022.

[4] Z. Ren, L. Qiu, J. Xu, and D. W. K. Ng, "Robust transmit beamforming for secure integrated sensing and communication," *IEEE Trans. Commun.*, vol. 71, no. 9, pp. 5549–5564, Sep. 2023.

[5] H. Hua, J. Xu, and T. X. Han, "Optimal transmit beamforming for integrated sensing and communication," *IEEE Trans. Veh. Technol.*, vol. 72, no. 8, pp. 10 588–10 603, 2023.

[6] X. Liu, T. Huang, and Y. Liu, "Transmit design for joint MIMO radar and multiuser communications with transmit covariance constraint," *IEEE J. Sel. Areas Commun.*, vol. 40, no. 6, pp. 1932–1950, Jun. 2022.

[7] T. A. Le and K. Navaie, "Downlink beamforming in underlay cognitive cellular networks," *IEEE Trans. Commun.*, vol. 62, no. 7, pp. 2212–2223, Jul. 2014.

[8] M. G. Khoshkholgh, K. Navaie, and H. Yanikomeroglu, "Access strategies for spectrum sharing in fading environment: Overlay, underlay and mixed," *IEEE Trans. Mobile Comput.*, vol. 9, no. 12, pp. 1780–1793, Dec. 2010.

[9] T. A. Le and K. Navaie, "On the interference tolerance of the primary system in cognitive radio networks," *IEEE Wireless Commun. Letters*, vol. 4, no. 3, pp. 281–284, Jun. 2015.

[10] R. Saruthirathanaworakun, J. M. Peha, and L. M. Correia, "Opportunistic sharing between rotating radar and cellular," *IEEE J. Sel. Areas Commun.*, vol. 30, no. 10, pp. 1900–1910, Nov. 2012.

[11] P. Stoica, J. Li, and Y. Xie, "On probing signal design for MIMO radar," *IEEE Trans. Signal Process.*, vol. 55, no. 8, pp. 4151–4161, Aug. 2007.

[12] T. A. Le, D. W. K. Ng, and X.-S. Yang, "A rank-one optimization framework and its applications to transmit beamforming," *IEEE Trans. Veh. Technol.*, vol. 73, no. 1, pp. 620–636, Jan. 2024.

[13] H. Dahrouj and W. Yu, "Multicell interference mitigation with joint beamforming and common message decoding," *IEEE Trans. Commun.*, vol. 59, no. 8, pp. 2264–2273, Aug. 2011.

[14] M. Bengtsson and B. Ottersten, "Optimal downlink beamforming using Semidefinite optimization," in *Proc. 37th Annu. Allerton Conf. Commun., Control, and Comput.*, 1999, pp. 987 – 996.

[15] CVX Research Inc., "CVX: Matlab software for disciplined convex programming, academic users," http://cvxr.com/cvx, 2015.

[16] Y. Huang and D. P. Palomar, "Rank-constrained separable Semidefinite programming with applications to optimal beamforming," *IEEE Trans. Signal Process.*, vol. 58, no. 2, pp. 644–678, Feb. 2010.