Ordering Based Decision Making – A Survey

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A B S T R A C T

Decision making is the crucial step in many real applications such as organization management, financial planning, products evaluation and recommendation. Rational decision making is to select an alternative from a set of different ones which has the best utility (*i.e.*, maximally satisfies given criteria, objectives, or preferences). In many cases, decision making is to order alternatives and select one or a few among the top of the ranking. Orderings provide a natural and effective way for representing indeterminate situations which are pervasive in commonsense reasoning. Ordering based decision making is then to find the suitable method for evaluating candidates or ranking alternatives based on provided ordinal information and criteria, and this in many cases is to rank alternatives based on qualitative ordering information. In this paper, we discuss the importance and research aspects of ordering based decision making, and review the existing ordering based decision making theories and methods providing future research directions.

Keywords: Decision making; Ordering relation; Preference; Lattice; Logic; Aggregation

1. Introduction

Decision making is the final and crucial step in many real applications such as organization management, financial planning, products evaluation, risk assessment and recommendation, which, in many cases, can be seen as the process for choosing the most appropriate one among a set of alternatives under provided criteria, objectives, or preferences [1-3]. Along with the social and economic development, it becomes more and more difficult to make decision based on simple personal judgements. Various decision making models and methods are then developed to support human for making decisions under complex situations, but it is still a hard task to make a good decision, especially in the complex, dynamic and uncertain socio-economic environment [4].

Orderings provide a very natural and effective way for representing and reasoning with indeterminate situations which are pervasive in commonsense reasoning involved in real-life decision problems. How to handle different ordering relationships in decision making is always an essential research problem [5, 6]. In general, an order is an arrangement of elements according to some defined standards or natural relationships, such as alphabetical order, numerical order, and power set order whose ordering relation is the inclusion relation between subsets. Ordinal information in real life, especially in decision making areas, includes ordinal attributes, preference relation and so on. For example, you might use A, B, C, D, and F to grade a student with the assumption that $A>B>C>D>F$; or you may "prefer beef to lamb" when ordering a meal, *i.e.*, you put beef before lamb in a preferential order. *Ordering based decision making* is therefore to find the suitable method for evaluating candidates or ranking alternatives based on provided ordinal information and criteria, and this in many cases is to rank alternatives based on preferential ordering information.

In ordering based decision making, it is always more natural and reasonable for decision makers to express a qualitative preferential ordering among alternatives than to provide quantitative preference degrees. Additionally, there are always many conditions or criteria in real decision making problems, not all of which can be satisfied simultaneously due to the uncertainty and complexity involved [7]. Sometimes, it may not always be feasible or realistic to acquire exact judgment on each attribute due to time restriction, lack of knowledge or data, limited expertise related to the problem domain and so on. For instance, in expressing preferences about movies, it is much easier for most people to express their preferences over two movies they have seen rather than describing preferences over attributes like director or main actors. Some people may not know well about the name of director or actors, even when they may have a preference for movies with good directors.

To conduct ordering based decision making, the first stage is to find some suitable structures for representing the ordinal information involved, which is known as information representation. We then need to choose the suitable aggregation algorithm or inference mechanism in order to aggregate or rank the alternatives according to the provided ordinal information. The final step is to choose the "best" alternative, which is normally made up by two phases: (a) The aggregation of ordering relations for obtaining a collective performance value on the alternatives, and (b) The exploitation of the collective performance value in order to establish a rank ordering among the alternatives for choosing the best one [8-11].

In order to represent ordinal information, many kinds of ordered structures have been defined, due to the diversity of orders in real problems, such as totally ordered structure, partially ordered structure and lattice structure [12-14]. On the other hand, totally ordered sets are widely used for ordering information representation in decision making due to their simplicity, but are often forced

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to make simplifying assumptions about reality when using only totally ordered sets. We use totally ordered sets to represent all ordering relations when we lack the ability or tool to handle nonlinear ones. Actually, most relations in real world are nonlinear, due to the fact that humans' intelligent activities, especially decision making, are always associated with many uncertainties. Incomparability is such a kind of uncertainty, which is mainly caused by ambiguity, conflicting opinions or missing information. For example, we always find it difficult to make a decision in real life when the decision is based on multiple criteria where conflicting opinions always exist. Partially ordered set or lattice structure is more suitable and flexible for information representation under these situations [15].

Partially ordered pieces of knowledge appear in many applications because of the dynamics of knowledge and when we merge multiple sources information [16, 17]. For example, one situation may be better in one dimension but worse in another. Partial orders offer more flexibility than total orders to represent incomplete knowledge. Moreover, they avoid comparing unrelated pieces of information. The following two examples [18] will show that partial order is ubiquitous in our daily decision making problems.

Example 1. (My dinner I) Consider a simple example shown in Fig. 1 that expresses my preference over dinner configurations, where S and W stand for the soup and wine respectively. An arrow going from alternative x_i to x_j indicates that x_i is preferred to x_j. The figure shows that I strictly prefer fish soup (S_f) to vegetable soup (S_v), while my preference between red (W_r) and white (W_w) wine is conditioned on the soup to be served: I prefer white wine if served a fish soup, and red wine if served a vegetable soup.

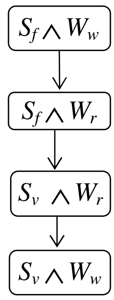

Fig. 1. Preference graph of "my dinner I".

The preferential relation shown in Example 1 is a totally ordered set, but it will become partial order when adding the main course M as another variable, which can be shown in Example 2.

Example 2. (My dinner II) My preference over the options for the main course is clear from the Fig. 2: I strictly prefer a steak M_{sc} to a fish fillet M_{fc}. In addition, I prefer not to have two fish courses in one dinner; thus my preference between vegetable and fish soup is conditioned on the main course: I prefer to open with fish soup if the main course is steak and with vegetable soup if the main course is fish fillet.

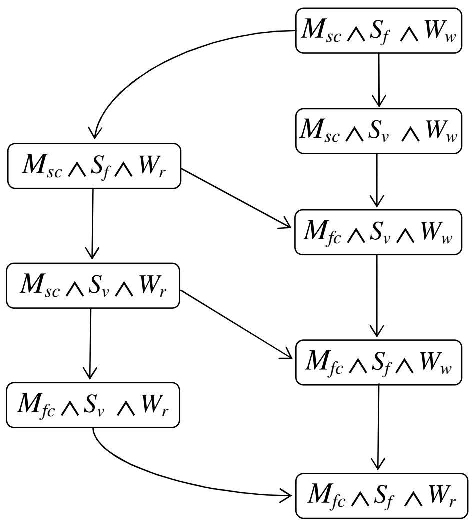

Fig. 2. Preference graph of "my dinner II".

This is a partially ordered set where $M_{sc} \wedge S_f \wedge W_r$ and $M_{sc} \wedge S_v \wedge W_w$ are incomparable, which can be easily elicited from real-life decision making problems.

As a special kind of partially ordered structure, lattice has been shown to be a suitable and efficient structure for representing ordering relationship in the real world due to its better properties and additional operations [12-14]. Although these additional properties and operations will in turn restrict its application areas, there are still many real problems which can be modelled by lattice structure. For example, when evaluating the quality of some products, one may express his opinions as "high", "very high", "low", and so on, that can be illustrated in Fig. 3 as a lattice structure [19]. Furthermore, lattice also plays an important role as the truth-value field of logic, that is, it has a close relation to logic and can serve as a bridge between real problem and logical foundation.

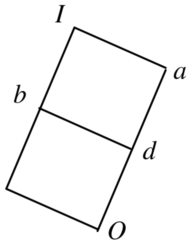

Fig. 3. A kind of lattice ordering structure
I=more high, *b*=high, *c*=less high, *a*=less low, *d*=low, *O*=more low.

After the representation of ordinal information involved, the next step is to use aggregation algorithm or inference mechanism to obtain the final decision, usually an ordering of alternatives. For getting the final decision result, current ordering based decision making methods are mainly from the information aggregation point of view, *i.e*., how to combine all the decision makers' opinions to obtain a final ordering or evaluation. For multi-criteria decision making, the judgments provided by decision makers for different criteria are usually assumed to preference relations in the same form [7, 20] or similar forms [21, 22]. While in many real cases, the provided preference relations are always in different forms which can be illustrated using the following example.

Example 3. In a movie ranking problem, the customers are asked to provide qualitative preferential orderings about the movies they have enjoyed. It is very common that one customer can only provide the rating for some movies, but not other if he/she did not get chance to watch yet. Ratings given by each customer therefore can be transferred into a preference relation among the movies, which, for example, might be simply illustrated in Fig. 4, where five customers are asked to express their preferences among five different movies *a*, *b*, *c*, *d*, *e*, and the arrow directed from alternative *x* to *y* indicates that *x* is preferred to *y*. Take the upper right one as an example, this means that this customer cannot express his/her preference between movies *b* and *c*, but it does not mean that he/she has no opinion about movie *b* or *c* individually, because he/she prefers *a* to *b*, and *a* to *c* as well.

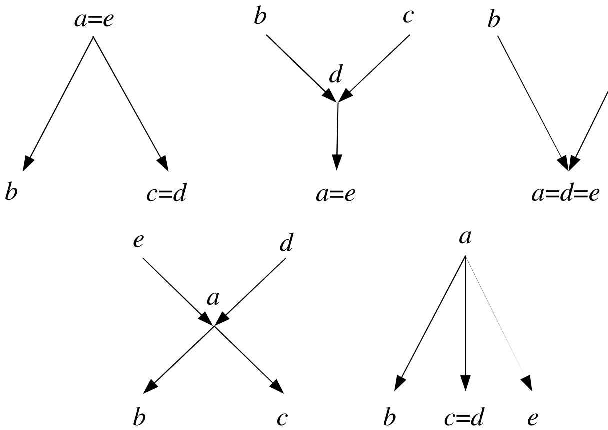

Fig. 4. Preferences in different partial ordering structures

Taking into account the fact that most existing ordering based decision making approaches are mainly focus on totally ordered information while the information involved in real decision making problems is mostly partially ordered, this paper has discussed the importance and necessity of decision making with partially ordered information throughout the Introduction. Although there are some decision making approaches that can deal with partially ordered information, they usually transform qualitative information into quantitative scale, which will cause loss of information and is time consuming. It will be more natural and reasonable to represent and reason about qualitative ordering information in its original form, *i.e*., through symbolic way.

From the viewpoint of symbolism, it is important and necessary to establish the logical foundation for decision making approach. As put by Zagare [23]: "*Without a logically consistent theoretical structure to explain them, empirical observations are impossible to evaluate; without a logically consistent theoretical structure to constrain them, original and creative theories are of limited utility; and without a logically consistent argument to support them, even entirely laudable conclusions … lose much of their intellectual force*." That is: Logic serves as the most important foundation and standard for justifying or evaluating the soundness and consistency of our methods. In summary, we want to highlight in this survey paper the decision making with partially ordered information from the algebraic and logic point of view.

Due to the fact that decision making is a very general research domain where almost all human involved activities are concerned, we are not able to cover all the related topics in this paper, and so this survey mainly reviews existing ordering based decision making approaches, especially those which consider partially ordered qualitative information or are logic-oriented.

Furthermore, this survey is mainly from the methodology point of view, that is, we mainly review the methods and the underlying theories, not the applications.

Based on the over 100 articles and books collected (searched via Google Scholar, IEEE Xplore, ScienceDirect, SpringerLink, Wiley Online Library, Web of Knowledge, and so on), three issues are examined, including: (i) Which approaches can handle partially ordered information? (ii) Which approaches are designed for treating qualitative information? (iii) Which approaches are from the algebraic and logic point of view?

The rest of this paper is organized as follows: Section 2 reviews the existing methods for representing ordinal qualitative information in decision making, which serves as the basis of most decision making approaches reviewed in this paper. Representation and aggregation methods for preference based decision making approaches are reviewed in Section 3, and the decision making methods from the logic point of view are reviewed in Section 4. Section 5 summarizes the reviewed methods through a table and draws some concluding remarks with some thinking into the future research directions.

2. Representation of Ordinal Qualitative Information

Qualitative information is frequently used in the area of decision-making, such as judgments/opinions from experts. Human beings always give their judgments/opinions about things using natural language (linguistic terms). Linguistic terms, not like numerical ones whose value are crisp numbers, are always vague and imprecise. Sometimes, it is difficult to clarify the boundary for some linguistic terms or words, but one can understand their common meaning well. Linguistic information involved with decision making problems is always in some ordering relation. For example, when we are evaluating the quality of a computer, the evaluations may be "bad," "acceptable," "good," "very good," and these evaluations are in an order according to their semantic meanings.

There are generally two types of ways for decision making with linguistic information: fuzzy set based method, and symbolic method. The conventional fuzzy set based method [8, 24-27] uses membership function or fuzzy number to represent linguistic information and need a linguistic approximation of the final computed result, which are time consuming and computationally complex [28]. Symbolic approaches [29-31] use symbols (usually in a structure) to represent linguistic information directly without the numerical approximation, and aggregate or reason about these symbols to obtain the final result.

One of the representative linguistic valued information processing approaches is fuzzy ordinal linguistic approach [29-32]. This method uses an ordered structure, linguistic labels with indexes, to represent the set of linguistic terms, with the assumption that the terms under discussion is totally ordered [33]. The 2-tuple linguistic representation (or computational) model [28, 34, 35], one of most popular extended fuzzy ordinal linguistic approaches, is a continuous linguistic representation model. In this model, the linguistic 2-tuple is used to represent the linguistic information, and it is a pair of values, (L_i, α_i), where $L_i \in S$ is a linguistic label and $\alpha_i \in [-0.5, 0.5)$ is a number, called the symbolic translation, which supports "difference of information" between the result obtained after aggregation and the closest one in the set of linguistic terms. Take the simple linguistic term set $S=\{s_{-4}$=extremely low, s_{-3}=very low, s_{-2}=low, s_{-1}=slightly low, s_0=fair, s_1= slightly high, s_2=high, s_3=very high, s_4=extremely high$\}$ as an example, $(s_2, 0)$ is "high", while $(s_2, -0.2)$ expresses the evaluation has a difference of -0.2 to "high", that is, it is 0.2 lower than "high".

These representative symbolic approaches use linguistic labels with indexes to represent linguistic terms and make operations on these indexes, and the representation and manipulation of linguistic terms are explored in a qualitative setting [32, 33], trying to avoid an underlying numerical approximation needed by fuzzy set based method. Suppose that $S=\{s_0, s_1, \ldots, s_g\}$ is a set of linguistic terms under consideration, which is an ordered structure, i.e., $s_i<s_j$ iff $i<j$. In the linguistic 2-tuple method, the 2-tuple representation (L_i, α_i) will be transformed into a number by combining the index i and the number α_i, which will be used for aggregation, then the result obtained after aggregation will be retransformed to a 2-tuple. The transformation between a 2-tuple and its equivalent numerical value $\beta \in [0, g]$, where $g+1$ is the cardinality of the linguistic term set S, is defined as:

$$\Delta : [0, g] \to S \times [-0.5, 0.5)$$
$$\Delta(\beta) = (s_i, \alpha), \quad \text{with} \begin{cases} s_i, & i = round(\beta) \\ \alpha = \beta - i, & \alpha \in [-0.5, 0.5) \end{cases} \tag{1}$$

where *round* is the usual round operation, i the index of the closest linguistic term, s_i to β, and α the value of symbolic translation.

Based on the idea of hesitant fuzzy sets [36], Rodríguez *et al.* [37] introduced the concept of hesitant fuzzy linguistic term sets which allow the decision makers to express their opinions using several linguistic terms instead of a single term. For example, the judgment of one decision maker on some alternative may be {slightly low, fair}. Hesitant fuzzy linguistic term sets are more appropriate for the situations where decision makers hesitate among several linguistic values for assessing the alternatives, than the traditional fuzzy linguistic approach which does not allow the decision makers to use more than one linguistic term to assess each alternative.

Although fuzzy ordinal linguistic approach has the advantages of computational simplicity without the need of membership function and avoiding loss of information [28, 38], it requires the ordering relation discussed should be in total order and can be represented by indexes. This limits the application of fuzzy ordinal linguistic approach to more general situations where partially ordered information often involved.

Hedge algebra [39], a linguistic information representation method from the algebraic point of view, is proposed as an ordered algebraic structure for modelling linguistic terms. In later years, Ho *et al.* proposed extended hedge algebras [40], refined hedge algebras [41], complete hedge algebras [42]. Generally speaking, linguistic hedges can be seen as some kind of linguistic modifiers such as "*very*," "*little*," "*possibly*," etc. on the prime terms such as "*high* and *low*," "*true* and *false*." These linguistic hedges can strengthen or weaken the meaning of the prime terms. Hedge algebra is then constructed by applying the set of hedges to the prime terms (also called generators), which is essentially a partially ordered set according to the natural meanings of its elements, and generally a lattice as simply shown in Fig. 3.

Hedge algebra takes the academic idea that there exists a semantic ordering relation among these linguistic terms, and linguistic hedges, which can strengthen or weaken the meaning of the prime terms, play a vital role for generating the algebraic structure [43]. Hedge algebras are logical algebras, so logic systems and the corresponding approximate reasoning methods can be built intuitively based on hedge algebras, but no such work has been done.

3. Preference Based Decision Making

Preference from the experts is the most commonly used ordinal information in real world decision making problems. Examples include risk aversion in economics and finance [44], quality assessment of service, food or textile products [45], dance competition adjudication, meta-search engine whose goal is to combine the preference relations of several WWW search engines [46], and so on. This kind of preference always appears as partially ordered structure [47]. For decision making under this situation, a preference aggregation procedure is always applied to combine these partial orders to produce an overall preference ordering, and this again can be a partial order.

Normally, the preference representation structures can be categorized into three types according to the decision making objective and the provided information [10, 48-52]:

(1) *Utility function*. A utility function is a function given by each decision maker which assigns each alternative a real number $u_i^k \in [0,1]$, utility value, which indicates evaluations of the decision maker k on the ith alternative. The utility values are mainly given by the decision makers subjectively according to some requirements.

(2) *Preference relation*. This preference representation structure asks the decision maker expresses his preferences on the set of alternatives based on pair comparison. This is usually described as a preference relation matrix whose element depicts the degree, usually numerical, of the decision maker's preference of one alternative to another.

(3) *Preference ordering*. In the case of preference ordering, each decision maker expresses his preferences on the set of alternatives as a preference ordering, which can be a total order which orders alternatives from the best to the worst or a partial order with some alternatives are incomparable.

Although utility function, preference relation and preference ordering are the three major representation models of preference, and they are applicable to different certain situations, there are also some overlaps among them. For example, Borda count [53, 54], the widely used preference based ordering method firstly asks decision makers to rank the alternatives, and then allocates absolute scores to the alternatives according to the rating, which are similar to utility values. There are also some transformation methods from one representation structure to another, such as the methods presented by Chiclana and Herrera *et al*. [10, 48, 49] for transforming utility values and preference orderings into preference relations. A simple method for getting the preference relation matrix $A^k = (a_{ij}^k)$ from a set of utility values given by an expert $U^k = \{u_1^k, \cdots, u_n^k\}$ is $a_{ij}^k = \frac{u_i^k}{u_j^k}$.

3.1 Decision making based on preference relations

Preference relations, generated from pairwise comparison of alternatives, are widely used to model experts' preferences in real decision-making problems. A preference relation R is usually modelled by a preference structure, a triplet (P, I, J) of three binary relations: strict preference, indifference or incomparability, on a finite set of alternatives X, which satisfy [55]:

1) P is irreflexive and asymmetrical;

2) I is reflexive and symmetrical;

3) J is irreflexive and symmetrical;

4) $P \cap I = P \cap J = I \cap J = \varnothing$;

5) $P \cup P^t \cup I \cup J = X^2$, where P^t is the transpose (or inverse) of P: $(x_i, x_j) \in P \Leftrightarrow (x_j, x_i) \in P^t$.

Here, Condition 5) is called the completeness condition. Take the upper left preference shown in Fig. 4 as an example, it can be expressed in preference structure as (P, I, J) where $P=\{(a, b), (a, c), (a, d), (e, b), (e, c), (e, d)\}$, $I=\{(a, a), (b, b), (c, c), (d, d), (e, e), (a, e), (c, d)\}$, $J=\{(b, c), (b, d)\}$.

Since preference structures are restricted to classical relations, preference degrees cannot be expressed, which is seen as an important drawback from the practical point of view [56, 57]. There are generally three forms of preference relations that extend the classical case:

(1) Multiplicative preference relations [10, 50, 58-60]: In a multiplicative preference relation, the decision maker's preferences on a set of alternatives X is represented by a positive matrix $A \subseteq X \times X$, $A=(a_{ij})$, whose element a_{ij} is the intensity of preference of alternative x_i to x_j, usually measured by a numerical ratio scale. The most popular ratio scale is suggested by Saaty [58, 59], known

as AHP (Analytic Hierarchy Process), which defines the 1-9 scale for measuring a_{ij}. a_{ij}=9 means that x_i is totally preferred to x_j; a_{ij}=1 shows the indifference of the decision maker's preference to x_i and x_j, and a_{ij}=2, …, 8 represents the intermediate preferences. It is usually supposed that the multiplicative preference relation is multiplicative reciprocal, *i.e.*, $a_{ij} \cdot a_{ji} = 1$, $\forall i, j \in \{1, \cdots, n\}$.

(2) Fuzzy (valued) preference relations [48, 52, 61-63]: In this case, the decision maker's preferences on the set of alternatives X is described by a fuzzy relation R (sometimes called as weak preference relation) on $X \times X$, with its associated membership function $\mu_R : X \times X \to [0,1]$, and this is usually represented by a $n \times n$ fuzzy relation matrix $R=(r_{ij})$ with $r_{ij} = \mu_R(x_i, x_j)$, $\forall i, j \in \{1, \cdots, n\}$. Here $r_{ij} = \mu_R(x_i, x_j)$ denotes the degree of preference of alternative x_i to x_j: r_{ij}=1 means that x_i is totally preferred to x_j; r_{ij}=0.5 shows the indifference of the decision maker's preference to x_i and x_j, and r_{ij}>0.5 means that the decision maker prefers x_i to x_j. Similarly, it is usually supposed that the preference matrix R is additive reciprocal, *i.e.*, $r_{ij} + r_{ji} = 1$, $\forall i, j \in \{1, \cdots, n\}$, and this shows that r_{ii}=0.5.

(3) Linguistic preference relations [64-67]: Unlike multiplicative preference relation and fuzzy preference relation using crisp numerical values to express the intensity of preference of one alternative to another, linguistic preference relation uses linguistic terms to indicate this preference level when numerical preferences are not available or difficult to obtain. Similarly, a linguistic preference relation on X is also denoted by a matrix $B=(b_{ij})$, whose element $b_{ij} \in S$, $S = \{s_\alpha \mid \alpha = -t, \cdots, -1, 0, 1, \cdots, t\}$ is the set of all linguistic terms considered, indicates the linguistic degree of preference of alternative x_i to x_j, and satisfies $b_{ij} \oplus b_{ji} = s_0$, $b_{ii} = s_0$, $\forall i, j \in \{1, \cdots, n\}$, where $\oplus$ is the operation of linguistic terms defined as $s_\alpha \oplus s_\beta = s_{\alpha+\beta}$.

A good number of studies have been made on fuzzy preference modelling or the axiomatic construction of fuzzy preference structures (see e.g. [56, 57, 62, 68-73]), which serves as the theoretical foundation of preference based decision making. The key issue for this question is how to decompose a weak preference relation R into a strict preference relation P, an indifference relation I, and an incomparability relation J such that (P, I, J) is a fuzzy preference structure, and the axiomatic construction is mainly based on De Morgan triplet (T, S, N), which consists of a t-norm T, its dual t-conorm S, and a strong negation N. A fuzzy preference structure on X is a triplet (P, I, J) of fuzzy relations satisfying

1) P and J are irreflexive, I is reflexive;

2) P is T-asymmetrical ($P \cap_T P^t = \varnothing$), I and J are symmetrical;

3) $P \cap_T I = P \cap_T J = I \cap_T J = \varnothing$;

4) $P \cup_S P^t \cup_S I \cup_S J = X^2$, where $P \cap_T I$ is the T-intersection of two sets P and I and $P \cup_S I$ is their S-union.

Condition 4) is one of the numerous completeness conditions for fuzzy preference structures, where the differences between crisp and fuzzy preference structures, and among different fuzzy preference structures mainly come from (please refer to [56] for more details).

In order to illustrate the preference relations based decision making approaches, we use the following simple but representative example, which is adapted from [22], through this section:

Example 4. Suppose the information management steering committee of a company, which comprises (1) E_1: the Chief Executive Officer, (2) E_2: the Chief Information Officer, and (3) E_3: the Chief Operating Officer, must prioritize for development and implementation a set of six information technology improvement projects x_j (j=1, 2,…, 6), x_1: Quality Management Information, x_2: Inventory Control, x_3: Customer Order Tracking, x_4: Materials Purchasing Management, x_5: Fleet Management, x_6: Design Change Management. The committee is concerned that the projects are prioritized from highest to lowest potential contribution to the firm's strategic goal of gaining competitive advantage in the industry. In assessing the potential contribution of each project, one main factor considered is productivity. This is a typical ordering based decision making problem by ranking the projects according to some criteria.

All the three methods can be used to represent the preference relations provided by the committee in Example 4 (project evaluation), and all the preference relations are expressed by 6×6 matrices. The only difference is that the elements of the preference relation matrices take different forms depending on the information the committee provided, numbers ranging from 1-9 for multiplicative preference relation, numbers ranging from 0-1 for fuzzy preference relation, and linguistic terms for linguistic preference relation. Take the linguistic preference relation for an example, the preference relation about the six projects provided the Chief Executive Officer could be

$$B_1 = \begin{bmatrix} s_0 & s_1 & s_{-2} & s_3 & s_1 & s_{-1} \\ s_{-1} & s_0 & s_{-1} & s_2 & s_0 & s_1 \\ s_2 & s_1 & s_0 & s_1 & s_{-2} & s_2 \\ s_{-3} & s_{-2} & s_{-1} & s_0 & s_2 & s_3 \\ s_{-1} & s_0 & s_2 & s_{-2} & s_0 & s_{-2} \\ s_1 & s_{-1} & s_{-2} & s_{-3} & s_2 & s_0 \end{bmatrix}, \tag{2}$$

where the matrix elements are from the linguistic term set S={s_{-4}=extremely low, s_{-3}=very low, s_{-2}=low, s_{-1}=slightly low, s_0=fair, s_1= slightly high, s_2=high, s_3=very high, s_4=extremely high}. The preference relations provided by other committee members can be obtained similarly as B_2 and B_3.

In order to avoid the difficulty of providing accurate numerical value for decision maker's preference degree, interval multiplicative preference relations [74] and interval fuzzy preference relations [75-77] were proposed which use interval numbers as the judgements of the decision maker's preference.

Wang *et al.* [7, 20, 78-80] proposed a hybrid method, called fuzzy linguistic preference relations, for representing preference relation under linguistic environments by expressing linguistic terms as fuzzy numbers $(P_{ij}^L, P_{ij}^M, P_{ij}^R)$, which can be seen as a special case of linguistic preference relation.

Decision making approaches with preference relations are usually from the aggregation point of view. Once we have the information expressed by preference relations uniformly, aggregation algorithms will be applied to obtain the collective preference relation from all the individual preference relations. An exploitation phase of the collective linguistic performance value will then be made to establish a rank ordering among the alternatives for choosing the best alternatives by using the principle of fuzzy majority or consensus [10].

The methods for aggregating preference relations are mainly based on the OWA (Ordered Weighted Averaging) operator proposed by Yager [81] and further developed operators, such as Linguistic OWA [64], Weighted OWA and Linguistic Weighted OWA [82], Ordered Weighted Geometric Averaging Operator [83], Induced Ordered Weighted Geometric Operators [50], Induced Linguistic OWA [84], continuous Ordered Weighted Geometric Operator [85], Lattice-based Linguistic-Valued Weighted Aggregation Operator [86], and some hybrid Weighted Averaging Operators [87]. Overviews of these aggregation operators can be found in [87-89]. The original and wide-used OWA operator aggregates a collection of labels by always assigning the *i*th weighing factor to the *i*th biggest label, which is the reason why it is called Ordered Weighted Averaging aggregation operator. For aggregating qualitative labels, the corresponding aggregation operators always use linguistic labels with indexes to represent linguistic terms and make operations on these indexes. Consider Example 4 and suppose that the preference relation matrices take the form of Eq. (2), then linguistic aggregation operators, e.g., linguistic weighted arithmetic averaging (LWAA) operator, can be used for aggregating the preference relations B_1, B_2 and B_3 to get the collective preference relation B whose element

$$b_{ij} = w_1 b_{ij}^{(1)} \oplus w_2 b_{ij}^{(2)} \oplus w_3 b_{ij}^{(3)} \tag{3}$$

where $w=(w_1, w_2, w_3)^T$ is the weighting vector of B_1, B_2 and B_3, $b_{ij}^{(k)}$ is the ij-th element of B_k. The detailed procedure can be found in [22] and other related articles.

Generally, decision making approaches with preference relations need the preference for each pair of candidates is known, which actually makes all of the alternatives a total order. In fact, incomparability relation in crisp preference structure is always treated as a special case of indifference relation [54, 90], while in fuzzy preference structure, the relation matrix cannot express the incomparability relation, while r_{ij}=0.5 indicates indifference relation between two alternatives. However, decision makers may only be able to provide their preferences on a subset of all the alternatives in many real problems, due to incompleteness of information, unclear evaluations, and so on. This kind of preference appears exactly as a partial order.

3.2 Decision making with Preference ordering

Many decision making applications, especially in socio-economic areas, are to order alternatives based preference, which is to rank alternatives by a group of people based on each member's preference on subsets of the alternatives [46]. Many of these approaches have been applied to Social Choice [91] area, which blends elements of welfare economics and voting theory.

Because preference in most of case of reality appears as partially ordered structure due to incompleteness of knowledge, ambiguity of opinions, and so on [47, 92], which makes the aggregation process much more complex and challenging. For decision making under this situation, a preference aggregation or inference procedure need to be applied to combine these partial orders to produce an overall preference ordering, and this again can be a partial order.

Soft constraints [93-95] are one of the popular methods for representing and aggregating quantitative preferences. Soft constraints were originally proposed to overcome the limitations of classical methods for constraint satisfaction problems (CSPs) under fuzzy or incomplete situations. Here, each soft constraint associates a value from a partially ordered set to a set of variables. These values can be interpreted as degrees of preference, levels of consistency, and probabilities. The set of preference values is modelled by a semiring structure, a domain with two operations, additive operation "+" for ordering alternatives and multiplicative operation "×" for combining preferences to obtain the result. A relation "≤" is also defined over the domain by $a \le b$ iff $a + b = b$ which makes the set a partially ordered set.

Soft constraints are the main tool for representing and reasoning about preferences in constraint satisfaction problems due to their expressive and powerful ability to support representing and reasoning about preferences. However, they require specifying a numeric semiring value for each variable assignment in each constraint diminishes their applicability to many situations which are qualitative in nature. In many applications, it is more natural for users to express preferences via generic qualitative (usually partial) preference relations over variable assignments [95]. For example, it is more natural to express the preference on the car as "I prefer silver car to black car", rather than "Silver car has preference 0.8 and black car has preference 0.4", which is required in soft constraints for assigning numeric preference values to variables.

In most real cases, decision makers are always asked to express their preferences over the decision alternatives via qualitative statements, such as "If the main course is beef or lamb, I prefer red wine to white wine", or "I prefer a seat near aisle to near window". Among methods for representing and reasoning with qualitative preferences, CP-nets [18, 21, 96, 97] is one of the most popular, where CP is the abbreviation of Conditional Preference or *ceteris paribus* (all other conditions being equal, or conditional preferential independence). A CP-net represents a preference in a straightforward form as p: x>y, which indicates that x is strictly preferred to y provided condition p, and all these conditional preferences about a certain feature with values, denoted as a node in a CP-net, will be associated with this node in a table form. For instance, the CP-net for Example 2 can be shown in Fig. 5, where the left side cells of the tables represent the provided conditions, i.e., my preference between vegetable and fish soup is conditioned on main course and my preference between red wine and white wine depends on the soup to be served. The condition for a certain feature depends on the preferences of other features, and these nodes (features) with mutual dependent relations to each other according to conditional preferences will be put in a partial order. However, although the representation of preference ordering is succinct, the main problem of CP-nets is that it is complex to make an optimal assignment of preference values to all the features, a NP hard problem under some assumptions, as well as that accurate utility values are difficult to decide for non-specialist users [21, 95, 98].

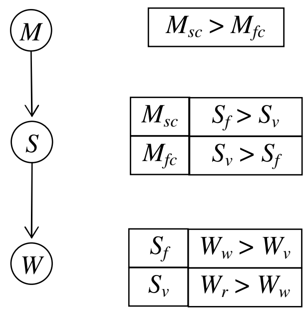

Fig. 5. CP-net for "my dinner II".

Some extensions of CP-nets have been developed to overcome the above mentioned defects. Utility CP-nets, or UCP-nets [99] is one of the extensions which uses numerical utility factors to replace the binary relationship between node values in CP-nets. Doing so, UCP-nets allow the node values to be able to retain their qualitative form, and only preferences are quantified with utility values. LCP-nets (Linguistic Conditional Preference networks) [100, 101] is another kind of extension by combining the linguistic approach into CP-nets, and this allows for the preference modelling of more qualitative statements such as "I prefer *a bit* Dell laptop to Sony laptop if their CPU speeds are *approximately* the same and RAM sizes are *more or less* the same".

One widely used, especially in Social Choice problems, solution for preference based ordering is the Borda count or Borda's Rule [53, 54]. Borda's Rule asks decision makers to rank the alternatives, and then allocates absolute scores to the alternatives. The higher an alternative is ranked, the more points it will receive. A simple solution is to assign one point to an alternative for each competitor ranked below it in the ranking. The alternative with the most total points is declared the winner. For instance, in Example 3 (movie ranking), Borda's Rule will ask each user to rank the movies he/she have enjoyed, usually in a total order, then allocate absolute scores to the movies, e.g., 5 to the most favourite, 4 to the second, …, and 1 to the last one. All the scores associated with the same movie will be added up accordingly, and the movie with the highest total points will be declared the most popular.

The primary advantage to this procedure is its ability to find a "fair" compromise since it includes more information from the decision makers than either plurality or majority rule [53, 102, 103], while the main drawback is absolute scores are usually difficult to decide and we cannot expect the same score always means the same to every decision maker.

Approval voting [104] is another popular Social Choice system, always used for elections. Under this mechanism, each voter is allowed to vote for as many of the alternatives as he/she wishes, and each voter may vote for any combination of alternatives and may give each alternative at most one vote. The alternative who receives the most votes is declared the winner.

Similar to Approval voting, Majority Judgment is also a single-winner voting system [105]. This voting system asks voters to freely grade each alternative in one of several named ranks, for instance from "excellent" to "reject", and the alternative with the highest median grade is the winner. If more than one alternative has the same highest median grade, all other alternatives are eliminated. Then, one copy of that median grade is removed from each remaining alternative's list of grades, and the new median is found, until there is an unambiguous winner. There are also many other voting systems, such as Plurality voting and Preferential voting. Most of them take the same drawback as that of Borda count, that is, it is usually not easy to allocate absolute scores to the alternatives in a consistent and fair way.

In order to overcome the difficulty of allocating absolute scores, Cohen *et al.* [106] developed an algorithm for generating an approximately optimal total order for all the alternatives from pair-wise preferences. The proposed methodology is a two-stage approach where the first stage learns a *preference function* PREF(u, v), which is a numerical measure of the certainty that u should be ranked above v. The preference function is a weighted combination of primitive preference functions obtained from ordering functions. An *ordering function* is a function f: $X \rightarrow S$, from the set of all alternatives X to a totally ordered set S, given by experts, and the *rank ordering*, a special kind of preference function, R_f is defined as

$$R_f(u,v) = \begin{cases} 1 & \text{if } f(u) > f(v), \\ 0 & \text{if } f(u) < f(v), \\ \frac{1}{2} & \text{otherwise.} \end{cases} \tag{4}$$

The final preference function is obtained in the form $\mathrm{PREF}(u,v) = \sum_{i=1}^{N} \omega_i R_i(u,v)$ by a weight allocation algorithm which uses the preference functions, or rank orderings, shown in Eq. (4) and provided weights ω_i (i=1, …, N). The second stage uses a greedy algorithm SCC-GREEDY-ORDER which is to assign each alternative v a potential value. Then the algorithm picks the alternatives one by one according to their potential values, and then an approximately optimal total order for all the alternatives is obtained according to the ranks from high to low.

This method provides a novel alternative ordering method which can obtain an approximately optimal total order. It, however, is essentially an indirect and more complex way for assigning score to each alternative, the potential value, than Borda's rule. It is also a bit unreasonable for assigning $1/2$ to the preference relation whenever there is no ordering relation between two elements without considering different causes.

Wang *et al*. [46] developed a new method for calculating the pair-wise preferences from the preference relations given by decision makers, which was applied by Augusto *et al*. [107] into situation assessment during disaster management. The main idea is to calculate the probability that each pair of alternatives should be placed in an order, which is then used as the preference function PREF(u, v) in Cohen's method to generate the potential value of each alternative. This probability is obtained by considering each preference as a sequence and calculating the number of all the common sub-sequences of the considered pair and every sequence in the set of preferences given by decision makers. All the probabilities are then fed to PREF(u, v) and the corresponding algorithm SCC-GREEDY-ORDER to generate the final approximately optimal total order. Let us consider the lower two preferences in Example 3 by omitting the equal preferences, which can be rewritten as a set of sequences: I={eac, dab; ab, ac, ad, ae}. Assuming equal weighting, we can calculate the probability G of each pair of movies should be placed in an order, e.g., $G(ab) = \frac{16}{nK}$, where K is a normalization factor, and n=6 is the number of sequences in I.

The method proposed in [46] has provided a new method for calculating preference value from a different point of view by taking each preference given by decision makers as a sequence, but it is not expressive under situations where some alternatives can be equally ranked as in Example 3, and the method for calculating the probability of a pair of alternatives should be placed in an order can be extended accordingly.

An interesting result given by Rossi *et al*. [47] is that "aggregating preferences cannot be fair", which is a generalization of Arrow's impossibility theorem for aggregating total orders [108]. This result is of course disappointing to some extent. The problem is: what does a "fair" aggregation mean. The requirements for a preference aggregating system to be "fair" given by in [47] are freeness, independence to irrelevant alternatives, monotonicity, and non-dictatorial, and the statement is: "*If there are at least two agents and three outcomes to order, a preference aggregation system cannot be fair if agents use partial orders with a unique top and unique bottom, and the result is a partial order with a unique top or bottom*." Fortunately, this is not a big problem under a fuzzy or qualitative context which can provide more flexibility [109]. In fact the aggregation result we need is one that can take all the opinions into account and reflect the opinions of most decision makers, that is, we are looking for a sound and acceptable consensus/collective result, not the best result. We can also try to look for the approximation of the optimal result although incompleteness and incomparability existing in preference aggregation [110].

4. Logic Based Decision Making

Logic is the foundation and standard for justifying or evaluating the soundness and consistency of the methods, including decision making methods [23]. In order to establish the rational reasoning approaches and intelligent support system to deal with both totally ordered information and non-totally ordered information, it is important and necessary to study the logical foundation with such kind of feature for them, which should be some kind of non-classical logical system [111].

Generally, logic can be used for modelling decision making problems in two different ways: syntactic and semantic. From the syntactic point of view, logic uses formulas and propositions to represent judgments from decision makers. For example, we consider a set of decision makers: E={e_1, e_2, …, e_n} whose judgments among a set of alternatives can be represented by the propositions of a logic system, p_1, p_2, etc. Such as, p_1 means that alternative 1 performs well in some specified property according to decision maker 2. The composite propositions, which are composed by the primitive propositions p_1, p_2, etc. with logical connectives $\neg$ (not), $\wedge$ (and), $\vee$ (or), $\rightarrow$ (if-then) and $\leftrightarrow$ (if and only if), can be used for modelling more complex judgments. Then different logical reasoning methods, such as MP (Modus Ponens) rule and fuzzy CRI (Compositional Rule of Inference) [24, 25], can be applied to reach the collective judgment. From the semantic side, the truth-value field of logic system, such as {0, 1} for classical logic or [0, 1] for fuzzy logic, is used for modelling the set of evaluations on the alternatives. Take Fig. 3 as a truth-value field example, the truth-value of p_1 is b means that the judgment of decision maker 2 on alternative 1 is *highly* true. This kind of truth-value can be used for modelling the uncertainties involved in the decision making process, and will change accordingly along with the syntactic inference process.

Mainly from the syntactic representation point of view, Das [112] developed a formal logic for reasoning about preferences by representing preference through the binary relation R among propositional formulae which represent the considered alternatives or actions. For example, $R(a, b)$ is interpreted as the alternative a is preferred to b. Wilson [92] developed a logic of soft constraints where the set of preferences is only assumed to be a partially ordered set, with a minimum element and a maximum element. This means that there are no additional restrictions and operations, which will restrict the representational power, needed for the set of preferences to form a lattice.

There are also some attempts to model decision making problems in logic framework by combining syntactic and semantic parts together. Among them, Benferhat *et al.* [16] proposed some reasoning methods with partial information by using extended possibilistic distribution in the framework of possibilistic logic. Namely, elements from a partially ordered set are associated with formulas or interpretations in the logic instead of numbers in [0, 1], and two definitions of possibilistic inference are presented by extending the one used in possibilistic logic. They [15, 113] also extended the possibilistic logic by defining new combination rules to aggregate multiple-source information, which provides a coherent way to represent and reason uncertain information from different sources. Inspired from hedge algebra and by analyzing semantic heredity of linguistic hedges, based on the extensive work on lattice implication algebras and the corresponding logic systems [114], Xu, *et al.* proposed linguistic truth-valued lattice implication algebra [115, 116], as simply shown in Fig. 3, for modelling ordinal linguistic information, and discussed the corresponding logic system [116, 117] and the approximate reasoning approaches based on it [118, 119]. Liu *et al.* [19] laid some basic ideas on lattice valued decision making, especially with linguistic information, along with some lattice structures for representing the ordinal linguistic information involved in decision making procedure. Lu *et al.* [120] adopted several simple temporal predicates into a linguistic-valued logic based reasoning system for dynamically modelling and aggregating information under uncertain qualitative situations, and applied this reasoning mechanism to some smart home applications.

Although these logic based methods are promising due to the strict theoretical foundation, there are still many efforts that need to be made in order to make them more applicable to real decision making problems, especially socio-economic areas, due to the complex structures and high computational complexity [98].

The book [5] gave a detailed introduction and analysis of some existing popular ordinal linguistic information processing approaches, including the above mentioned fuzzy ordinal linguistic approaches, algebraic based and logic based methods.

5. Conclusions and Perspectives

Ordinal information is usually involved in the process of decision making such as ordinal attributes and preference relations, and always appears as qualitative and partially ordered. Ordering based decision making, usually on how to rank alternatives based on given ordering information, is drawing much attention recently. Table 1 illustrates the strengths and weaknesses of some typical approaches related to ordering based decision making reviewed in this paper. The first five are preference based alternative ordering methods, and the rest three are mainly for qualitative information processing.

Table 1. The pros and cons of existing ordering based decision making methods

	Strengths	Weaknesses
Soft constraints	Expressive and powerful in computational machinery for reasoning about them [94]	Difficult on global combination of quantitative preferences [95]
CP-nets	Elicitation of CP-nets from users is very intuitive [96]	The complexity of reasoning with them [54, 95]
Borda count and other Social Choice approaches	Take all the opinions into account, intuitive and easy to implement [54]	Can only be used for total order, and absolute scores are usually difficult to decide [106]
Cohen's method	It can obtain an approximately optimal total order [106]	It can only deal with preferences in total ordering [46]
Wang's method	Intuitive and easy to use [46]	It is not expressive under situations where some alternatives can be equally ranked [107]
Fuzzy ordinal linguistic approach	Easy to use by manipulating directly on linguistic labels with indexes without underlying numerical approximation [32]	It can only deal with totally ordered information [5]
Hedge algebra	Expressive for modelling the semantic ordering relation among linguistic terms [43]	No further steps made into logic and information processing methods [5]
Logic based method	Strict theoretical foundation and direct reasoning about information without numerical approximation [111]	Complexity of the theoretical results makes it somewhat difficult for non-specialist users [98]

All the methods in Table 1 can deal with qualitative information except Soft constraints, but Borda's Rule, Cohen's method and Wang's method deal with qualitative information by assigning numerical score or transforming it into quantitative value. Fuzzy ordinal linguistic approach, Borda's rule and Cohen's method can only deal with totally ordered information as shown in Example 1, and Hedge algebra mainly focuses on the algebraic representation of ordinal linguistic information.

As discussed above, ordinal information in decision making always appears in qualitative form and partially ordered, while most ordering based decision making methods can only deal with totally ordered information as shown in Examples 1 and 4. Although there are already some methods which can deal with partially ordered qualitative information, most of them usually simply transform partially ordered information into totally ordered structure, and qualitative information into quantitative scale, which are time consuming and will cause loss of information. How to aggregate the ordinal evaluations provided by different decision makers, always take different forms (mainly partially ordered) as shown in Example 3, to get a "fair" final decision result is still an ongoing and open research direction. There are some potential solutions or research directions for this problem, such as:

1) A new algebra-oriented structure should be developed to represent ordinal information in decision making. This kind of structure should be able to model partially ordered information, along with additional operations such that direct computations/reasoning between ordinal terms is possible. This computations/reasoning process should have much lower computational complexity than that of CP-nets.

2) Ordering based decision making should be formalized in a logic framework. The process of decision making which is to draw some conclusion based on given information, can be essentially interpreted as a reasoning process. Logic serves as the most important foundation and standard for justifying or evaluating the soundness and consistency of reasoning methods. Therefore, it is necessary to develop ordering based decision making approach based on approximate reasoning from the logical point of view.

3) New aggregation approach should be developed for dealing with different kinds of uncertainties accompanying the process of ordering based decision making. These kinds of uncertainties consist of the degrees of credibility or belief associated with the preferences given by different decision makers, consistency level of different opinions, and so on, which are very common in real decision problems, especially in the complex and dynamic socio-economic environment.

The complexity and dynamics of real world decision making problems require more advanced tools to develop more appropriate decision making approaches which can successfully deal with partial orders, adaptability, uncertainty and more solid theoretical foundation. It is clear from the research reported here that, due to the challenging problems that remain to be solved, there is still substantial work to be done on ordering based decision making to see its prosperity both in theoretical researches and practical applications. However, given its importance for a number of important areas and real world applications, we are optimistic these challenges will attract considerable attention and be eventually overcome.

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