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DEVELOPMENT AND APPLICATION

OF LEARNING CURVES

by

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A Thesis submitted to the Council for

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ACKNOWLEDGEMENTS

Some three years have passed since the Post Office initiated the Research Project which has resulted in this thesis. During that time I have had the privilege of working with and observing many people, all of whom gave of their best to ensure a successful conclusion to the Project. At this point in time I feel rather inadequate in my attempts to express my thanks, and hope that those not mentioned by name will appreciate the thought, if not the deed.

My first thanks must go to the Post Office, who, through Mr. N. Gandon initiated the Training Research Project. His colleague; our Project Manager Mr. Frank Rippon, assisted us in many ways, not the least being his ability to convince everyone that what Dick Lamb and myself were attempting to do was relevant to training!

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Especial thanks to Dick Lamb, my colleague in this Research - his comments and analyses proved of great value in this study. In addition, while the results of the tape recorded tests for telephonists are not reported here, I must thank Dick's wife, Maureen; my wife, Eirwen, and Mrs. F. Gaunt for their enthusiastic help in their production.

Finally, it is only fitting that I thank all the Post Office staff who assisted us - the trainees, operators, supervisors, exchange superintendents - without their help and willing assistance this thesis could never have been written.

DECLARATION

I declare that with the exception of some experimental results made available by R. T. Lamb and other observations taken from the literature, the work submitted in this thesis is the result of my own investigation and has not been submitted in candidature for any other degree.

Candidate

SUMMARY

As part of a Research Project into the Cost Effectiveness of Training, various experiments were held in Telephone Exchanges in the Midland , Eastern and London Telecommunications Regions. The purpose of the experiments was to investigate the work load of telephonists, to see how the amount of time spent on the elements of the task done might vary as the training of the telephonists proceeded and also to attempt to compare two methods of training. Data from these experiments and from other sources in the literature was used to compare the efficiency of a selection of models of learning.

The method of comparison was based on an extension of an iterative 2-parameter curve fitting algorithm which uses a Taylor Series approximation to the function of the model of learning investigated.

The resulting analysis allowed a tentative choice of what might be called the "best" model, which was then used in a more detailed examination of further data obtained on telephonists. In the event, the curve fitting analysis was found to be complex, as was the apparently simple task of "telephonist". Time did not allow an extension of the study into other tasks performed by Post Office personnel.

Conclusions Drawn

Part I Studies of Data Available in the Literature

(1) The model which resulted in the best fit most consistently was the Wiltshire Model. However the Wiltshire Model only gave the solution in 31 of the 88 studies contained in the first part of the thesis. The second order model was the most regular method of obtaining the curve fit working in 87 of the 88 cases.

(2) The de Jong model and Logmathematical Models gave consistently the worst fits.

(3) Little difference could be detected in the remaining models.

(4) The most practical model (because the parameters may be defined in understandable terms) is the time constant model sometimes known as the Bevis model. This model worked in 77 of the 88 studies. The second order model is a logical extension of the Bevis model, and may fit the data more accurately, but requires a more complex curve fit procedure.

Part II Studies on GPO Data

(5) Despite the apparent advantages of the Bevis model, the accuracy with which the Bevis Model predicts the parameter values is not good enough to consider its use for a comparison of different training methods which might be used by the Post Office and hence allow an evaluation of the cost effectiveness of training. This may be due to insufficient or inaccurate data; or the model may not be a true reflection of the learning process that occurs.

(6) Telephonists learn to do their work in two stages, a training stage and an experience gaining stage, which may be defined by two learning curves.

(7) The method of evaluating the work done by a trainee telephonist in the early stages of training is inaccurate. The inaccuracy is probably due to the high variability in the presentation of calls to the trainee.

(8) The problem of curve fitting to tasks which are not truly repetitive but contain elements which are repetitive, is complicated because of the difficulty of establishing an accurate performance measuring system. The cost of the work needed to do this is likely to be prohibitive.

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1. INTRODUCTION

1.1. The Training Research Project.

During the summer of 1971, the Post Office approved the setting up of a Research Project to investigate the Effectiveness of Training. Two members of Post Office staff were recruited and commenced academic and experimental work in the following November at Hendon College of Technology (now part of Middlesex Polytechnic). Extracts from the Terms of Reference for the study are given in Appendix A.

At an early stage of the research it was agreed that although the researchers would work together on experiments of common interest, the emphasis and/or interpretations they might place on the results obtained could usefully be guided in different directions. As a result, some of the experiments quoted in this thesis are the combined efforts of the two researchers, while others are individual attempts to prove a particular point in question. Most of these experiments are described in detail in Lamb's thesis[1] which deals with the Evaluation of Telephonist Training. This thesis, which concentrates on a comparison of learning curve models and the possible application of one model to the evaluation of training effectiveness does not go into such detail, to avoid duplication and also to keep the size of the thesis within reasonable bounds.

1.2. The Nature of the Problem.

Learning, or Progress Curves, have been in use for some 35 years as indicators of the improvement of skill in repetitive tasks. Originally they were developed to measure Industrial progress, i.e. the improvement in performance or the reduction in cycle times in production with passage of time. The development of Learning Curve theory was thus due to technologists and it was only at a later stage that psychologists made attempts to derive forms of equation which could be used to depict individual learning.

The pressure on the technologists was an economic one, they wished to establish reasonable estimates of future manufacturing costs so that competitive tendering was possible. In doing so, they were concerned with a mass learning effect, i.e. how the works personnel would improve their skill as a group while manufacturing several thousand or more items over periods of months, if not years. Consideration of individual performance, in which learning takes place in days or weeks for a simple repetitive task, (which would normally be an element of the complete industrial process), did not enter into their calculations.

Such models that were developed by the technologists were empirical - no formal theory of the acquisition of skill was used to assist in the derivation of the learning equations used to depict the model. Psychologists, on the other hand, used formally developed theories to derive models for individual learning and used truly repetitive tasks in their experiments. No mention of fitting their models to industrial work has been noted, although the large number of variables which can affect the observations such as motivation to do well, variation in presentation of the task, individual differences and observational error have been discussed.

Many learning curve models have therefore been proposed, and all have some factors in their favour. While the models developed by the technologists have been used to depict learning over a long period, and those developed by psychologists have been used to depict the learning of simple tasks, little attempt appears to have been made to fit curves to training data. Can a "best" model for such a purpose be selected from those available? If such a selection can be made, can the model then be used to compare methods of training (and hence allow a cost effectiveness study to be made) say by examination and analysis of the parameters? This study attempts an answer to these questions.

2. HISTORY OF PREVIOUS WORK.

2.1. Introduction.

Very many attempts have been made to fit curves to data relating to learning. The general approach has been empirical, in that researchers appear to have made personal judgements on what curve will fit (usually on the basis of fitting by inspection) and then tried it out on a mathematical basis.

Formulae have also been developed on a psychological basis. The discussion which follows will list the equations considered in this research, in historical sequence, and show that some suggested formulae are based on different forms of the same equation.

2.1.1. Robertson's Equation.

Morecombe[2] states that the first equation proposed as suitable to fit to learning data was that of Robertson in 1915. The equation is

$$\log \left\{ \frac{y_i}{a - y_i} \right\} = k\,(x_i - x_o) \qquad 2.1.1.(a)$$

No definition of y_i, a, k, x_i and x_o is given but it is presumed that

y_i is cycle time for the i^{th} operation

a, k are constants

x_i is the x^{th} repetition of the task

x_o is the first observation point.

If some algebraic manipulation is done on the equation, we get:

$$\ln\left\{\frac{y_i}{a-y_i}\right\} = k(x_i - x_o) \qquad 2.1.1.(b)$$

$$\frac{y_i}{a-y_i} = e^{k(x_i-x_o)} \qquad 2.1.1.(c)$$

$$y_i = (a-y_i)\, e^{k(x_i-x_o)} \qquad 2.1.1.(d)$$

$$y_i + y_i e^{k(x_i-x_o)} = a\, e^{k(x_i-x_o)} \qquad 2.1.1.(e)$$

$$y_i\left[1 + e^{k(x_i-x_o)}\right] = a\, e^{k(x_i-x_o)} \qquad 2.1.1.(f)$$

$$y_i = \frac{ae^{k(x_i-x_o)}}{1 + e^{k(x_i-x_o)}} \qquad 2.1.1.(g)$$

and dividing the numerator and denominator by $e^{k(x_i-x_o)}$

$$y_i = \frac{a}{\left(\frac{1}{e^{k(x_i-x_o)}} + 1\right)}$$

$$= \frac{a}{1 + e^{-k(x_i-x_o)}} = \frac{a}{1+e^{kx_o-kx_i}} \qquad 2.1.1.(h)$$

and as kx_o will be a constant (say b)

then $$y_i = \frac{a}{1 + e^{b-kx_i}} \qquad 2.1.1.(i)$$

which is the form of the Pearl and Reed equation suggested in 1925 and can also be shown to be the Bevis equation in a different form (to be discussed later).

2.1.2. Moore's Equation.

Morecombe[3] also quotes Moore's equation, suggested in 1932.

This is $\log y_i = a + b.c^{x_i}$ 2.1.2.(a)

which may be used to define the variation of output or cycle time, according to the signs of the parameters. However, if we consider the logistic curve $y_i = a - b.c^{x_i}$ 2.1.2.(b)

(where the parameters are positive and which defines output/time) and let $Y_c = (a-b)$ and $Y_f = b$

Then $Y_c = a - Y_f$

and $a = Y_c + Y_f$

$$y_i = Y_c + Y_f - Y_f.c^{x_i}$$

$$= Y_c + Y_f\left[1 - c^{x_i}\right] \quad 2.1.2.(c)$$

If c is now made equal to $e^{-1/\tau}$, $c^{x_i} = (e^{-1/\tau})^{x_i}$

$$y_i = Y_c + Y_f\left[1 - e^{-x_i/\tau}\right] \quad 2.1.2.(d)$$

which is the Bevis equation, with x_i substituted for t_i.

Thus Moore's equation when used to define output data is effectively a Bevis equation plotted to a logarithmic y-scale.

2.1.3. Wright's Equation.

A different form of equation was proposed by Wright[4] in 1936. Morecombe[5] quotes this article on factors affecting the cost of airplanes and shows the equation as

$$\bar{t} = t_1 n^{-m} \qquad 2.1.3.(a)$$

where $\bar{t}$ = the cumulative average direct labour manhours for any quantity n

t_1 = the number of direct labour manhours to manufacture the first unit produced

n = the number of completed units

m = an exponent (typically of value .322)

Now let $\bar{t} = Y_i$, $n = X_i$, $m = n$, $t_1 = A$ (a constant)

then $\bar{t} = t_1 n^{-m}$ is transformed to

$$Y_i = AX_i^{-n} \qquad 2.1.3.(b)$$

Note that Y_i or $\bar{t}$ is calculated from $\dfrac{\sum_1^N t_i}{x_i}$

so that the curves, if plotted, are to a modified y-scale.

2.1.4. Crawford's Equation.

Crawford (quoted by Morecombe)[6] seems to have felt that the equation

$$t_n = t_1 n^{-m} \qquad 2.1.4.(a)$$

fitted his firm's experience better,

where t_n = the unit cost, or the direct labour hours for unit number n.

t_1 = direct labour hours for the first unit

n = number manufactured

m = an exponent (still typically of value .322)

Converting Crawford's equation for use on x/y axes by letting $t_n = y_i$, $t_1 = A$, $n = x_i$, $m = n$ gives the same form as before, i.e. $y_i = Ax_i^{-n}$ 2.1.4.(b)

2.1.5. de Jong's Equation.

It was not until 1957 that de Jong[7,8,9] proposed a further modification to Wright's and Crawford's equations. He came to the conclusion that there existed an "incompressible" component in the cycle time taken to complete an operation. Conversely, this also implies a maximum output above which a worker would not be able to go. In his series of articles, de Jong considered the reduction in cycle time of experienced workers in many industries and came to the conclusion that an equation of the form

$$y_i = t_1 M - t_1(1-M)\, x_i^{-n} \qquad 2.1.5.(a)$$

best expressed the reduction in cycle time, where

y_i = cycle time

t_1 = time required for the first cycle of a batch

M = the factor of incompressibility ($0 \leq M \leq 1$)

n = the exponent of reduction.

Now let $B = t_1 M$; $A = -t_1(1-M)$

$$y_i = B + A x_i^{-n} \qquad 2.1.5.(b)$$

This equation is still in a form which expresses the reduction in cycle time, for when $x = 1$, $y = B + A$ and when $x = \infty$, $y = B$.

If the sign of A is changed

$$\text{i.e.} \quad y = B - Ax^{-n} \qquad 2.1.5.(c)$$

then when $x = 1$, $y = B-A$

and when $x = \infty$, $y = B$

which form is suitable for expressing output as a function of x.

2.1.6. American Government Equation.

Nadler and Smith[10] quote a variation on the same theme. After extensive study by the Stanford Research Institute it was found that $y_i = a(x_i + B)^n$ 2.1.6.(a)

appeared to be a more suitable equation to fit to progress functions or learning curves. In that equation

y_i = direct manhours per unit

x_i = the cumulative number accepted

a = the cost of the first unit when $B = 0$

n = a reduction exponent

B = a constant which could be expressed as the number of units theoretically produced prior to the first unit acceptance.

Note again that the equation may be modified to depict output or cycle times.

i.e. $$y_i = a\left[x_i + B\right]^{-n}$$ 2.1.6.(b)

for cycle time data

$$y_i = a\left[x_i + B\right]^{n}$$ 2.1.6.(c)

for output data.

2.1.7. Glover's Equation.

Glover[11,12] suggests an equation of the form

$$\Sigma y_i + c = a(\Sigma x_i)^m$$ 2.1.7.(a)

and gives an extensive mathematical treatment which shows that given certain conditions the equation reduces to the same form as Wright's equation. For the purposes of this analysis let $\Sigma y_i = Y_i$, $\Sigma x_i = X_i$, $m = -n$, $c = -B$, $a = -A$.

Hence $$Y_i - B = -A X_i^{-n}$$ 2.1.7.(b)

$$Y_i = B - A_i X^{-n}$$ 2.1.7.(c)

Therefore this is de Jong's equation to a different scale.

2.1.8. Wiltshire's Equation.

Recently, Wiltshire[13] has suggested an equation of the form

$$y_i = ke^{-\alpha x_i^n} + c$$ 2.1.8.(a)

where y_i = cycle time for i^{th} cycle

x_i = no. of repetitions of cycle and

k, α, n, c are constants.

He gives a detailed series of results based on the cycle times of the elements of assembly tasks and also the cycle times for the complete assembly. This equation is an innovation in that it is a new form. It cannot be manipulated algebraically into a form discussed previously.

2.1.9. Bevis's Equation.

Bevis[14] considered some of the previous models discussed, but also suggested the model

$$y_i = Y_f \left(1 - e^{-\frac{(x_i - 1)}{x_f}}\right) + c \qquad 2.1.9.(a)$$

where y_i = rate of production

x_i = time in days

c = initial rate of production

x_f = the time constant for a particular curve

Y_f = Difference in the rate of output between the initial rate of output 'c' and the maximum rate of y_i.

Hitchings[15] investigated the modified form of the above equation

$$y_i = Y_c + Y_f \left(1 - e^{-t_i/\tau}\right) \qquad 2.1.9.(b)$$

where Y_c = c

t_i = x_i

$$\tau = x_f$$

It is this form which is of interest, for whereas Bevis assumed that the initial output observed was the 'constant' Y_c, Hitchings accepted that that initial value could be in error, and attempted an iterative curve fitting method to sets of Bevis's data, based on the variation of the two parameters Y_f and τ as Y_c was given set values. The iterative technique developed will be discussed later. Now consider the form of equation

$$y_i = \frac{k}{1 + e^{a+bx_i}} \qquad 2.1.9.(c)$$

(the Pearl and Reed curve mentioned earlier)

$$\frac{1}{y_i} = \frac{1 + e^{a+bx_i}}{k} \qquad 2.1.9.(d)$$

$$= \frac{1}{k} + \frac{e^{a}}{k} . \quad e^{bx_i} \qquad 2.1.9.(e)$$

which is of the form

$$\frac{1}{y_i} = A + Be^{cx_i} \qquad 2.1.9.(f)$$

Now let $A = Y_c + Y_f$, $B = -Y_f$

then $$\frac{1}{y_i} = Y_c + Y_f - Y_f e^{cx_i} \qquad 2.1.9.(g)$$

$$= Y_c + Y_f \left[1 - e^{cx_i} \right] \qquad 2.1.9.(h)$$

which is the Bevis Equation with $c = -1/\tau$, and the inverse of y_i. Hence the Pearl and Reed equation, when used on cycle time data, is the inverse of the Bevis Equation.

2.2. An Alternative Approach: Psychological Models of Learning.

In the same period that researchers were proposing various empirical models to account for variations in performance during learning, other researchers were attempting to develop models, and hence equations, based on a psychological approach to the problem. Restle and Greeno[16] give a modern analysis of several models, two of which are of interest from the point of view of this study.

2.2.1. A model for replacement learning.

Without going into the detailed theory used to develop the equation, it can be said that the replacement model is based on the idea that information related to the activity being learnt replaces information not related to that activity and that "learning" thus follows the equation

$$P_n = a - (a-b)(1-\theta)^{n_i-1} \qquad 2.2.1.(a)$$

where P_n = the probability of success on the n^{th} trial

a = the maximum probability of success

b = the initial probability of success

n_i = No. of trials

θ = a proportion.

Over the series of trials, once the probability of success has reached its maximum value, we have also reached the maximum possible performance of the subject, i. e. maximum output. Hence, replacing probability by performance (or output) will not affect the nature of the work. Note that the equation relates output (o/p) to cumulative output (Σ o/p) since n_i = total number of trials. Now we have

$$P_i = o/p_i = y_i = a - (a-b)(1-\theta)^{n_i-1} \qquad 2.2.1.(b)$$

where o/p_i is the output on the i^{th} trial.

Let $a = Y_c + Y_f$

and $b = Y_c$

$$y_i = Y_c + Y_f - (Y_c + Y_f - Y_c)(1-\theta)^{n_i-1} \qquad 2.2.1.(c)$$

$$= Y_c + Y_f(1 - (1-\theta)^{n_i-1}) \qquad 2.2.1.(d)$$

Let $(1 - \theta) = e^{-1/\tau}$

$$y_i = Y_c + Y_f(1 - (e^{-1/\tau})^{n_i-1}) \qquad 2.2.1.(e)$$

$$= Y_c + Y_f(1 - e^{-\frac{n_i-1}{\tau}}) \qquad 2.2.1.(f)$$

Compare this equation with equation 2. 1. 9. (b)

$$y_i = Y_c + Y_f(1-e^{-t/\tau}) \qquad \text{the Bevis Equation}$$

Thus the replacement model is very similar to that of Bevis.

2. 2. 2. A model for accumulative learning.

Restle and Greeno[17] also discuss a model for accumulative learning, in which all information on the activity being learnt is accumulated. This results in the following equation, which may be related to performance as well as probability of success at trial n.

$$P_n = \frac{b + \theta a(n_i - 1)}{1 + \theta(n_i - 1)} \quad 2.2.2.(a)$$

where P_n, a, b, θ, n_i stand for the same as before.

Set $(n_i - 1) = X_i$

$$y_i = \frac{b + \theta a X_i}{1 + \theta X_i} = b\frac{\left[1 + \frac{\theta a}{b} . X_i\right]}{1 + \theta X_i} \quad 2.2.2.(b)$$

$$= \frac{b(1 + \theta X_i - \theta X_i + \frac{\theta a}{b} . X_i)}{1 + \theta X_i} \quad 2.2.2.(c)$$

$$= b - \frac{b\theta X_i + \theta a X_i}{1 + \theta X_i} \quad 2.2.2.(d)$$

$$= b - \frac{\theta X_i \left[b - a\right]}{1 + \theta X_i} \quad 2.2.2.(e)$$

$$= b - \frac{(b-a)}{1 + \frac{1}{\theta X_i}} \quad 2.2.2.(f)$$

$$= b - \frac{1}{\frac{1}{(b-a)} + \frac{\theta}{(b-a)} X_i} \quad 2.2.2.(g)$$

which is of the form $y_i = b - \frac{1}{c + g X_i}$ 2.2.2.(h)

(the equation to a mathematical hyperbola). Note again, however, that as n is the total number of trials, we can also plot o/p against Σo/p, to obtain our learning curve.

2.2.3. Modification of the above equations to depict learning to a base of time.

Restle and Greeno[18] expand their analysis to show how the above equations may be modified to account for varying speeds of learning. The resulting equations may be used to depict the variation of output with time during the learning process, but need four parameters to do so. Computer analysis in those two cases was not attempted.

2.3. Other Mathematical Forms of Equations to Fit Learning Data.

Obviously there are an infinite variety of mathematical equations which might be used to define learning data. Ezekiel[19] discusses some forms which are basically geometrical and trigonometrical. In this study, no attempt has been made to justify the use of the following equations to depict such learning data, some, in fact, were not pursued, due to their being so similar in form to other equations which were studied.

2.3.1. Modification to the equation for the Basic Hyperbola.

This equation is $y_i = b - \dfrac{1}{c + gx_i}$ 2.3.1.(a)

where b, c and g are constants. It is a modification to the basic form of hyperbola commonly quoted $(y_i = b - \frac{c}{x_i})$ and is similar in form to the accumulative model discussed earlier.

2.3.2. The same modification to a logarithmic scale.

This equation is $\log y_i = b - \dfrac{1}{c + gx_i}$ 2.3.2.(a)

and may be a better "fit" to the data. Other forms such as

$$y_i = b - \frac{1}{c + g \log x_i} \quad 2.3.2.(b)$$

$$\log y_i = \frac{1}{c + g \log x_i} \quad 2.3.2.(c)$$

were not pursued.

2.3.3. A mathematical form using hyperbolic expressions.

This form was of interest because it offered the possibility of curve fitting to data which had previously given problems. The data related to "slow" learners and commonly gave an "S" curve which has been noted previously. Unfortunately, the use of 4 parameters eventually resulted in computing problems, and the model was not pursued. The equation proposed was :

$$y_i = A + B \tanh (Dx_i - C) \quad 2.3.3.(a)$$

where A, B, C and D are constants.

2.3.4. A cubic model.

Thomas[20] has quoted Miller's equation

$$y_i = A + BX_i + CX_i^2 + DX_i^3 \quad 2.3.4.(a)$$

where A, B, C and D are constants and

X_i = cumulative number of units produced.

and suggested that a regression analysis might be used to calculate the parameters.

2.3.5. Gompertz's Equation.

Morcombe [21] quotes Stanley's reference to the Gompertz curve and analyses it in some detail. The form is new to this discussion, although Wiltshire's equation has some resemblance to the form of the equation, which is

$$y_i = ka^{b^{x_i}} \qquad 2.3.5.(a)$$

where k, a and b are constants.

2.4. A Second Order Model.

As a result of considering the nature of the preceding forms of equation, the author felt that an attempt to develop a learning curve equation which would be a second order, rather than a first order equation, was justified.

2.4.1. Three hypothetical experiments.

The development of the equation may best be explained by considering the following three hypothetical experiments. In all the experiments, the purpose is the same, to get the subject S to sort out a deck of playing cards into red and black piles as quickly as possible. However, S is told before commencing the experiment to sit down at a table and wait for instructions. When the instructions are given to him, he is told, he is not allowed to ask any questions of his instructor.

Consider the situation that would occur if E (the experimenter) then came in and said to S "Πάρε σέ παρακαλῶ τήν τράπουλα καί βάλε τά χαρτοπαίγνια, ὅσον τό δυνατόν πιό γρήγορα, σέ δύο σειρές· ἡ μία σειρά κόκκινα καί ἡ ἄλλη σειρά μαῦρα χαρτιά."

Presumably S, unless he understood Greek, would be at a complete loss on what to do.

Similarly, the situation that might occur if E came in and said to S, "Please take this pack of cards and sort them out" in English is that S would perhaps sort them out into suits. E would then say, in English "That's incorrect, please shuffle the cards and do it differently!" After shuffling the cards, S would then make a second, and perhaps several more attempts before sorting out the cards into the correct categories. At that stage E would say "That's correct, please shuffle the cards and do it again, but more quickly" and S would then proceed to repeat the process until E was satisfied that full proficiency had been attained.

In the third experiment, E would say to S "Please take this pack of cards and sort them out into piles of red and black cards as quickly as possible" in English, whereupon S would proceed to do the experiment (hopefully in the correct manner!), repeating as frequently as necessary.

Now what are the differences in the three experiments? Experiments 1 and 3 do not differ in the amount of information given to S, because the same presentation was used to tell S what to do, yet S would presumably do far worse in Expt. 1 than in Expt. 3. Experiment 2 had less information to begin with and then built up to the same content as 1 and 3, as S's understanding of what was required of him increased and, in the same way, S's performance increased. Because S can do relatively badly at the commencement of Expt. 2, there is an implication that there is a lower limit to the amount of information needed before even a simple task can be done correctly. Yet this is not the complete explanation, because in experiment 1, S was given all the necessary information, albeit in a form which S may not have understood (i. e. in Greek).

This, it seems, is the crucial point, that the performance of a task does not depend solely on the amount of information available, but also on the understanding of that information.

2.4.2. A possible relationship between Understanding and Information.

Let us assume, for the moment, that we can measure "understanding" on a U-scale - how does U vary with I (information)? What we can say is that while there can be some understanding if information related to the operation of the task is being received, if most of that information is changing from one cycle to another, then only confusion results. Once a "pattern" has been established and the information received from one cycle to another is relatively constant in content, then reinforcement learning may take place.

The total amount of understanding measured could therefore be dependent not only on the amount of information received, but on the rate of change of that information. Mathematically one might say

$$U = kI + k' \frac{dI}{dt} \qquad 2.4.2.(a)$$

2.4.3. A possible relationship between Information and Output Performance.

How does our subject gain the information from which he attains understanding? Crossman[22], suggests a theory of trial and error learning based on an earlier theory of Thorndike and also discusses what an operator measures to account for the acquisition of speed skill. He comes to the conclusion that the internal measurement of time by the operator is unlikely, but suggests that the work done by the operator is a possible suitable alternative.

A study of data from Pickering and MacAulay[23] also suggests that trial and error learning is taking place.

For example, in Table I, it can be seen that the cycle time for the complete operation is showing a general trend downwards, yet the elemental times do not necessarily show this trend. There are even large increases in some elemental times (over the period of trials), which nevertheless allow a reduction in the total cycle times because other elemental times are reduced by a larger total amount. It is as if the subject is able to make an assessment of his performance, as he varies his method of "grasping", "moving" etc., the variation being done by trial and error.

TABLE I

"200 TRIALS ON THE PURDUE PEGBOARD FOR SUBJECT 6"

Trial	Element Reach	Element Grasp	Element Move	Element Position	Cycle Time Total
1	.0905	.5564	.3691	.7136	1.7295
2	.0941	.5109	.3795	.6359	1.6205
3	.0727	.5300	.3477	.6750	1.6255
5	.0687	.5543	.3538	.6495	1.6262
10	.0700	.5282	.3732	.6023	1.5736
15	.0532	.5963	.2716	.6563	1.5774
25	.0900	.4873	.3773	.7214	1.6759
50	.1132	.5527	.3427	.5782	1.5868
100	.0436	.4441	.3627	.6200	1.4705
150	.0600	.4195	.3545	.4836	1.3177
200	.0857	.3513	.3474	.5053	1.2878

Morcombe,[24] as a result of his simulated assembly task experiment, also came to the conclusion that the incentive to improve on cycle time resulted in the successive selection of better methods by "trial and error".

So we come to the conclusion that some mechanism is at work which allows comparison of performance, without being certain what that mechanism is. Some relationship must exist between the performance (or output) of the subject, and the information he obtains from his performance of the task.

Once again, if the subject is skilled, he completes many operations in a given time, and thus generates large amounts of information. In addition, if he makes a mistake, and so, for a short period his output is dramatically reduced, he takes particular note of that mistake, vowing "not to do that again!" (don't we all!). Thus the mathematical connection between output and information could be :-

$$I = k'' \, o/p + k''' \frac{d\, o/p}{dt} \qquad 2.4.3.(a)$$

i.e. $I \propto$ output and also $\propto$ rate of change of output.

$$\text{whence} \quad \frac{dI}{dt} = k'' \frac{d\, o/p}{dt} + k''' \frac{d^2\, o/p}{dt^2} \qquad 2.4.3.(b)$$

and from equation 2.4.2.(a) $\left[U = kI + k' \frac{dI}{dt} \right]$

$$U = kk'' \, o/p + kk''' \frac{d\, o/p}{dt} + k'k'' \frac{d\, o/p}{dt} + k'k''' \frac{d^2\, o/p}{dt^2}$$

2.4.3.(c)

$$\frac{U}{kk''} = o/p + \frac{(kk''' + k'k''')}{kk''} \frac{d\,o/p}{dt} + \frac{k'k'''}{kk''} \frac{d^2\,o/p}{dt^2}$$

2.4.3.(d)

This equation is a second order differential equation, and a solution may be found using methods commonly applied in the analysis of feedback control systems. In this case, we assume learning is taking place and that the final output will be at a steady value Y_f, having started (at t = 0), at a value = 0.

The experimenter, by asking his subject to do the task "as quickly as possible" may be said to be demanding a step increase in output from his subject of value Y_f.

From equation 2.4.3.(d) the characteristic equation can be written as

$$p^2 + \frac{D}{J} p + \frac{k}{J} = 0 \qquad 2.4.3.(e)$$

where $D = kk'' + k'k''$

$J = kk''$

$k = k'k'''$

The solutions to this equation in this form are given by Chestnut and Mayer[25] as

(a) $n > 1$

$$y_i = Y_f - \frac{Y_f}{2\sqrt{n^2-1}} \left[(n + \sqrt{n^2-1})\, e^{-(n-\sqrt{n^2-1})\,\omega_o t_i} - (n - \sqrt{n^2-1})\, e^{-(n+\sqrt{n^2-1})\,\omega_o t_i} \right] \qquad 2.4.3.(f)$$

which is termed the over-damped condition.

(b) $n < 1$

$$y_i = Y_f - \frac{Y_f}{\sqrt{1-n^2}} e^{-n \omega_o t_i} \sin \left[\sqrt{1-n^2}\, \omega_o t_i + \phi \right] \qquad 2.4.3.(g)$$

$$\text{where } \phi = \tan^{-1} \frac{\sqrt{1-n^2}}{n}$$

which is the underdamped condition

and

(c) $n = 1$

$$y_i = Y_f - Y_f(1 + \omega_o t_i)\, e^{-\omega_o t_i} \qquad 2.4.3.(h)$$

which is the critically damped condition.

In all the equations $\omega_o = \sqrt{\frac{k}{J}}$ and $n = \frac{D}{2\sqrt{kJ}}$

If the initial condition is assumed to have some value, it is only necessary to include the term $+Y_c$ in all the equations.

The solutions given are second order equations which connect output with time. Chestnut and Mayer show the effect on the transient part of the curve as n is varied, and it appears that, for $n > 1$, the resulting curve could simulate the S-type learning curve which is occasionally encountered.

At a later stage in this study, it was decided to concentrate on only that equation which had 2 parameters, for the addition of a constant value Y_c to the equation then increased the number of parameters to 3. The model selected thus became the critically damped model:

$$y_i = Y_c + Y_f(1 - (1 + \omega_o t_i)\, e^{-\omega_o t_i}) \qquad 2.4.3.(i)$$

The similarity with the Bevis model is obvious.

3. WHICH MODEL?

3.1. A Historical/Computational Review.

The reader will have observed that Chapter 2 dealt with learning curve models from a historical viewpoint - the models were dealt with in rough chronological order. One can also see that the computing requirements of the day also had some influence, for Robertson's, Moore's, and Pearl and Reed's models would be computationally cumbersome when dealing with large amounts of data on hand calculating machines.

This, no doubt, led to the general acceptance of Wright's model when he proposed it in 1936. Based on aircraft production figures, it was quite a good first order approximation to the learning curve generated by a large number of people employed on a production line. In addition, by use of log/log scales, straight line fits could be obtained, allowing good prediction for relatively long periods ahead.

de Jong[26], however, realised that such an approximation was not appropriate to shorter term learning curves, because the mathematical implication of the equation $y = Ax^{-n}$ is that as x increases, so y goes to zero, and one would not expect a production worker to reduce his cycle time to zero!

Thus de Jong postulated the model

$$y_i = t_1 M - t_1(1-M)x_i^{-n}, \qquad \text{equation 2.1.5.(a)}$$

which has been shown to be of the form

$$y_i = B + Ax_i^{-n}$$ equation 2.1.5.(b)

From the computational viewpoint this equation is still difficult to fit when using hand calculating machines so that it is quite relevant to note that it is only recently that alternative forms of learning curve, having the same features as the de Jong model (asymptotic approach to a finite value) have been proposed.

Modern computors, of course, make rapid calculating facilities available, so that it seems opportune to discuss the mathematical requirements of such types of learning curve and attempt to establish that model which gives the best fit.

3.2. The Connection between the Shape of the Learning Curve and the Parameter Values.

If one considers the information available, it can be seen that the shape of the learning curves predicted by most of the learning curve models is hyperbolic and asymptotic. Because of this, it is possible to define more exactly the nature of the parameter values. As an example consider the model

$$y_i = Y_c + Y_f\,(1-e^{-t_i/\tau})$$ [equation 2.1.9.(b)]

it can be seen that Y_c is a constant value at $t_i = 0$ and that Y_f is a transient value which adds to Y_c as $t_i \rightarrow \infty$. When t_i reaches ∞, then $y_i = Y_c + Y_f$ (its maximum value).

The shape of the curve is then assumed to be as in diagram 1

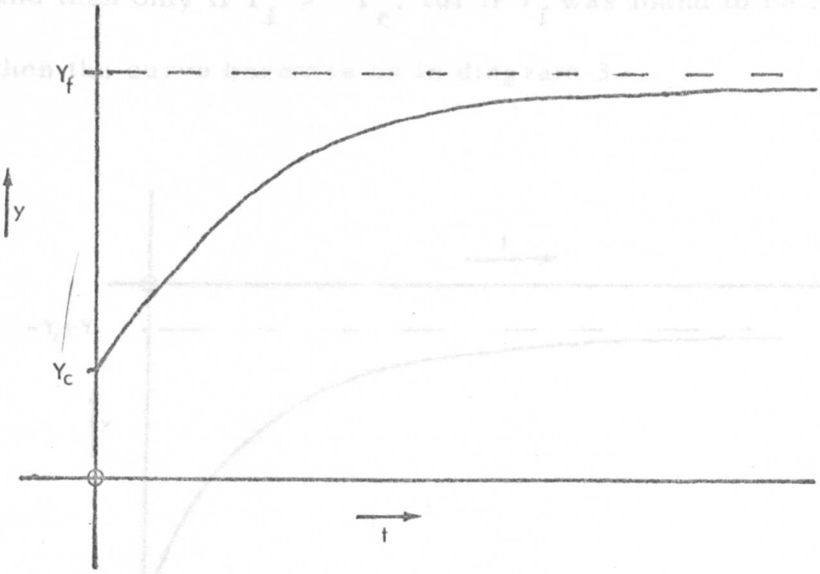


DIAGRAM 1

SHAPE OF BEVIS LEARNING CURVE WITH Y_c + ve, Y_f + ve AND τ + ve

"Assumed" because it has not yet been defined whether Y_c and Y_f are positive or negative numbers. If, on completing a curve fitting programme, it was found that Y_c was negative, the curve would be of shape shown in diagram 2.

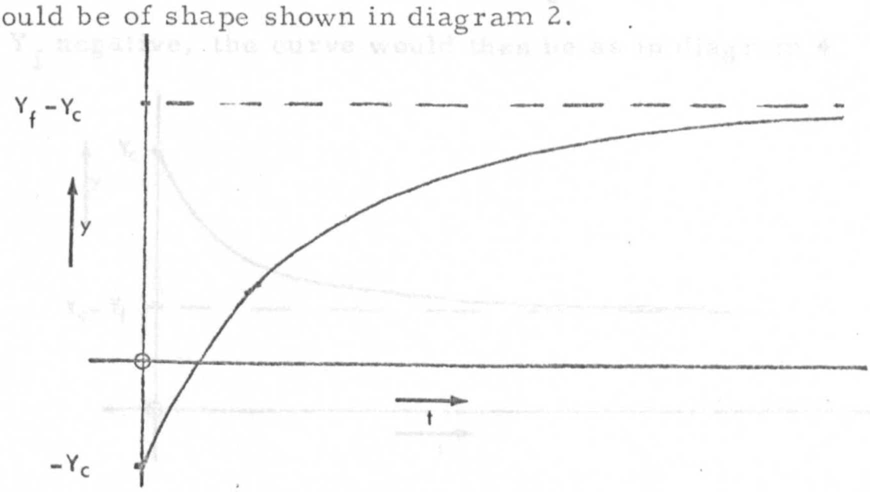


DIAGRAM 2

SHAPE OF BEVIS LEARNING CURVE WITH Y_c -ve, Y_f +ve, τ +ve.

and then only if $Y_f > Y_c$, for if Y_f was found to be less than $|Y_c|$ then the curve becomes as in diagram 3

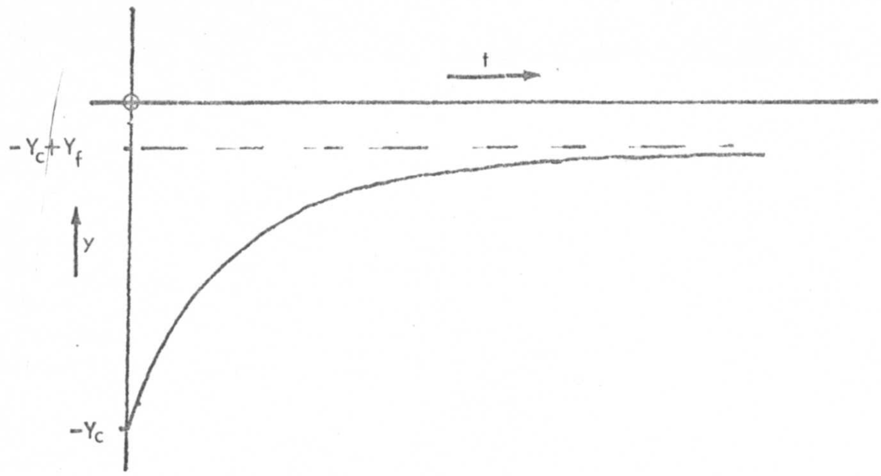


DIAGRAM 3

SHAPE OF BEVIS LEARNING CURVE WITH Y_c -ve, Y_f +ve,

τ + ve, $Y_f < |Y_c|$

As a further alternative if Y_c were found to be positive, and Y_f negative, the curve would then be as in diagram 4

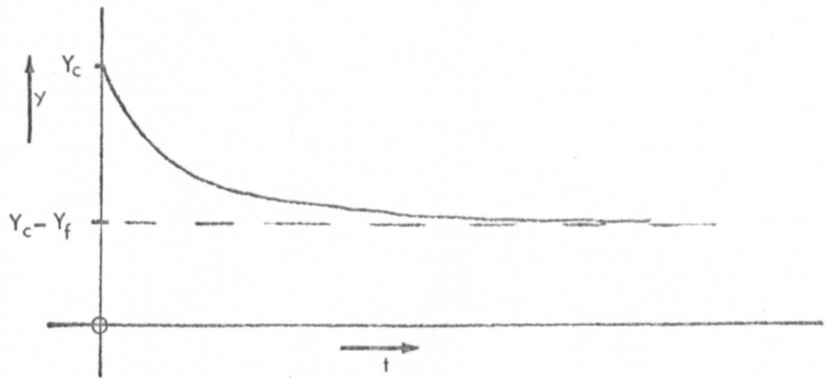


DIAGRAM 4

SHAPE OF BEVIS LEARNING CURVE WITH Y_c +ve, Y_f -ve, τ +ve.

and obviously the other alternative of Y_c negative and Y_f negative, results in similar changes. In this short analysis, no consideration has been given to a change in sign of τ ; it has been assumed positive. If, as a result of inaccurate data, a curve fitting programme hunted to any of the alternatives to the +ve Y_c, +ve Y_f, +ve τ then while the predicted curve might fit the data points well, it is unlikely that extrapolation outside the range of data points used would be accurate.

The possibility of poor extrapolation also results in the rejection of models such as the cubic model discussed earlier, and similarly, the possible use of a polynomial of any higher degree as a model, because such models predict values of 0 or $\pm\infty$ as $x \rightarrow \infty$. The Wright, Crawford and the American Government model are also rejected on these grounds, although it is emphasised once again, that for very long term learning curves, these models may be quite good approximations to the initial stages of learning.

3.3. Choice of Models for Investigation.

As a result of these considerations, a short-list of nine models was selected for assessment. These were: -

1. The Bevis Model $y_i = Y_c + Y_f(1-e^{-t_i/\tau})$ 2.1.9.(b)

2. The Gompertz Model $y_i = ka^{b^{x_i}}$ 2.3.5.(a)

3. The Mathematical Model $y_i = b - \dfrac{1}{c + gx_i}$ 2.3.1.(a)

4. The Wiltshire Model $y_i = c - ke^{-\alpha x_i^n}$ 2.1.8.(a)

5. The Accumulative Model $y_i = \dfrac{b + \theta\, a(n_i - 1)}{1 + \theta\, a(n_i - 1)}$ 2.2.2.(a)

6. The Replacement Model $y_i = a - (a-b)(a-\theta)^{n_i - 1}$ 2.2.1.(a)

7. The de Jong Model $y_i = B - Ax_i^{-n}$ 2.1.5.(b)

8. The Log-mathematical Model $\log y_i = b - \dfrac{1}{c + gx_i}$ 2.3.2.(a)

9. The Second Order Model $y_i = Y_c + Y_f(1 - (1 + \omega_o t_i)e^{-\omega_o t_i})$ 2.4.3.(i)

By the use of a computer to reduce the vast amount of computation, it is possible to attempt a series of curve-fitting exercises, using the same data for each model. An assessment can then be made of the most suitable model.

4. MEASURING THE "GOODNESS OF FIT".

4.1. The Nature of the Problem.

How is one to say if one equation, curve-fitted to a set of data points is a better fit than a second equation? The standard practice when curve fitting to only one set of data is to use the method of least squares to obtain the best fit. Then if the sum of errors squared for one fitted equation is greater than that for an alternative equation, a choice may be made - the alternative equation is considered to be the better fit. If several models exist, the same argument applies and a choice may be made.

In the more complicated case where several sets of data exist, how is one to differentiate between the possible equations? The sets of data need not necessarily relate to the same operations, and hence the data may be measured to different orders of scale. For example, consider the two sets of data relating to cigar rolling[27] and hemming[28] given in Table II.

Now the sum of errors squared for curve fitted models to the Bevis data is almost certainly going to be greater than that for the Morcombe data, yet one cannot be sure that one fit is better than the other. Obviously there is a need to "normalise" the results in some way so that a comparison may be made.

An examination of possible methods indicates that this problem may be solved by using one of three statistics:

TABLE II

OUTPUT DATA FOR TWO TASKS FROM DIFFERENT SOURCES

Bevis Mean Score 15 Subjects Rolling		Morcombe Mean Score 23 Subjects Hemming			
Day	Mean Output	Day	Mean Output	Day	Mean Output
1.0	1670	2.5	18.5	27.5	62.0
5.0	2314	5.0	37.0	30.0	64.0
10.0	2574	7.5	44.0	32.5	65.5
15.0	3314	10.0	51.0	35.0	67.0
20.0	3889	12.5	54.0	37.5	68.5
25.0	4055	15.0	57.0	40.0	70.0
30.0	4205	17.5	57.5	42.5	70.0
35.0	4243	20.0	58.0	45.0	70.0
		22.5	59.0	47.5	70.0
		25.0	60.0	50.0	70.0

(a) The Validity statistic

(b) The Chi-square statistic

(c) The 'R' statistic.

4.2. The Validity Statistic.

Consider the equation quoted in the TELFIT 1 computer manual[29] as a Validity statistic.

$$\text{Validity} = \left\{ 1 - \sqrt{\frac{\sum_1^N \left\{ \frac{y_i - y_e}{y_i} \right\}^2}{(N-1)}} \right\} \times 100 \qquad 4.2.(a)$$

Assume that we attempt to calculate parameters to obtain the maximum validity, and also calculate that validity (obviously for a perfect fit, Validity = 100).

Then

$$\text{Val} = \left\{ 1 - \sqrt{\frac{\sum_1^N \left(\frac{y_i - y_e}{y_i} \right)^2}{(N-1)}} \right\} \times 100 \text{ is a maximum} \qquad 4.2.(b)$$

$$\frac{\text{Val}}{100} = \left\{ 1 - \sqrt{\frac{\Sigma (\quad)^2}{(n-1)}} \right\} \text{ is a maximum} \qquad 4.2.(c)$$

$$1 - \frac{\text{Val}}{100} = \sqrt{\frac{\Sigma (\quad)^2}{(N-1)}} \text{ is a minimum} \qquad 4.2.(d)$$

$$\left(1 - \frac{\text{Val}}{100}\right)^2 = \frac{\sum_1^N (\quad)^2}{(N-1)} \text{ is a minimum} \qquad 4.2.(e)$$

$$\left(1 - \frac{\text{Val}}{100}\right)^2 \times (N-1) = \sum_1^N \left(\frac{y_e - y_i}{y_i} \right)^2 \text{ is a minimum} \qquad 4.2.(f)$$

Now it will be shown later that it is possible to develop an algorithm to "hunt" for parameters which will give this condition i.e.

$$\text{Minimum} \quad \sum_1^N \left(\frac{y_e - y_i}{y_i} \right)^2$$

4.3. The Chi-square Statistic.

The calculation of χ^2 from $\chi^2 = \sum_{1}^{N} \frac{(y_i - y_e)^2}{y_e}$ 4.3.(a)

is a more accepted method of establishing the "goodness of fit" of a model to data[30]. In addition it is possible to calculate the probability that such a value of χ^2 would be obtained. Later it will be shown that it is a much more difficult problem to develop an algorithm to hunt for best parameters to give minimum χ^2, than is the problem to hunt to parameters for maximum validity or least sum of errors squared.

4.4. The 'R' Statistic.

Kendal and Yule[31] discuss the problem of comparison of 'fits' to data and suggest the use of the statistic 'R'. The relationship suggested is

$$R^2 = 1 - \frac{U}{n\,\alpha_y^2} \qquad 4.4.(a)$$

where U = sum of squares of residuals (sum of errors squared)

n = No. of data points

α_y^2 = Variance of observed values of Y.

R is shown to lie between 0 and 1.0, with good 'fits' having R values $\longrightarrow$ 1.0.

In the example quoted by Kendal and Yule, curve fits to two sets of data are compared, using the R value as a criterion, and a choice is made as to which is the "best" fit.

4.5. Choice of Statistic to be used in the Analysis.

As a result of the analysis to be discussed later, it was decided that the curve-fitting routines developed would be arranged to calculate not only the sum of errors squared (to allow comparison of models fitted to one set of data), but also to calculate R. Given sufficient time it was hoped to analyse the values obtained for R for all sets of data.

4.6. Statistical Analysis of the Values Obtained of the Sum of Errors Squared.

If one model is consistently a better fit for the sets of data examined, then, on average, the value of sum of errors squared should be lower than that found for other models. If all the values obtained are ranked, a suitable test for significance is Kendall's Coefficient of Concordance W.

Siegel[32] quotes an example which examines the three independent sets of ranks given by executives to six applicants and tests whether the ranking of the applicants shows a measure of agreement among the judges. In this study the 'judges' are the sets of data, and the 'applicants' are the models for which curve fitting has been undertaken. The sum of errors squared for all models for each set of data is ranked and W computed as follows:-

(i) Calculate the sum of ranks for each model, R_j.

(ii) Calculate the mean value of all the R_j.

Each of the R_j may be expressed as a deviation from the mean value, and it can be shown that the greater the deviations, the greater is the degree of association among the sets of ranks.

(iii) Calculate the sum of squares of the deviations.

Then $$W = \frac{12s}{k^2 (N^3 - N)} \qquad 4.6.(a)$$

where s = the sum of squares of the observed deviations from the mean of R_j

i.e. $$s = \Sigma \left(R_j - \frac{\Sigma R_j}{N}\right)^2 \qquad 4.6.(b)$$

k = No. of sets of data

N = No. of models.

Siegel also shows that for reasonably large N, k then the expression given above is approximately distributed as χ^2 with (N-1) degrees of freedom.

Thus if the value of χ^2 so calculated exceeds the value quoted in the χ^2 table for a particular level of significance and a particular value of degrees of freedom = (N-1), then the null hypothesis that the k rankings are unrelated may be rejected at that level of significance.

5. THE CURVE-FITTING PROGRAM AND DATA FILE.

5.1. Choice of Bevis, Finnear and Towill algorithm.

During the course of this study, three iterative methods of curve-fitting were found in the literature. Bevis[33], in his thesis, discussed the problem with special reference to calculating the parameters of the Bevis equation, but later, Bevis, et al.[34] developed a 2 parameter-algorithm which "hunted" to the best parameter values. Hitchings[35] used this algorithm extensively in his study on Dynamic Learning Curve Models, while later, Sriyananada[36] discussed the same problem using the Kalman Filter technique.

Towill[37] has also noted the method of Ba Hli and discussed the calculation of parameter values to the same data (Bevis's).

Unfortunately, the Kalman filter technique, and that of Ba-Hli appear to be usable only in the case of the Bevis Equation, and not, for example, if fitting to the Wiltshire or de Jong Equations. For this reason, the Bevis, Finnear and Towill algorithm was extended to cover more than 2 parameters, and programmes developed to calculate "best" parameters for the various forms of equation selected for study.

5.2. Derivation of Basic Formulae using the Bevis et al Analysis.

If the data to be studied has N data points, then the output rate is represented by the series $Y_1 \dots Y_N$. At each data point, the corresponding series using a particular control law is given by $\bar{Y}_i \dots \bar{Y}_N$. The total error squared is then

$$E^2 = \sum_1^N E_i^2 = \sum_1^N (Y_i - \bar{Y}_i)^2 \qquad 5.2.(a)$$

To seek the values of the parameters which minimise equation 5.2.(a), the usual least squares minimisation analysis is unwieldy, and Bevis et al[38] suggested using a Taylor series expansion in an iterative loop, as the resulting equations are then linear and easily solved. The method is explained as follows: -

Let the estimated value of $\bar{Y}_i$ at time t_i be represented by the function $f(a, b, c, t_i)$

$$\text{i.e.} \quad \bar{Y}_i = f(a, b, c, t_i) \qquad 5.2.(b)$$

where a, b, c are three parameters.

Expanding Equation 5.2.(b) about the estimated value of $\bar{Y}_i$, using current best estimates of a, b & c ($\bar{a}$, $\bar{b}$ & $\bar{c}$ respectively), terms above first order being ignored, yields

$$\bar{Y}_i \simeq f(\bar{a}, \bar{b}, \bar{c}, t_i) + \frac{\delta f}{\delta a} \Delta a + \frac{\delta f}{\delta b} \Delta b + \frac{\delta f}{\delta c} \Delta c \qquad 5.2.(c)$$

where $\frac{\delta f}{\delta a}$, $\frac{\delta f}{\delta b}$ and $\frac{\delta f}{\delta c}$ are the partial derivatives of $f(a, b, c, t_i)$ with respect to a, b and c respectively and Δa, Δb and Δc are small increments ("correction factors") in a, b and c.

After Q iterative loops of the routine, adequate estimates of $\bar{a}$, $\bar{b}$ and $\bar{c}$ are obtained and since Δa, Δb and Δc then become negligible, equation 5.2.(c) reduces to

$\bar{Y}_{iQ} = f(\bar{a}_Q, \bar{b}_Q, \bar{c}_Q, t_i)$ and prediction is complete.

From equations 5.2.(a) and 5.2.(c), an estimate of the sum of error squared at any time in the iterative process is given by

$$E^2 = \sum_1^N \left\{ Y_i - f(\bar{a}_r, \bar{b}_r, \bar{c}_r, t_i) - \frac{\delta f}{\delta a} \Delta a_r - \frac{\delta f}{\delta b} \Delta b_r - \frac{\delta f}{\delta c} \Delta c_r \right\}^2 \quad 5.2.(d)$$

where $\bar{a}_r$, $\bar{b}_r$, $\bar{c}_r$ are the r^{th} estimates of the parameters a, b, c.

Since Equation 5.2.(d) is linear in Δa, Δb and Δc, the usual mean square error minimisation procedure may now be adopted.

For if $\frac{\delta E}{\delta \Delta a} = 0$, $\frac{\delta E}{\delta \Delta b} = 0$, $\frac{\delta E}{\delta \Delta c} = 0$

and if we let

$$(Y_i - f(\bar{a}_r, \bar{b}_r, \bar{c}_r, t_i)) = \Delta Y_{ir}$$

$\left(\text{i.e. the } r^{th} \text{ estimate of } \Delta Y_i \right) = \text{PDY}$

and let $\frac{\delta f}{\delta a} = \text{P01}$; $\frac{\delta f}{\delta b} = \text{P02}$; $\frac{\delta f}{\delta c} = \text{P03}$

then
$$2 \sum_1^N \left[\text{PDY} - \text{P01}\,\Delta a_r - \text{P02}\,\Delta b_r - \text{P03}\,\Delta c_r \right] \left[- \text{P01} \right] = 0 \quad 5.2.(e)$$

$$2 \sum_1^N \left[\text{PDY} - \text{P01}\,\Delta a_r - \text{P02}\,\Delta b_r - \text{P03}\,\Delta c_r \right] \left[- \text{P02} \right] = 0 \quad 5.2.(f)$$

$$2 \sum_{1}^{N} \left[PDY - P01 \Delta a_r - P02 \Delta b_r - P03 \Delta c_r \right] \left[- P03 \right] = 0 \qquad 5.2.(g)$$

(on differentiating to obtain $\frac{\delta E}{\delta \Delta a}$, $\frac{\delta E}{\delta \Delta b}$, $\frac{\delta E}{\delta \Delta c}$)

Equations 5.2.(e), 5.2.(f), 5.2.(g) may now be rearranged to give

$$h_1 = \alpha_1 \Delta a_r + \beta_1 \Delta b_r + \gamma_1 \Delta c_r \qquad 5.2.(h)$$

$$h_2 = \alpha_2 \Delta a_r + \beta_2 \Delta b_r + \gamma_2 \Delta c_r \qquad 5.2.(i)$$

$$h_3 = \alpha_3 \Delta a_r + \beta_3 \Delta b_r + \gamma_3 \Delta c_r \qquad 5.2.(j)$$

which are 3 simultaneous equations in Δa_r, Δb_r and Δc_r and can be solved by the usual methods. In the 3 equations 5.2.(h), 5.2.(i), 5.2.(j)

$$\alpha_1 = \sum_{1}^{N} P01.P01 \qquad \beta_1 = \sum_{1}^{N} P02.P01 \qquad \gamma_1 = \sum_{1}^{N} P03.P01$$

$$\alpha_2 = \sum_{1}^{N} P01.P02 \qquad \beta_2 = \sum_{1}^{N} P02.P02 \qquad \gamma_2 = \sum_{1}^{N} P03.P02$$

$$\alpha_3 = \sum_{1}^{N} P01.P03 \qquad \beta_3 = \sum_{1}^{N} P02.P03 \qquad \gamma_3 = \sum_{1}^{N} P03.P03$$

$$h_1 = \sum_{1}^{N} P01.PDY$$

$$h_2 = \sum_{1}^{N} P02.PDY$$

$$h_3 = \sum_{1}^{N} P03.PDY$$

Solution of the 3 equations 5.2.(h), 5.2.(i), 5.2.(j) give estimates for the increments Δa_r, Δb_r and Δc_r, which allow the new parameter estimates $\bar{a} + \Delta a_r$, $\bar{b} + \Delta b_r$ and $\bar{c} + \Delta c_r$ to be used when the iterative process is repeated.

What does this analysis imply? It implies that whatever three parameter equation is used to define the data points, three simultaneous equations may be set up for an iterative procedure, provided that the equation used may be differentiated with respect to the three parameters. Logically the analysis could be extended to n parameters, but it is likely that the difficulty of estimating the parameter values sufficiently accurately to obtain rapid convergence would be too great.

5.3. Derivatives required for all programmes used.

All the equations used in this study, and the derivatives of those equations (with respect to the various parameters) are given below.

5.3.1. Bevis Equation Derivatives.

$$y_i = Y_c + Y_f (1 - e^{-t_i/\tau})$$

$$= Y_c + Y_f (1 - e^{-t_i Z}) \quad \text{where } Z = 1/\tau$$

$$\frac{\delta y_i}{\delta Y_c} = 1 \qquad 5.3.1.(a)$$

$$\frac{\delta y_i}{\delta Y_f} = (1 - e^{-t_i Z}) \qquad 5.3.1.(b)$$

$$\frac{\delta y_i}{\delta Z} = t_i Y_f e^{-t_i Z}$$ 5.3.1.(c)

5.3.2. Wiltshire Equation Derivatives.

$$y_i = c - ke^{-\alpha x_i^n}$$

$$\frac{\delta y_i}{\delta c} = 1$$ 5.3.2.(a)

$$\frac{\delta y_i}{\delta k} = -e^{-\alpha x_i^n}$$ 5.3.2.(b)

$$\frac{\delta y_i}{\delta n} = \alpha . x_i^n . k. e^{-\alpha x_i^n} . \ln (x_i)$$ 5.3.2.(c)

for if $y_i = ke^{-\alpha x_i^n}$

$$\ln (y_i) = \ln k - \alpha x_i^n$$ 5.3.2.(d)

Now let $p = \alpha x_i^n$

$$\ln p = n \ln(x_i) + \ln \alpha$$ 5.3.2.(e)

$$\frac{1}{p} dp = \ln (x_i) dn$$ 5.3.2.(f)

$$\frac{dp}{dn} = p \ln (x_i) = \alpha x_i^n \ln (x_i)$$ 5.3.2.(g)

From equation 5.3.2.(d)

$$\frac{1}{y_i} dy_i = \frac{d (\ln k - \alpha x_i^n)}{dn}$$

$$= -\alpha x_i^n \ln (x_i) dn$$ 5.3.2.(h)

$$\frac{dy_i}{dn} = -\alpha \cdot x_i^{n} \cdot \ln(x_i) \cdot k. e^{-\alpha x_i^{n}} \qquad 5.3.2.(i)$$

$$\frac{\delta\left(-ke^{-\alpha x_i^{n}}\right)}{\delta n} = \alpha x_i^{n} \ln(x_i).\ k.\ e^{-\alpha x_i^{n}} \qquad 5.3.2.(j)$$

and $$\frac{\delta y_i}{\delta \alpha} = x_i^{n} \cdot k.\ e^{-\alpha x_i^{n}} \qquad 5.3.2.(k)$$

5.3.3. de Jong Equation Derivatives.

We have shown earlier that this equation is of the form

$$y_i = B - A x_i^{-n}$$

$$\frac{\delta y_i}{\delta B} = 1 \qquad 5.3.3.(a)$$

$$\frac{\delta y_i}{\delta A} = -x_i^{-n} \qquad 5.3.3.(b)$$

$$\frac{\delta y_i}{\delta n} = A.\ \ln(x_i)\, x_i^{-n} \qquad 5.3.3.(c)$$

for if $y_i = Ax_i^{-n}$

$$\ln y_i = \ln A - n \ln x_i \qquad 5.3.3.(d)$$

$$\frac{1}{y_i} \cdot dy_i = -\ln x_i\, dn \qquad 5.3.3.(e)$$

$$\frac{\delta y_i}{\delta n} = -y_i \cdot \ln x_i = -Ax_i^{-n} \cdot \ln x_i \qquad 5.3.3.(f)$$

$$\frac{\delta y_i}{\delta n} = \frac{\delta\left(B - Ax_i^{-n}\right)}{\delta n} = +A.\ x_i^{-n} \ln(x_i) \qquad 5.3.3.(g)$$

5.3.4. Gompertz Equation Derivatives.

$$y_i = ka^{b^{x_i}}$$

$$\frac{\delta y_i}{\delta k} = a^{b^{x_i}} \quad 5.3.4.(a)$$

$$\frac{\delta y_i}{\delta a} = k.\ b^{x_i}\ .\ a^{(b^{x_i}-1)} \quad 5.3.4.(b)$$

$$\frac{\delta y_i}{\delta b} = x.\ b^{(x_i-1)}\ .\ k.\ a^{b^{x_i}}\ .\ \ln a \quad 5.3.4.(c)$$

for if $y_i = ka^{b^{x_i}}$

$$\ln y_i = \ln k + b^{x_i} \ln a \quad 5.3.4.(d)$$

$$\frac{1}{y_i}\ .\ dy_i = d\,(b^{x_i} \ln a)\ db \quad 5.3.4.(e)$$

Now let $q = b^{x_i} \ln a$

$$\ln q = \ln (\ln a) + x_i \ln b \quad 5.3.4.(f)$$

$$\frac{1}{q}\ .\ dq = x_i\ \frac{1}{b}\ .\ db \quad 5.3.4.(g)$$

$$\frac{dq}{db} = x_i\ \frac{1}{.\ b}\ .\ q = x_i\ \frac{1}{b}\ .\ b^{x_i} \ln a \quad 5.3.4.(h)$$

As $\frac{1}{y_i} dy_i = d\,(b^{x_i} \ln a)\ db$ from 5.3.4.(e)

Then $\frac{1}{y_i}\ dy_i = x_i\ \frac{1}{b}\ .\ b^{x_i} \ln a\ .\ db$ 5.3.4.(i)

$$\frac{dy_i}{db} = x_i \frac{1}{b} . b^{x_i} . \ln a . k . a^{b^{x_i}} \qquad 5.3.4.(j)$$

$$= x_i b^{(x_i - 1)} . \ln a . k . a^{b^{x_i}} \qquad 5.3.4.(k)$$

5.3.5. Mathematical Equation Derivatives.

$$y_i = b - \frac{1}{c + gx_i} = b - (c + gx_i)^{-1}$$

$$\frac{\delta y_i}{\delta b} = 1 \qquad 5.3.5.(a)$$

$$\frac{\delta y_i}{\delta a} = (c + gx_i)^{-2} \qquad 5.3.5.(b)$$

$$\frac{\delta y_i}{\delta g} = (c + gx_i)^{-2} . x_i \qquad 5.3.5.(c)$$

Note that these expressions are valid for use in the other mathematical equations used, e.g.,

$$\ln(y_i) = b - \frac{1}{c + g.x_i}$$

for all that is required is to substitute $\ln(y_i)$ for y_i in all the necessary equations in the computer programme developed.

5.3.6. Replacement Equation Derivatives.

$$y_i = P_n = a - (a-b)(1-\theta)^{n_i - 1}$$

$$\frac{\delta\, y_i}{\delta\, a} = 1 - (1 - \theta)^{n_i - 1} \qquad 5.3.6.(a)$$

$$\frac{\delta\, y_i}{\delta\, b} = (1 - \theta)^{n_i - 1} \qquad 5.3.6.(b)$$

$$\frac{\delta\, y_i}{\delta\, \theta} = -(a-b)\,(n_i - 1)\,(1-\theta)^{n_i - 2} \qquad 5.3.6.(c)$$

5.3.7. <u>Accumulative Equation Derivatives.</u>

$$y_i = \frac{b + \theta a\,(n_i - 1)}{1 + \theta\,(n_i - 1)}$$

$$\frac{\delta\, y_i}{\delta\, b} = \frac{1}{1 + \theta\,(n_i - 1)} \qquad 5.3.7.(a)$$

$$\frac{\delta\, y_i}{\delta\, a} = \frac{\theta\,(n_i - 1)}{1 + \theta(n_i - 1)} \qquad 5.3.7.(b)$$

$$\frac{\delta\, y_i}{\delta\, \theta} = \frac{(n_i - 1)\,(a-b)}{[1 + \theta\,(n_i - 1)]^2} \qquad 5.3.7.(c)$$

for

$$\frac{\delta\, y_i}{\delta\, \theta} = \frac{[1 + \theta(n_i - 1)]\, a(n_i - 1) - [b + \theta a(n_i - 1)]\,.(n_i - 1)}{\{1 + \theta\,(n_i - 1)\}^2} \qquad 5.3.7.(d)$$

$$= \frac{[n_i - 1]\;[a + \theta a\,(n_i - 1) - b - \theta a\,(n_i - 1)]}{\{1 + \theta\,(n_i - 1)\}^2} \qquad 5.3.7.(e)$$

$$= \frac{\{n_i - 1\}\;\{a - b\}}{\{1 + \theta\,(n_i - 1)\}^2} \qquad 5.3.7.(f)$$

5.3.8. Second Order Equation Derivatives. (3 parameter form with $\tau = 1/\omega_o$)

$$y_i = Y_c + Y_f(1 - (1 + t_i/\tau)\, e^{-t_i/\tau})$$

$$= Y_c + Y_f(1 - (1 + t_i Z.)\, e^{-t_i Z}) \qquad \text{if } Z = 1/\tau$$

$$\frac{\delta y_i}{\delta Y_c} = 1 \qquad 5.3.8.(a)$$

$$\frac{\delta y_i}{\delta Y_f} = 1 - (1 + t_i Z)e^{-t_i Z} \qquad 5.3.8.(b)$$

$$\frac{\delta y_i}{\delta Z} = t_i^2 Z.\ Y_f e^{-t_i Z} \qquad 5.3.8.(c)$$

for if $y_i = Y_c + Y_f(1 - (1 + t_i Z)\, e^{-t_i Z}$

$$\frac{\delta y_i}{\delta Z} = +t_i \,.\, Y_f(1 + t_i Z)\, e^{-t_i Z} - Y_f e^{-t_i Z} \,.\, t_i \qquad 5.3.8.(d)$$

$$= Y_f \,.\, t_i \,.\, e^{-Zt_i} \left[1 + Zt_i - 1 \right] \qquad 5.3.8.(e)$$

$$= Y_f \,.\, Z \,.\, t_i^2 e^{-Zt_i} \qquad 5.3.8.(f)$$

5.4. Application to the Validity and the X^2 Statistics.

In section 5.1. the application of the Bevis, Finnear, Towill curve-fitting algorithm to the solution of three parameter models was discussed with special reference to using the criterion of "least sum of errors squared". The same considerations discussed previously in 5.2. apply even if the criterion is changed to maximum validity, or minimum χ^2 values (as discussed in Chapter 4). Naturally there is some modification to the equations derived, because it is not required to solve for parameters which minimise

$$E^2 = \sum_{1}^{N} (Y_i - \bar{Y}_i)^2 \qquad \text{(equation 5. 2. (a))}$$

but for parameters which minimise

$$\sum_{1}^{N} \left\{ \frac{Y_i - Y_e}{Y_i} \right\}^2$$

for the validity statistic (from equation 4. 2. (f))

or for parameters which minimise

$$\sum_{1}^{N} \left\{ \frac{Y_i - Y_e}{Y_e} \right\}^2$$

for the χ^2 statistic (from equation 4. 3. (a))

Because of the different form of these functions it is not necessarily true that parameters which minimise the sum of errors squared will also minimise the other statistics, although they may be approximately the same value.

Consider the function

$$\text{Val} = \sum_{1}^{N} \left(\frac{Y_i - Y_e}{Y_i} \right)^2 = \sum_{1}^{N} \left\{ \frac{Y_i - f(\bar{a}, \bar{b}, \bar{c}, t_i) - \frac{\delta f}{\delta a} \Delta a - \frac{\delta f}{\delta b} \Delta b - \frac{\delta f}{\delta c} \Delta c}{Y_i} \right\}^2 \qquad 5.4. (a)$$

In this case we again have a function linear in Δ a, Δ b and Δ c so the usual minimisation procedures apply. However, an examination of the analysis in Section 5.2. shows that while the equations for the solution of the above function will be very similar, they are slightly more complex computationally.

In the case of

$$\chi^2 = \sum_{1}^{N} \left\{ \frac{Y_i - Y_e}{Y_e} \right\}^2$$

or $$\chi^2 = \sum_{1}^{N} \left\{ \frac{Y_i - f(\bar{a}, \bar{b}, \bar{c}, t_i) - \frac{\delta f}{\delta a}\Delta a - \frac{\delta f}{\delta b}\Delta b - \frac{\delta f}{\delta c}\Delta c}{f(\bar{a}, \bar{b}, \bar{c}, t_i) - \frac{\delta f}{\delta a}\Delta a - \frac{\delta f}{\delta b}\Delta b - \frac{\delta f}{\delta c}\Delta c} \right\}^2 \qquad 5.4.(b)$$

the situation is not solveable by the existing technique because the expression is not linear in Δa, Δb and Δc.

It would appear that the only circumstance which would result in all three possible methods iterating and hunting to the same parameter values is that in which the data is exactly correct, and also follows the law defined by the suggested equation to the model. Small errors in the data could very well result in slightly different parameters being indicated by the three methods. Thus, the statistic chosen for this comparitive study was the "least sum of errors squared".

5.5. Setting up the Data File.

To deal with the large number of data-sets involved, it was found necessary to create a computer data file in the form of card images (TEDSFILEI). To ensure that each set of data could be called up as required, it was given a TITLE CARD and TITLE CARD NUMBER. See the example below.

T0137 BLACKBURN AVERAGE SCORE OF S2. OPERATION:- CROSSING OUT E'S.

The 'T' confirms that the card is a title card, 0137 is the title card number. The remaining information relates to the source of the data and what operation was involved.

Each set of data cards also included a card giving the number of pairs of (X, Y) data points recorded. The (X, Y) data points which followed were punched 4 pairs to each card. A print-out of some data sets has been shown in diagram 5 so that the above explanation can be followed, and so that the explanation of the operation of the curve-fitting programme can be followed.

5.6. Some Notes on the Estimation of the Parameters.

It was decided in the early stages of the analysis that there was a need for fair accuracy in the parameter estimates, otherwise the iterative procedure eventually failed. Erroneous parameter estimates usually resulted in large changes in those parameter estimates, which could result in the creation of such large numbers in the numerical calculations that the computer store became overloaded. Because it was fairly easy to estimate the starting point of the curve by eye, it was decided to include estimates of the 'start' and 'final' values of the learning curve, and calculate the parameters

```
T0138 BLACKBURN AVERAGE SCORE OF S3                    OPERATION:- CROSSING OUT "E"'S
 32
1.0 127.0 2.0 127.0 3.0 141.0 4.0 148.0
5.0 153.0 6.0 145.0 7.0 152.0 8.0 150.0
9.0 151.0 10.0 155.0 11.0 164.0 12.0 166.0
13.0 163.0 14.0 167.0 15.0 155.0 16.0 162.0
17.0 163.0 18.0 158.0 19.0 150.0 20.0 157.0
21.0 164.0 22.0 164.0 23.0 171.0 24.0 172.0
25.0 180.0 26.0 168.0 27.0 172.0 28.0 163.0
29.0 191.0 30.0 177.0 31.0 167.0 32.0 180.0
T0139 BLACKBURN AVERAGE SCORE OF S4                    OPERATION:- CROSSING OUT "E"'S
 23
1.0 152.0 2.0 155.6 3.0 170.0 4.0 176.3
5.0 179.6 6.0 180.4 7.0 180.2 8.0 183.1
9.0 190.5 10.0 200.0 11.0 198.7 12.0 193.5
13.0 194.2 14.0 199.3 15.0 188.0 16.0 198.0
17.0 211.2 18.0 209.6 19.0 204.1 20.0 218.2
21.0 210.0 22.0 196.1 23.0 202.7
T0140 BLACKBURN AVERAGE SCORE OF S1                    OPERATION:- CODE SUBSTITUTION
 35
1.0 22.2 2.0 25.6 3.0 30.0 4.0 28.9
5.0 32.8 6.0 32.8 7.0 40.6 8.0 40.6
9.0 41.7 10.0 41.7 11.0 48.3 12.0 48.9
13.0 49.4 14.0 50.0 15.0 51.7 16.0 55.6
17.0 53.3 18.0 58.9 19.0 51.7 20.0 52.8
21.0 60.0 22.0 62.2 23.0 62.8 24.0 66.1
25.0 65.0 26.0 63.9 27.0 71.7 28.0 71.7
29.0 70.6 30.0 73.3 31.0 73.3 32.0 72.8
33.0 73.9 34.0 77.2 35.0 78.9
```

DIAGRAM 5

COMPUTER PRINTOUT OF DATA USED FOR CURVE FITTING EXERCISE from these values. This procedure was followed for all 3-parameter models, unfortunately the procedure did not work well for the Wiltshire model (4 parameters) and was discontinued.

As an example, consider the Gompertz model

$$y_i = ka^{b^{x_i}} \qquad \text{(equation 2.3.5.(a))}$$

when $x_o = 0$ (start)

$$\text{then } y_o = ka^{b^0} = ka^1 = k.\,a. \qquad 5.6.(a)$$

If $o < b < 1$, then when $x_i \longrightarrow \infty$ (final)

$$y_\infty = ka^{b^\infty} = ka^o = k.\,1 = k \qquad 5.6.(b)$$

Thus the 'final' estimate = k

$$\text{and } \frac{\text{'start'}}{\text{'final'}} = \frac{ka}{k} = a$$

Given also one of the data points (x_n, y_n) where n denotes the n^{th} point

$$\text{then } Y(N) = k.\,a^{b^{x(N)}} \qquad 5.6.(c)$$

$$\frac{Y(N)}{k} = a^{b^{x(N)}} \qquad 5.6.(d)$$

$$\ln \frac{Y(N)}{k} = b^{x(N)} \ln(a) \qquad 5.6.(e)$$

$$b^{x(N)} = \frac{\ln \frac{Y(N)}{k}}{\ln(a)} \qquad 5.6.(f)$$

$$b = \left\{ \frac{\ln \frac{Y(N)}{k}}{\ln(a)} \right\}^{1/x(N)} \qquad 5.6.(g)$$

Thus all three parameters may be estimated from the 'start' estimate, the 'final' estimate, and the n^{th} data point. Similar calculations were made for all the 3-parameter programs used and the formulae included in the programs. The derivations of other formulae are given in Appendix B.

5. 7. Operation of the Curve-Fitting Routine.

To operate the curve fitting routine, a set of "estimation" cards was included at the end of the programme which defined the title number of the data set to be used, estimates of the "final" (i. e. the asymptotic value) and the "start" values, and whether the data needed to be modified or not. (Some data sets were included which recorded cycle-time data, hence if these were going to be used in the analysis, the cycle-time data needed to be converted to output data).

Sample cards are shown below in diagram 6. The first card which would be read by the card reader is the card reading "2". This indicates the number of cards which follow containing data set requirements. The third card indicates the need to modify the data ("MOD").

The program thus hunts for the card image relating to title card TO158 and enters the curve fitting routine once it is established that the correct data set has been found.

In all curve-fitting routines such as this, a test needs to be

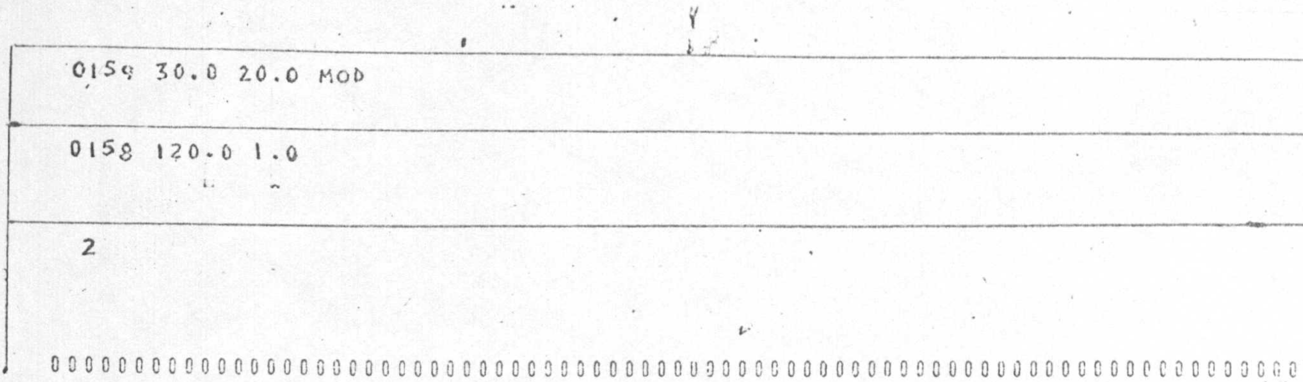


DIAGRAM 6

PUNCHED CARDS TO DEFINE PARAMETER ESTIMATES AND DATA MODIFICATIONS

included to prevent the iterative procedure from continuing indefinitely. The limit for this analysis was set at 15 iterations. Limits also need to be set, either on the change in the estimated parameter values, or on the change in the sum of errors squared, from one iteration to another. If this is not done the routine will continue to iterate 15 times, regardless of any high value of accuracy attained.

As a further check on the accuracy of the program an artificial set of data was created for each model. On each run of the program, the test data set was included so that the iterative calculations and the calculation for R could be checked. The flow diagram which follows gives the full sequence of the program developed.

PULLOUT

5.8. Computer Program Flow Diagram.

NOTE

At suitable points in this program, a test was included to see if arithmetic errors (caused by poor parameter estimation), had resulted in overflow of computer stores (lines 0064, 0080, 0084, 0088, 0092, 0096, 0099, 0101, 0103, 0105, 0111). If the test confirmed that an error had occurred, the program switched to the next set of data.

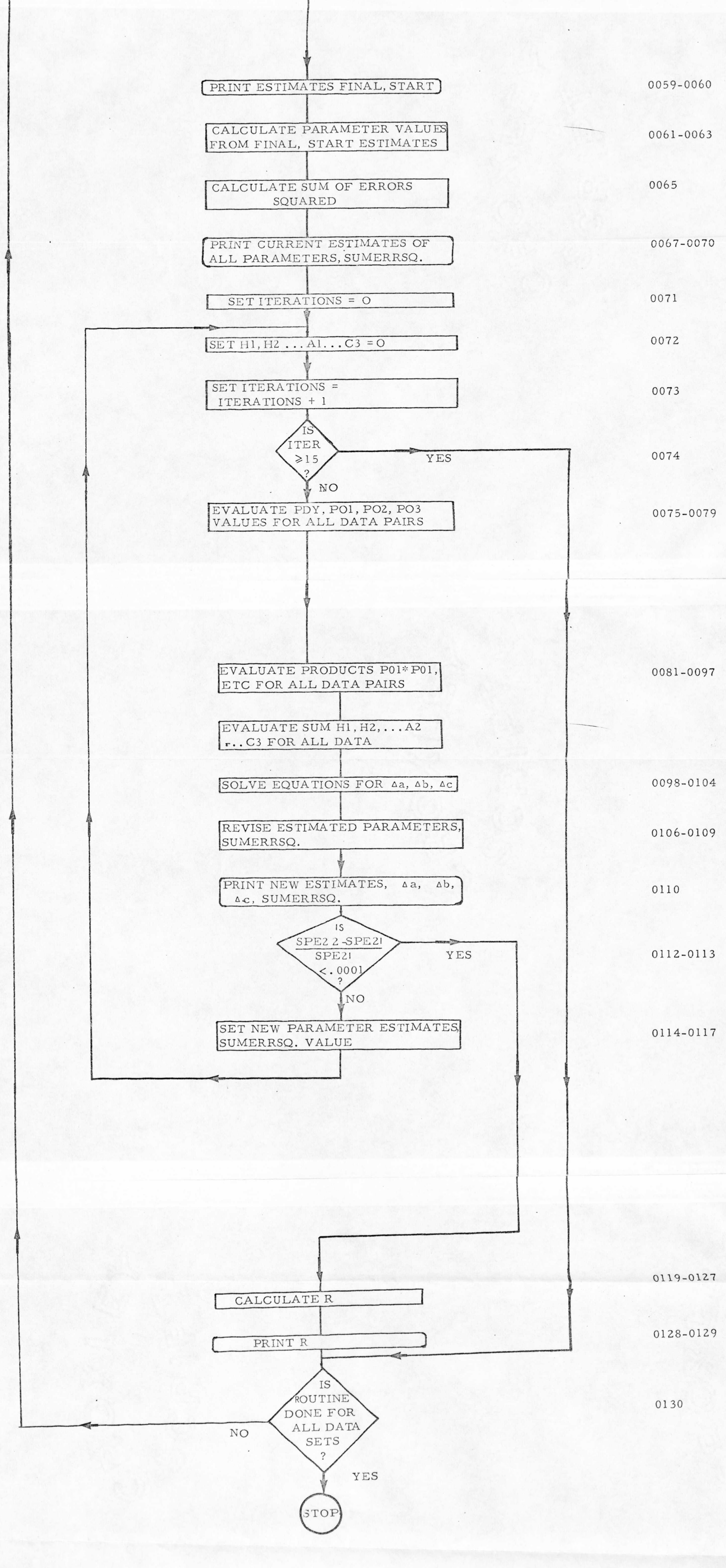


DIAGRAM 7

COMPLETE FLOW DIAGRAM FOR OPERATION OF CURVE FITTING ROUTINES

5.9. An Example of a Typical Computer Programme Used.

A typical programme with printout of test data and the iterations for one data set is included at this point. The reader will note that the programme requires very little alteration to make it suitable for a different three parameter curve fitting problem.

FORTRAN COMPILATION BY #XFAT MK 4C   DATE 20/02/74

```
          LIST(LP)
          PROGRAM(FIT8)
          OUTPUT2=LP0
          INPUT6=CR0
          USE 5=ED0/FORMATTED(MIDDXCOMMON1)
          USE 1=ED1/FORMATTED(TEDSFILE1)
          END


          MASTER CURVE FIT 8
          COMMON NULL
          EXTERNAL OFLOW
          REAL NK,NA,NB,MOD,MODX,MY,K
          DIMENSION T(10),X(250),Y(250),PO1(250),PDY(250),PO2(250),
         1PO3(250)
          DATA END,MOD/8HEND    ,8HMOD    /,TT/8HT       /
          CALL FTRAP(OFLOW)
          CALL DATE(DY)
      300 READ(6,1000)T
          CALL COMP8(END,T(1),L)
          IF (L.EQ.1) GO TO 6666
          WRITE(5,1000)T
          GOTO 300
     6666 REWIND 5
          READ(5,701) NIDENT
      701 FORMAT(I0)
          DO 10 KID=1,NIDENT
```

```
      NULL=0
      READ(5,702) IDENT,A,B,MODX
  702 FORMAT(1X,14,2F0.0,A3)
  710 READ(1,703) T
  703 FORMAT(10A8)
      L=1
      CALL COMP(L,TT,1,T(1),1)
      IF (L.NE.1) GOTO 710
      BACKSPACE 1
      READ (1,704) L
  704 FORMAT(1X,I4)
      IF (L.NE.IDENT) GOTO 710
      BACKSPACE 1
      READ(1,1000) T
 1000 FORMAT(10A8)
      READ (1,1001) N
 1001 FORMAT (I5)
      READ(1,1002) (X(I),Y(I),I=1,N)
 1002 FORMAT(8F0.0)
      WRITE(2,1003) DY,T
 1003 FORMAT(1H1,60X,1HX,39X,6HDATE:-,A8/
     160X,1HB,/
     231X,29HCURVE FITTING TO EQN. Y = K.A ////,24X,10A8)
      WRITE(2,1004)
 1004 FORMAT(////31X,5HGROUP,9X,1HX,13X,1HY)
      WRITE(2,1005) (I,X(I),Y(I),I=1,N)
 1005 FORMAT(30X,I5,5X,F9.4,5X,F9.4)
```

```
      CALL COMP8(MOD,MODX,L)
      IF(L.NE.1) GOTO 2
      DO 1 I=1,N
    1 Y(I)=3600/Y(I)
      WRITE(2,1004)
      WRITE(2,1005) (I,X(I),Y(I),I=1,N)
    2 WRITE(2,1007) A,B
 1007 FORMAT(////5X,7HFINAL= ,F10.3,9H  START= ,F10.3)
      K=A
      A=B/K
      B=(ALOG(Y(N)/K)/ALOG(A))**(1/X(N))
      IF(NULL.EQ.1) GOTO 10
      SPE21=SUMERRSQ(K,A,B,Y,N,X)
      DK,DA,DB=0
      WRITE(2,1008) DK,DA,DB,K,A,B,SPE21
 1008 FORMAT(///,' DK= ',1PE14.7,' DA= ',1PE14.7,' DB= ',1PE14.7,//,
     1' K=  ',1PE14.7,' A=  ',1PE14.7,' B=  ',1PE14.7,' SPE21= ',
     21PE14.7)
      ITER=0
  100 H3,A3,C1,C2,C3,H1,H2,A1,A2,B3,B1,B2=0
      ITER=ITER+1
      IF(ITER-15)0,10,10
      DO 5 I=1,N
      PDY(I)=Y(I)-K*(A**(B**X(I)))
      P01(I)=A**(B**X(I))
      P02(I)=(B**X(I))*K*(A**(B**X(I)-1))
      P03(I)=X(I)*(B**(X(I)-1))*K*ALOG(A)*(A**(B**X(I)))
      IF(NULL.EQ.1) GOTO 10
      H1=H1+P01(I)*PDY(I)
      H2=H2+P02(I)*PDY(I)
      H3=H3+P03(I)*PDY(I)
```

```
      IF(NULL.EQ.1) GOTO 10
      A1=A1+P01(I)*P01(I)
      A2=A2+P01(I)*P02(I)
      A3=A3+P01(I)*P03(I)
      IF(NULL.EQ.1) GOTO 10
      B1=B1+P02(I)*P01(I)
      B2=B2+P02(I)*P02(I)
      B3=B3+P02(I)*P03(I)
      IF(NULL.EQ.1) GOTO 10
      C1=C1+P03(I)*P01(I)
      C2=C2+P03(I)*P02(I)
      C3=C3+P03(I)*P03(I)
      IF(NULL.EQ.1) GOTO 10
    5 CONTINUE


      D=(A1*B2*C3+A2*B3*C1+A3*B1*C2-A1*B3*C2-A2*B1*C3-A3*B2*C1)
      IF(NULL.EQ.1) GOTO 10
      DK=(H1*B2*C3+H2*B3*C1+H3*B1*C2-H1*B3*C2-H2*B1*C3-H3*B2*C1)/D
      IF(NULL.EQ.1) GOTO 10
      DA=(A1*H2*C3+A2*H3*C1+A3*H1*C2-A1*H3*C2-A2*H1*C3-A3*H2*C1)/D
      IF(NULL.EQ.1) GOTO 10
      DB=(A1*B2*H3+A2*B3*H1+A3*B1*H2-A1*B3*H2-A2*B1*H3-A3*B2*H1)/D
      IF(NULL.EQ.1) GOTO 10
      NK=K+DK
      NA=A+DA
      NB=B+DB
      SPE22=SUMERRSQ(NK,NA,NB,Y,N,X)
      WRITE(2,1008) DK,DA,DB,NK,NA,NB,SPE22
```

```
      IF(NULL.EQ.1) GOTO 10
      Z=(SPE21-SPE22)/SPE21
      IF(ABS(Z).LT.0.0001  ) GOTO 500
      K=NK
      A=NA
      B=NB
      SPE21=SPE22
      GOTO 100
  500 SY=0
      DO 6 I=1,N
    6 SY=SY+Y(I)
      MY=SY/N
      SUME2=0
      DO 7 I=1,N
    7 SUME2=SUME2+(Y(I)-MY)**2
      VAR=SUME2/(N-1)
      R=SQRT(1-SPE22/(N*VAR))
      WRITE(2,1009) R
 1009 FORMAT(///80X,3HR= ,F12.9)
   10 CONTINUE
      STOP
      END

END OF SEGMENT, LENGTH   864, NAME  CURVEFIT8
```

```
      FUNCTION SUMERRSQ(K,A,B,Y,N,X)
      REAL K
      DIMENSION X(N),Y(N)
      SUMERRSQ=0
      DO 1 I=1,N
    1 SUMERRSQ=SUMERRSQ+(K*(A**(B**X(I)))-Y(I))**2
      RETURN
      END

END OF SEGMENT, LENGTH    71, NAME  SUMERRSQ


      SUBROUTINE OFLOW(NIL)
      COMMON NULL
      WRITE(2,100)NIL
  100 FORMAT(////,' ERROR ',I3  ////)
      NULL=1
      RETURN
      END

END OF SEGMENT, LENGTH    29, NAME  OFLOW
```

CURVE FITTING TO EQN. $Y = K.A^{B^X}$

T0134 BLACKBURN AVERAGE SCORE OF S6 OPERATION:-CARD SORTING

GROUP	X	Y
1	1.0000	30.4000
2	2.0000	28.0000
3	3.0000	32.3000
4	4.0000	37.8000
5	5.0000	41.6000
6	6.0000	42.4000
7	7.0000	47.2000
8	8.0000	47.2000
9	9.0000	50.6000
10	10.0000	60.0000
11	11.0000	59.2000
12	12.0000	58.3000
13	13.0000	64.6000
14	14.0000	68.9000
15	15.0000	60.9000
16	16.0000	65.6000
17	17.0000	68.9000
18	18.0000	82.4000
19	19.0000	76.4000
20	20.0000	85.7000
21	21.0000	84.0000
22	22.0000	63.6000
23	23.0000	77.8000
24	24.0000	97.7000
25	25.0000	91.3000
26	26.0000	91.3000
27	27.0000	105.0000

```
FINAL=    120.000   START=     25.000

DK=  0.0000000E-01 DA=  0.0000000E-01 DB=  0.0000000E-01
K=   1.2000000E 02 A=   2.0833333E-01 B=   9.1279414E-01 SPE21=  3.9951855E 03

DK=  7.6966174E-01 DA=  1.7483129E-02 DB=  2.9121669E-02
K=   1.2076966E 02 A=   2.2581646E-01 B=   9.4191581E-01 SPE21=  1.1185415E 03

DK=  1.9145699E 01 DA= -3.3657471E-02 DB=  8.2013653E-03
K=   1.3991536E 02 A=   1.9215899E-01 B=   9.5011717E-01 SPE21=  1.0670544E 03

DK=  6.1964703E 00 DA= -2.6834554E-03 DB=  7.3181489E-04
K=   1.4611183E 02 A=   1.8947554E-01 B=   9.5084899E-01 SPE21=  9.6412048E 02

DK=  3.4337792E-01 DA= -2.1219203E-04 DB=  1.0245539E-04
K=   1.4645521E 02 A=   1.8926334E-01 B=   9.5095144E-01 SPE21=  9.6406353E 02

                                                                R=  0.959922544
```

6. SOURCES OF DATA.

6.1. Blackburn's Study on the Acquisition of Skill.

In 1936, H.M.S.O. published a long report by Blackburn[39] dealing with an analysis of learning curves. In that report Blackburn considered the various methods then existing for depicting learning, the "plateau" effect, (in which performance apparently reaches a maximum, but then rises to a new maximum), and also the problem of whether there was a general learning curve equation. While Blackburn confined himself for the most part to the consideration of other experimenters' work, he also conducted experiments with his own volunteer subjects and recorded the results. Five experiments were performed: card sorting, maze learning, code substitution, crossing out E's, and addition. With the exception of maze learning, it could be said that these were simple learning experiments, in which the subjects would approach their maximum output reasonably quickly.

In Blackburn's experiments, not all subjects took part in all the experiments, and similarly not all subjects took the same number of tests. To avoid biasing any averaged curves because one or two subjects took more tests than the others, average curves were calculated for which all subjects had taken the same number of tests e.g. 7 subjects took 20 tests or more on card sorting, therefore an averaged curve was calculated for the 7 subjects for 20 tests. 4 subjects took 30 tests or more in the same card sorting experiment; these results were similarly averaged for 30 tests. In this way averaged results for 4, 6 or 7 subjects were found for the data. Averaged and individual data is given in Appendix C and full details of Blackburn's experiments in Appendix D.

6.2. Morcombe's thesis on Motor Skill Learning Models.

As part of his thesis, Morcombe[40] undertook a laboratory experiment in which the learning of a simulated simple assembly task was studied. Six subjects performed 20 tasks each at one sitting in which 54 square and triangular pieces had to be fitted into the shape of a perfect rectangle. Cycle time data for each trial is recorded in Appendix C, as is an averaged cycle time which was calculated for this study. For the purposes of the curve fitting exercise, this cycle time data was converted to output data within the computer programme used.

6.3. Blankenship and Taylor's Study of Machine Operators.

In their article, Blankenship and Taylor[41] examined the learning curves of operators in 3 machine processes; covering, trimming and hemming. Data is not given in the article, but Morcombe and Corlett[42] have interpolated points on the curves given and further discussed the results. The curves given are the averaged outputs of the workers, smoothed to reduce variability. The data is recorded in Appendix C.

6.4. Bevis's Thesis on Industrial Learning.

In his thesis, Bevis[43] examined several different learning situations in industry, including tack-welding of small components (operation 'B'), jointing short lengths of wire on to components (operation 'C'), making cigars at two different factories (rolling and bunching). In addition, data was quoted relating to one subject who assembled small machined component parts. In all cases, averaged data is quoted and is given in Appendix C.

6.5. Hackett and Lamb's Study of Telephonist Training.

As part of their study of telephonist training the author and his associate were given permission to examine the training records of nearly 100 telephonists employed by the Post Office. The data obtained indicates the amount of work done by the trainee in one hour for a series of tests during the training period of 5 weeks. The test is held at a switchboard, and the trainee handles live traffic (which is highly variable in content), so a scoring system has been developed by the Post Office that weights the score of each call dealt with according to the difficulty of the call handled. The units used are called "valued calls". The data is thus given in the form of the number of valued calls handled in one hour on a day of training. Included in some of the data is an observation made at a much later stage - this is a full efficiency check. All the data is given in Appendix E.

The average number of valued calls/hour handled on each day for all trainees was calculated and the mean data used to establish a best fit curve using the Bevis model. Predictions for each day of training were calculated and the individual scores ranked into high, low or medium categories using a computer programme. This allowed an estimation to be made of the trainee's overall performance during training and on the full efficiency test by using ranking scores of high = 3, medium = 2, low = 1 for all rankings.

The data sets could then be split up into those containing consistently high, medium or low scores. Such data sets contained 9 or 10 sets of trainees' data which were then averaged and used in this study. The averaged data found is presented in Appendix C.

At a later stage in the study of telephonist training, a series of experiments were held in which observations were made on a sample of trainee telephonists. Brief details of the experiments are given here, further information may be found in Lamb[44].

Lamb had previously made a series of observations on experienced telephonists in which he had confirmed that experienced telephonists performed their work to a common "activity profile". The activity profile was established by using a technique based on activity sampling. The task to be performed was split into elements such as Dialling, Operating Keys, Plugging In, Timing, Speaking, Listening, etc. and a record was taken of each activity in progress at ten second intervals during an observational period of 1 hour. An analysis of 6 hours observations made on several telephonists allowed the derivation of an "activity profile"; which can be defined as "the way in which an experienced telephonist divides her time while working at a switchboard".

Lamb hypothesised that a naive trainee would have an entirely different activity profile and that that activity profile would change, over a period of time, to a profile similar to that of experienced telephonists. To test this hypothesis, an experiment was arranged in which both researchers observed trainee telephonists at various exchanges and at a Training School, making frequent half-hourly and (at a later stage) hourly periods of observations when the trainees held their practice periods at the switchboard. After training was completed, further hourly periods of observations were taken at less frequent intervals. In all periods of observation activities were recorded at 6 second intervals, using an audible cue generated by a transistorised circuit and fed into earphones used by the researchers.

The observational requirements in the above experiment were slightly different to those previously established. In addition to noting the Dialling, Timing etc. categories of activity, a further subdivision to account for Procedural Instruction was required to allow for assistance given to the trainee by her trainer. This assistance might be relevant to an activity e.g. pointing out that a key should be operated, or it could be relevant to the whole call e.g. recapitulation of the procedure to be followed on a particular type of call. This further breakdown of the activities allows manipulation of the data to give a 5 day running average of occurrences of activities, set out to show the amount of work done by the trainee (Own Initiative) and the amount of Procedural Instruction received by the trainee. The resulting measures can be regarded as indicating the performance of the trainee at a particular element of the task during the period of observation, because as the trainee becomes more expert in her job, so instruction relevant to that element of the task should go down.

Now the total of Own Initiative + Procedural Instruction gives the total number of observations of any one element of activity. If the ratio

$$\frac{\text{Own Initiative}}{\text{Own Initiative + Procedural Instruction}} \times 100\% \text{ is}$$

calculated for each element in a period of observation, then the percentage value should rise to 100% over the period of training, because Procedural Instruction should fall to zero. The percentage calculated might then follow a learning curve.

The data gathered in the above experiment was modified in the above manner for one trainee telephonist to give a 5 day running average performance on the various elements of the task. Data is presented in Appendix C as a percentage Own Initiative performance for each day of training, so that the maximum performance attainable is 100%.

During the observational periods, records of the type of call, and of the difficulties that occurred were also made, so that at a later date, a "valued calls" total for the work done could be established. Appendix C includes data made available by Lamb relating to the performance of one trainee during this period of intensive observation.

7. ANALYSIS OF RESULTS

7.1. Relative Success Rates for Fitting Each Model.

A count of successful curve fitting runs for each model gives the following table. 88 sets of data were used.

TABLE III

RELATIVE SUCCESS RATES FOR EACH CURVE FITTING ROUTINE

MODEL	NO. OF SUCCESSFUL RUNS
BEVIS	77
GOMPERTZ	84
MATHEMATICAL	69
WILTSHIRE	31
ACCUMULATIVE	75
REPLACEMENT	81
DE JONG	37
LOG MATHEMATICAL	76
SECOND ORDER MODEL	87

From the table, it can be seen that there was little success in fitting the Wiltshire and de Jong models, but relatively high success rates for the other models. The failure of the Wiltshire model is probably due to the difficulty of estimating reliable parameter values. It was found in practice that unless the parameters were reasonably close to their 'best' values, the program overloaded, resulting in obviously incorrect parameter values, or error messages.

The relative failure of the de Jong model does not appear to have an explanation, unless the accuracy and quantity of data was insufficient to allow easy curve fitting.

7.2. Calculation of the Coefficient of Concordance.

Because of the poor success rates in fitting the Wiltshire and de Jong models, there were only 10 sets of data for which all models were curve fitted. The values of the sum of errors squared were ranked and a computer program written to calculate W and χ^2. For 9 models and 10 sets of data, χ^2 was significant ($p < .001$).

To establish which model was causing the effect, the computer program was extended to eliminate each of the models in turn from the rankings, correct the rankings affected by the elimination, and to recalculate W and χ^2.

The first printout for the 9 model/10 data sets case is repeated on the following pages. The integer values shown under each model heading are the ranks of that model for each set of data. Columns of zeros indicate that that particular model is excluded from the calculation.

KENDALL COEFFICIENT OF CONCORDANCE : W

M(1) = THE BEVIS MODEL

M(2) = THE GOMPERTZ MODEL

M(3) = THE MATHEMATICAL MODEL

M(4) = THE WILTSHIRE MODEL

M(5) = THE ACCUMULATIVE MODEL

M(6) = THE REPLACEMENT MODEL

M(7) = THE DE JONG MODEL

M(8) = THE LOGMATHEMATICAL MODEL

M(9) = THE SECOND ORDER MODEL

SET	M(1)	M(2)	M(3)	M(4)	M(5)	M(6)	M(7)	M(8)	M(9)
102	5	6	4	2	1	8	7	3	9
106	5	7	2	4	1	8	6	3	9
109	4	7	2	1	5	8	6	3	9
110	2	4	7	1	3	6	9	8	5
122	2	3	4	1	9	8	7	5	6
165	6	7	4	1	2	8	5	3	9
170	3	6	5	2	1	4	9	7	8
171	4	3	7	2	6	1	9	8	5
174	5	4	7	2	6	3	9	8	1
185	2	3	4	1	7	8	9	5	6

W= 4.0800000E-01 CHISQ= 3.2640000E 01 D.OF FREEDOM= 8

KENDALL COEFFICIENT OF CONCORDANCE : W

SET	M(1)	M(2)	M(3)	M(4)	M(5)	M(6)	M(7)	M(8)	M(9)
102	0	5	4	2	1	7	6	3	8
106	0	6	2	4	1	7	5	3	8
109	0	6	2	1	4	7	5	3	8
110	0	3	6	1	2	5	8	7	4
122	0	2	3	1	8	7	6	4	5
165	0	6	4	1	2	7	5	3	8
170	0	5	4	2	1	3	8	6	7
171	0	3	6	2	5	1	8	7	4
174	0	4	6	2	5	3	8	7	1
185	0	2	3	1	6	7	8	4	5

W= 3.9428571E-01 CHISQ= 2.7600000E 01 D.OF FREEDOM= 7

KENDALL COEFFICIENT OF CONCORDANCE : W

SET	M(1)	M(2)	M(3)	M(4)	M(5)	M(6)	M(7)	M(8)	M(9)
102	5	0	4	2	1	7	6	3	8
106	5	0	2	4	1	7	6	3	8
109	4	0	2	1	5	7	6	3	8
110	2	0	6	1	3	5	8	7	4
122	2	0	3	1	8	7	6	4	5
165	6	0	4	1	2	7	5	3	8
170	3	0	5	2	1	4	8	6	7
171	3	0	6	2	5	1	8	7	4
174	4	0	6	2	5	3	8	7	1
185	2	0	3	1	6	7	8	4	5

W= 4.2714286E-01 CHISQ= 2.9900000E 01 D.OF FREEDOM= 7

KENDALL COEFFICIENT OF CONCORDANCE : W

SET	M(1)	M(2)	M(3)	M(4)	M(5)	M(6)	M(7)	M(8)	M(9)
102	4	5	0	2	1	7	6	3	8
106	4	6	0	3	1	7	5	2	8
109	3	6	0	1	4	7	5	2	8
110	2	4	0	1	3	6	8	7	5
122	2	3	0	1	8	7	6	4	5
165	5	6	0	1	2	7	4	3	8
170	3	5	0	2	1	4	8	6	7
171	4	3	0	2	6	1	8	7	5
174	5	4	0	2	6	3	8	7	1
185	2	3	0	1	6	7	8	4	5

W= 4.2809524E-01 CHISQ= 2.9966667E 01 D.OF FREEDOM= 7

KENDALL COEFFICIENT OF CONCORDANCE : W

SET	M(1)	M(2)	M(3)	M(4)	M(5)	M(6)	M(7)	M(8)	M(9)
102	4	5	3	0	1	7	6	2	8
106	4	6	2	0	1	7	5	3	8
109	3	6	1	0	4	7	5	2	8
110	1	3	6	0	2	5	8	7	4
122	1	2	3	0	8	7	6	4	5
165	5	6	3	0	1	7	4	2	8
170	2	5	4	0	1	3	8	6	7
171	3	2	6	0	5	1	8	7	4
174	4	3	6	0	5	2	8	7	1
185	1	2	3	0	6	7	8	4	5

W= 2.7952381E-01 CHISQ= 1.9566667E 01 D.OF FREEDOM= 7

KENDALL COEFFICIENT OF CONCORDANCE : W

SET	M(1)	M(2)	M(3)	M(4)	M(5)	M(6)	M(7)	M(8)	M(9)
102	4	5	3	1	0	7	6	2	8
106	4	6	1	3	0	7	5	2	8
109	4	6	2	1	0	7	5	3	8
110	2	3	6	1	0	5	8	7	4
122	2	3	4	1	0	8	7	5	6
165	5	6	3	1	0	7	4	2	8
170	2	5	4	1	0	3	8	6	7
171	4	3	6	2	0	1	8	7	5
174	5	4	6	2	0	3	8	7	1
185	2	3	4	1	0	7	8	5	6

W= 4.6666667E-01 CHISQ= 3.2666667E 01 D.OF FREEDOM= 7

KENDALL COEFFICIENT OF CONCORDANCE : W

SET	M(1)	M(2)	M(3)	M(4)	M(5)	M(6)	M(7)	M(8)	M(9)
102	5	6	4	2	1	0	7	3	8
106	5	7	2	4	1	0	6	3	8
109	4	7	2	1	5	0	6	3	8
110	2	4	6	1	3	0	8	7	5
122	2	3	4	1	8	0	7	5	6
165	6	7	4	1	2	0	5	3	8
170	3	5	4	2	1	0	8	6	7
171	3	2	6	1	5	0	8	7	4
174	4	3	6	2	5	0	8	7	1
185	2	3	4	1	7	0	8	5	6

W= 4.6000000E-01 CHISQ= 3.2200000E 01 D.OF FREEDOM= 7

KENDALL COEFFICIENT OF CONCORDANCE : W

SET	M(1)	M(2)	M(3)	M(4)	M(5)	M(6)	M(7)	M(8)	M(9)
102	5	6	4	2	1	7	0	3	8
106	5	6	2	4	1	7	0	3	8
109	4	6	2	1	5	7	0	3	8
110	2	4	7	1	3	6	0	8	5
122	2	3	4	1	8	7	0	5	6
165	5	6	4	1	2	7	0	3	8
170	3	6	5	2	1	4	0	7	8
171	4	3	7	2	6	1	0	8	5
174	5	4	7	2	6	3	0	8	1
185	2	3	4	1	7	8	0	5	6

W= 3.3571429E-01 CHISQ= 2.3500000E 01 D.OF FREEDOM= 7

KENDALL COEFFICIENT OF CONCORDANCE : W

SET	M(1)	M(2)	M(3)	M(4)	M(5)	M(6)	M(7)	M(8)	M(9)
102	4	5	3	2	1	7	6	0	8
106	4	6	2	3	1	7	5	0	8
109	3	6	2	1	4	7	5	0	8
110	2	4	7	1	3	6	8	0	5
122	2	3	4	1	8	7	6	0	5
165	5	6	3	1	2	7	4	0	8
170	3	6	5	2	1	4	8	0	7
171	4	3	7	2	6	1	8	0	5
174	5	4	7	2	6	3	8	0	1
185	2	3	4	1	6	7	8	0	5

W= 4.2857143E-01 CHISQ= 3.0000000E 01 D.OF FREEDOM= 7

KENDALL COEFFICIENT OF CONCORDANCE : W

SET	M(1)	M(2)	M(3)	M(4)	M(5)	M(6)	M(7)	M(8)	M(9)
102	5	6	4	2	1	8	7	3	0
106	5	7	2	4	1	8	6	3	0
109	4	7	2	1	5	8	6	3	0
110	2	4	6	1	3	5	8	7	0
122	2	3	4	1	8	7	6	5	0
165	6	7	4	1	2	8	5	3	0
170	3	6	5	2	1	4	8	7	0
171	4	3	6	2	5	1	8	7	0
174	4	3	6	1	5	2	8	7	0
185	2	3	4	1	6	7	8	5	0

W= 4.3047619E-01 CHISQ= 3.0133333E 01 D.OF FREEDOM= 7

The values calculated for W and χ^2 are given in Table IV below. It can be seen that in the first test, where no model was omitted, χ^2 is significant at $< .001$, and that when models are omitted one by one, removal of the Wiltshire model causes the largest reduction in W and χ^2. Examination of the rankings indicates that the Wiltshire model consistently gives the best fit.

TABLE IV

VALUES OF W AND χ^2 FOUND FOR 10 DATA-SETS, 9 MODELS, WITH ONE MODEL DELETED FROM RANKINGS

Model Omitted	W	χ^2	Degrees of Freedom	Significance <
NONE	0.408	32.64	8	.001
BEVIS	0.394	27.60	7	.001
GOMPERTZ	0.427	29.90	7	.001
MATHEMATICAL	0.428	29.97	7	.001
WILTSHIRE	0.280	19.57	7	.01
ACCUMULATIVE	0.467	32.67	7	.001
REPLACEMENT	0.460	32.20	7	.001
DE JONG	0.336	23.50	7	.005
LOGMATHEMATICAL	0.429	30.00	7	.001
SECOND ORDER MODEL	0.430	30.13	7	.001

Even so, the value of χ^2 remains significant at < .01, suggesting that a further test could be done on the remaining 8 models by once again removing a model and establishing if this caused a large change in W and χ^2.

The test was repeated using the same 10 data sets, and the results are set out in Table V. Computer printout has not been included as the resulting text would become too bulky. In this case removal of the de Jong model causes the greatest reduction in the χ^2 value.

TABLE V

VALUES OF W AND χ^2 FOUND FOR 10 DATA SETS, 8 MODELS, WITH ONE MODEL DELETED FROM RANKINGS

Model Omitted	W	χ^2	Degrees of Freedom	Significance <
NONE	0.280	19.57	7	.01
BEVIS	0.246	14.74	6	.025
GOMPERTZ	0.301	18.04	6	.01
MATHEMATICAL	0.287	17.23	6	.01
ACCUMULATIVE	0.311	18.64	6	.005
REPLACEMENT	0.329	19.71	6	.005
DE JONG	0.183	10.97	6	.10
LOGMATHEMATICAL	0.287	17.23	6	.01
SECOND ORDER MODEL	0.281	16.89	6	.01

Once the effect of the Wiltshire model had been established, it was possible to extend the scope of the test by examining 23 sets of data. (As only 8 models were being considered, more sets of data had been curve fitted by those 8 models). Results are tabulated below in Table VI. As can be seen, the de Jong model still causes the greatest reduction in χ^2 values. In this case, examination of the rankings shows that the de Jong model consistently gave the worst fits.

TABLE VI

VALUES OF W AND χ^2 FOUND FOR 23 DATA SETS, 8 MODELS, WITH ONE MODEL DELETED FROM RANKINGS

Model Omitted	W	χ^2	Degrees of Freedom	Significance <
NONE	0.107	17.23	7	.02
BEVIS	0.108	14.91	6	.025
GOMPERTZ	0.123	16.94	6	.01
MATHEMATICAL	0.0884	12.20	6	.10
ACCUMULATIVE	0.134	18.56	6	.005
REPLACEMENT	0.139	19.16	6	.005
DE JONG	0.0485	6.69	6	.50
LOGMATHEMATICAL	0.0870	12.00	6	.10
SECOND ORDER MODEL	0.135	18.58	6	.005

Removal of the de Jong model reduced the number of models to be considered to 7, but also increased the data-sets available to be ranked to 54. The calculation was repeated, values of W and χ^2 being shown in Table VII below. In this instance, the removal of the logmathematical model causes the greatest reduction in χ^2. Again, this model consistently gave the worst fits when the rankings were examined.

TABLE VII

VALUES OF W AND χ^2 FOUND FOR 54 DATA SETS, 7 MODELS, WITH ONE MODEL DELETED FROM RANKINGS

Model Omitted	W	χ^2	Degree of Freddom	Significance $<$
NONE	0.0684	22.17	6	.005
BEVIS	0.0714	19.28	5	.005
GOMPERTZ	0.0743	20.05	5	.005
MATHEMATICAL	0.0627	16.94	5	.005
ACCUMULATIVE	0.0664	17.92	5	.005
REPLACEMENT	0.0967	26.11	5	.001
LOGMATHEMATICAL	0.0287	7.76	5	.20
SECOND ORDER MODEL	0.0840	22.69	5	.001

The analysis was extended to one more case - six models and 61 data sets. Results are shown in Table VIII. While the elimination of the second order model would cause the greatest reduction in χ^2, no further analysis could be attempted due to the unsuccessful curve fit attempts. The second order model was consistently giving the worst fits for this set of rankings.

TABLE VIII

VALUES OF W AND χ^2 FOUND FOR 61 DATA SETS, 6 MODELS, WITH ONE MODEL DELETED FROM RANKINGS

Model Omitted	W	χ^2	Degrees of Freedom	Significance <
NONE	0.0386	11.78	5	.05
BEVIS	0.0475	11.58	4	.025
GOMPERTZ	0.383	9.35	4	.10
MATHEMATICAL	0.051	12.44	4	.02
ACCUMULATIVE	0.0278	6.78	4	.20
REPLACEMENT	0.0419	10.23	4	.05
SECOND ORDER MODEL	0.0265	6.47	4	.20

7.3. Comparison of "Best Fit" Start and Final Values.

A further comparative assessment of the models investigated may be made by studying the 'start' and 'final' values. This method if obviously better than comparing the parameter values, because the parameters do not necessarily have similar meanings from one model to another. In this section, only a selection of data sets, with their 'start' and 'final' values, are compared. A complete list of parameter, 'start' and 'final' values is given in Appendix F, and a comparison of 'start' and 'final' values on a Model basis is given in Appendix G.

Consider the best fit 'start' and 'final' values obtained for the curve fitting of data set No. 106 (Mean score of 4 subjects taken from Blackburn's[45] data Operation: - Addition) given in Table 9 below.

TABLE IX

A TYPICAL DATA-SET, WITH A COMPARISON OF 'START' AND 'FINAL' VALUES FOUND FOR EACH MODEL

			Data		Data	
Model	Final	Start	X	Y	X	Y
BEVIS	134.91	61.00	1.0	73.3	15.0	124.7
GOMPERTZ	133.92	64.34	2.0	74.2	16.0	125.7
			3.0	90.7	17.0	127.9
MATHEMATICAL	150.22	52.21	4.0	97.8	18.0	130.9
WILTSHIRE	139.29	45.88	5.0	102.7	19.0	130.9
ACCUMULATIVE	145.02	57.57	6.0	109.9	20.0	130.2
			7.0	113.2	21.0	131.2
REPLACEMENT	133.40	66.57	8.0	117.9	22.0	134.3
DE JONG	422.88	67.75	9.0	114.7	23.0	135.6
			10.0	118.4	24.0	134.9
LOGMATHE-MATICAL	150.99	54.78	11.0	120.9	25.0	135.3
			12.0	121.8	26.0	130.7
SECOND ORDER MODEL	132.32	72.86	13.0	128.1	27.0	139.9
			14.0	123.7		

It is immediately obvious that the de Jong prediction of the final value is very much higher than the other predictions. The 'start' values, as is to be expected, are reasonably the same. However, this feature of the de Jong model predicting much higher final values is fairly general, as 25 of the predicted final values obtained for the de Jong model (out of 37) were the highest values obtained from the successful curve fitting runs.

Other examples may be found in Appendix G where the predictions of final values were not sensibly the same (as in the above example). Consider the results for data set No. 0116 (BEVIS[46], Mean Score of 15 subjects Operation: - Bunching (Plant A)) given in Table X below.

TABLE X

A SECOND EXAMPLE OF A DATA SET, WITH "FINAL" VALUES FOUND FOR EACH MODEL

Model	Final	Data X	Data Y
BEVIS	7338.01	1.0	1800.0
GOMPERTZ	5733.79	2.0	2015.0
MATHEMATICAL	11607.18	4.0	2321.0
WILTSHIRE	4857.07	6.0	2829.0
ACCUMULATIVE	7591.04	8.0	3085.0
REPLACEMENT	5484.52	10.0	3703.0
DE JONG	-	12.0	4084.0
LOGMATHEMATICAL	11788.0	14.0	4225.0
		16.0	4515.0
SECOND ORDER MODEL	5230.15	18.0	4617.0

If one wishes to set a standard for output on this operation, what value does one choose? In this case it is suggested that the mean value might well be a reasonable choice, but the range of values found emphasises the danger of selecting any one learning curve model and slavishly applying the results to calculate work study standards.

7.4. Discussion and Conclusions

In this comparitive study of the fits of various models to a selection of learning data, it has been shown that the Wiltshire Model is most consistently the best fit. This is, perhaps, obvious, when one realises that the Wiltshire model has four parameters, and the remaining models only three. Mathematically, one would expect a four parameter model to be a better fit than a three parameter model, unless the three parameter model was an exact representation of the learning data. It suggests that a useful further study might be one in which various four parameter models were compared.

Rather more surprising is the discovery that the de Jong model gave consistently the worst results. Not only was the difficulty in establishing the parameters more evident than for the other three parameter models, but those results that were obtained also predicted "final" values which were, in general, much higher than the "final" values predicted by the other models.

As such, this result throws doubt on the usefulness of the de Jong model (in comparison with the other models) when individual learning curves are to be analysed. Further work appears necessary to confirm both the above results.

It was not unexpected that the mathematical model was not rejected by the analysis, because it has been shown that that model is very similar to the Accumulative Model. The logarithmic form of the same model was shown to be the worst of the remaining models, however, and this suggests that manipulation of the x, y scales is not an improvement. It would be of interest to study the results of fitting the selected models to logarithmic scales and comparing them on that basis.

In the final table, the second order model appeared to be the worst of the 6 remaining models, indicating that perhaps the general shape of the critically damped model is not the most suitable choice. Further investigations on four parameter models would allow the overdamped and underdamped solutions of the second order equations to be compared with other four parameter models and previous results. A further experiment of interest would be to compare the damping factors found for individuals learning of different tasks. Might they be the same?

A "best" model cannot be selected from those remaining (Bevis, Gompertz, Accumulative, Replacement, Mathematical), purely on the basis of the statistical test done. Given very much larger quantities of learning data this might be possible, but in this instance it seems best policy to choose a model which has parameters which may be defined in understandable terms. This criterion indicates the Bevis model as the most suitable choice because the three parameters Y_c, Y_f and τ may be easily defined in terms which are acceptable.

8. AN EXAMINATION OF THE LEARNING DATA FOR INDIVIDUAL TELEPHONISTS

8.1. Introduction.

In the earlier chapters of this thesis it has been shown that there is little to choose between several learning curve models. The Bevis model was suggested as the most suitable model, but not directly as a result of the objective assessment attempted. To investigate the use of the Bevis model as applied to data related to skills acquired by Post Office employees, it was decided to attempt to fit the Bevis model to all the individual data-sets obtained from Oxford and North West Telephone Areas mentioned earlier in Section 6.5, and also to examine in rather more detail the fitting of the model to data obtained by personal observation.

8.2. Fitting the Bevis Model to Recorded Data.

The attempt to fit the Bevis model to the data sets was made in two ways. In the first curve fitting attempt, all available data was used. In the second, the last data point was eliminated from those data sets which included a full efficiency check and curve-fitting again attempted. A comparison of the two sets of results showed that there was little correspondence between them.

A total of 87 data sets were used in the investigation. 73 data-sets included the full efficiency check. 44 pairs of results were obtained for the complete and one-data-point-omitted data sets. Only 9 of those pairs resulted in parameters which were all positive, and only 5 pairs gave parameters which were equal to within ± 25% for each curve fitting attempt.

15 pairs of results included negative Y_f and negative τ values for that data set with less data. 7 further pairs of results included values for Y_c or Y_f which were unlikely to be accurate.

e.g. Set 249
$Y_c = -5837$
$Y_f = 6020$
$\tau = 1.10$

Set 282
$Y_c = -33.73$
$Y_f = 593.5$
$\tau = 46.8$

A complete set of all results obtained is included in Appendix H.

8.3. Discussion.

Why were these results so poor? The first possibility is to suggest that not enough data was available accurately to predict the true parameter values and also that observational error might cause this result. In addition, the effect of a data point at some time in the far future with very few observations in the intervening period would force the curve fitting routine to hunt to parameters which would predict that value. Consider data set 231 given in Table XI below.

TABLE XI

DATA SET 231 - VALUED CALLS FOR T31, EXCHANGE A	
Day	Valued Calls
5.0	98.0
7.0	108.0
10.0	155.0
13.0	165.0
15.0	168.0
18.0	205.0
156.0	225.0

The above data is typical of the data investigated. The results for the two curve fitting runs were:-

	Y_c	Y_f	τ	Final
ALL DATA POINTS	1.50	225.34	9.64	226.84
LESS DAY 156	41.54	321.62	26.68	363.16

In the first run, the effect of the observation on day 156 is to cause the prediction of parameter values Y_c and Y_f which total 226.84, very close to the observed value of 225.0. If the last data point is removed the prediction then becomes $Y_c + Y_f$ = final = 363.16. The predicted τ values, 9.64 and 26.68 are not reasonably similar, thus the two predictions do not agree.

A further examination of data set 231, however, shows that the subject was learning well up to day 18, and that the predicted values of Y_c and Y_f may well be reasonably accurate for the data to day 18. What happened in the period day 18 - day 156?

8.4. An Examination of More Detailed Learning Data.

We can investigate the question posed in the previous section by considering the results when the data obtained by intensive observation of trainees (mentioned in Section 6.5) is curve-fitted. The data obtained was made up into data sets which covered

(i) The first 3 weeks of training

(ii) To the end of training (5 weeks)

(iii) All observations.

(iv) Experience data only (all observations less training data),

and is given in the above form in Appendix I.

Now if the learning curve follows the one equation during the observational period for a telephonist, then as long as sufficient and accurate data is available, the parameter values formed by a curve-fitting approach will be similar. If the values are not reasonably the same then the implication is that there has been a change in the learning process, and that the learner is on a new learning curve. All the results obtained are given in Table XII, blank spaces indicating failure to curve fit successfully.

TABLE XII

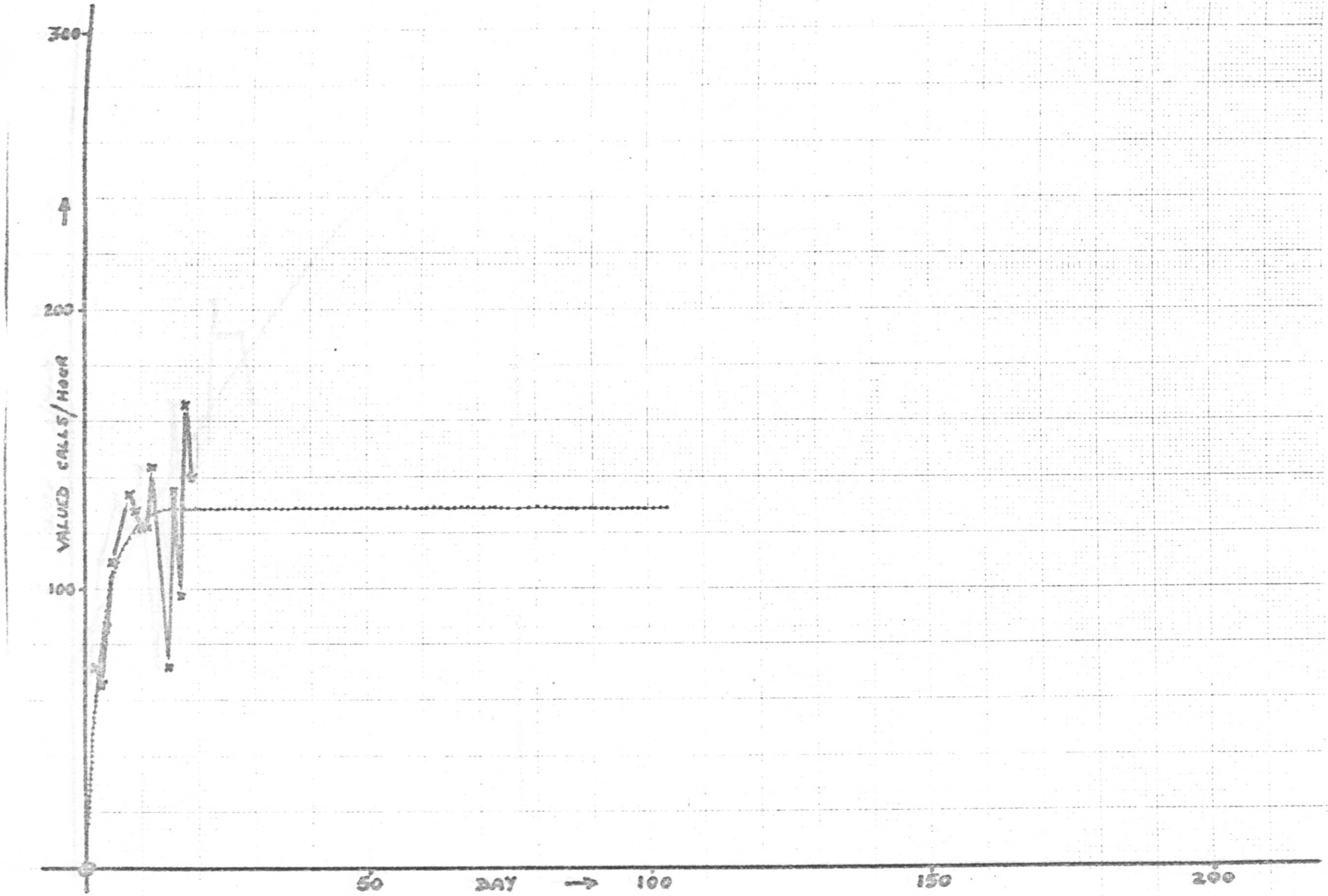
ALL PARAMETER VALUES FOUND FOR BEVIS MODEL CURVE-FITTED TO DATA OBTAINED BY DIRECT OBSERVATIONS

Trainee	Set	Y_c	Y_f	τ
JJ	301			
JJ	302	86.79	102.13	15.60
JJ	303	92.50	164.71	36.23
JJ	304			
KF	305	44.65	288.06	47.46
KF	306	21.12	112.16	7.11
KF	307	60.90	157.11	39.60
KF	308	144.12	72.67	24.98
LS	309	-27.75	133.25	1.64
LS	310	76.97	89.79	34.02
LS	311	79.68	130.03	61.11
LS	312	135.09	73.46	56.57
SJ	313	54.93	128.33	5.91
SJ	314	60.32	125.12	6.53
SJ	315	95.60	159.23	25.05
SJ	316			
EB	317	-0.87	129.26	3.01
EB	318	72.66	325.65	73.41
EB	319	67.15	156.94	26.48
EB	320	175.34	52.47	33.86
KN	321			
KN	322	67.66	138.37	24.76
KN	323	63.26	123.00	18.23
KN	324			
JC	325	-106.81	205.79	2.29
JC	326	68.66	123.19	8.48
JC	327	41.22	192.58	42.85
JC	*			

* JC resigned at an early date and only 2 observations were made after completion of training. Curvefitting is thus pointless in this case.

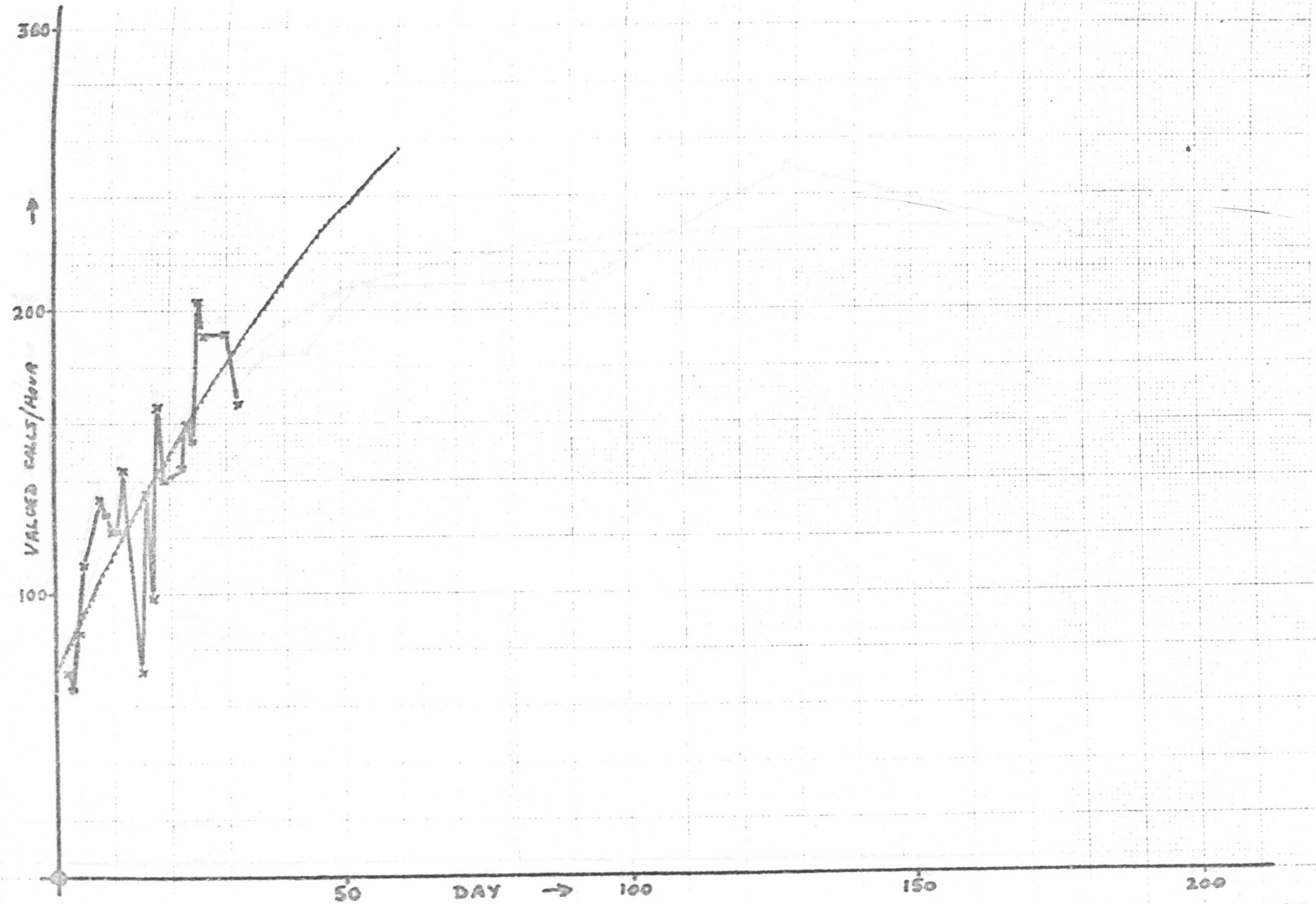
Comparison of the results shows that the considerations discussed previously still apply. Curve fits to data for only the early stages of training predict "final" values which are not consistent with what eventually occurs, and curve-fits for data covering longer time periods do not agree with previous estimates. Some of the results are shown in graphical form in diagrams 8-19 following: -

Diagram 8.



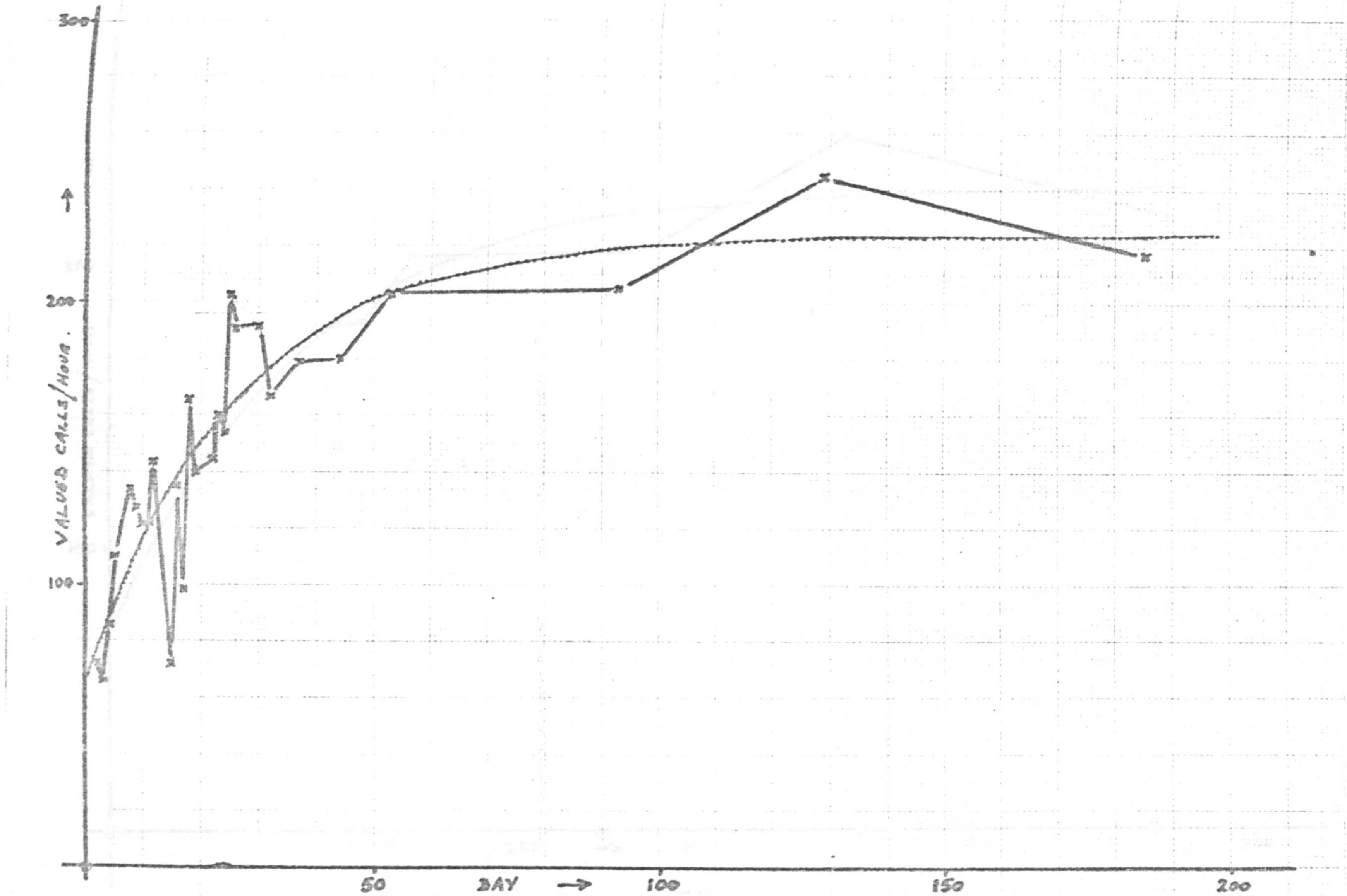
Learning Curve for EB, to end of first 3 weeks

Diagram 9



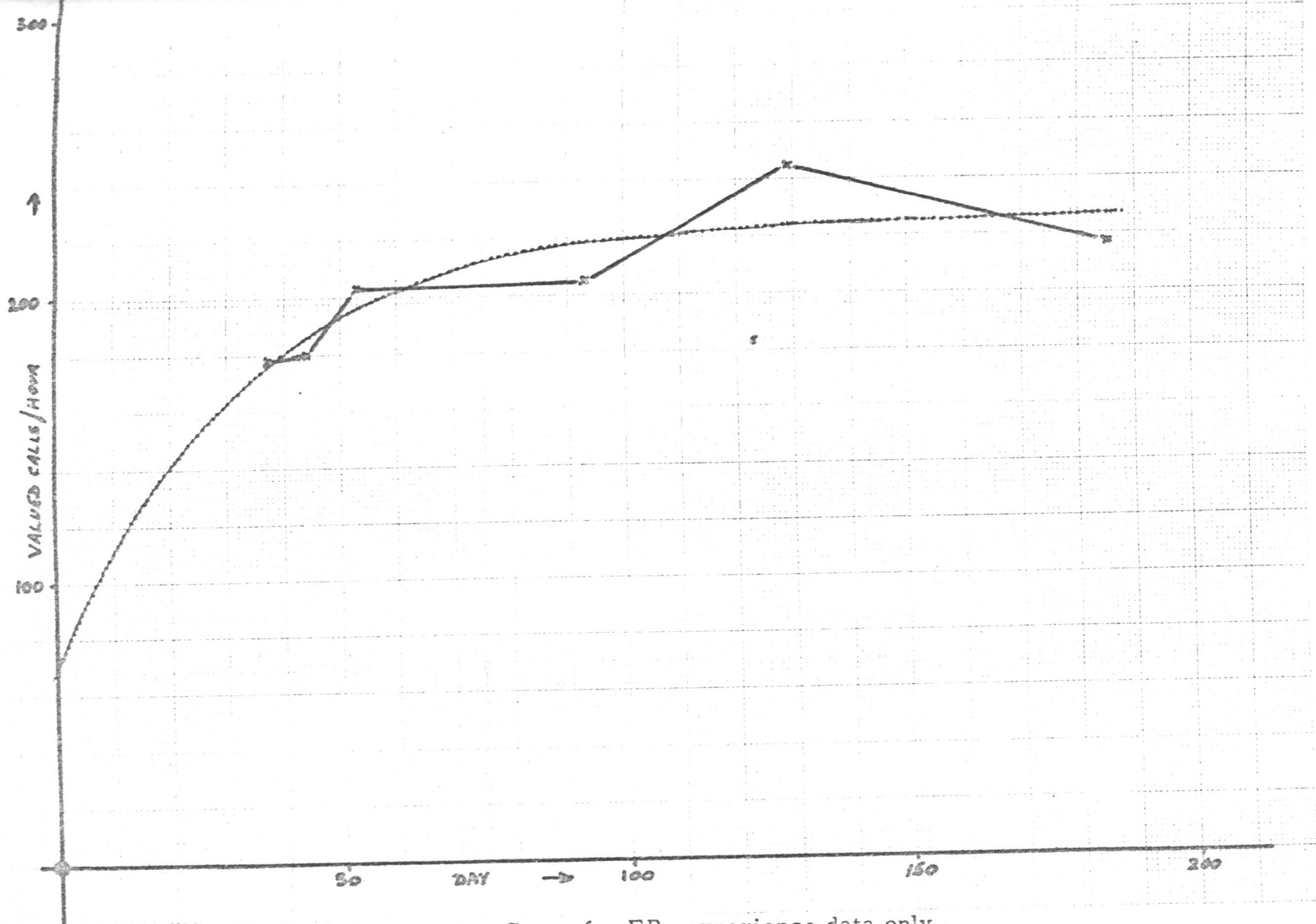
Learning Curve for EB, to end of training

Diagram 10



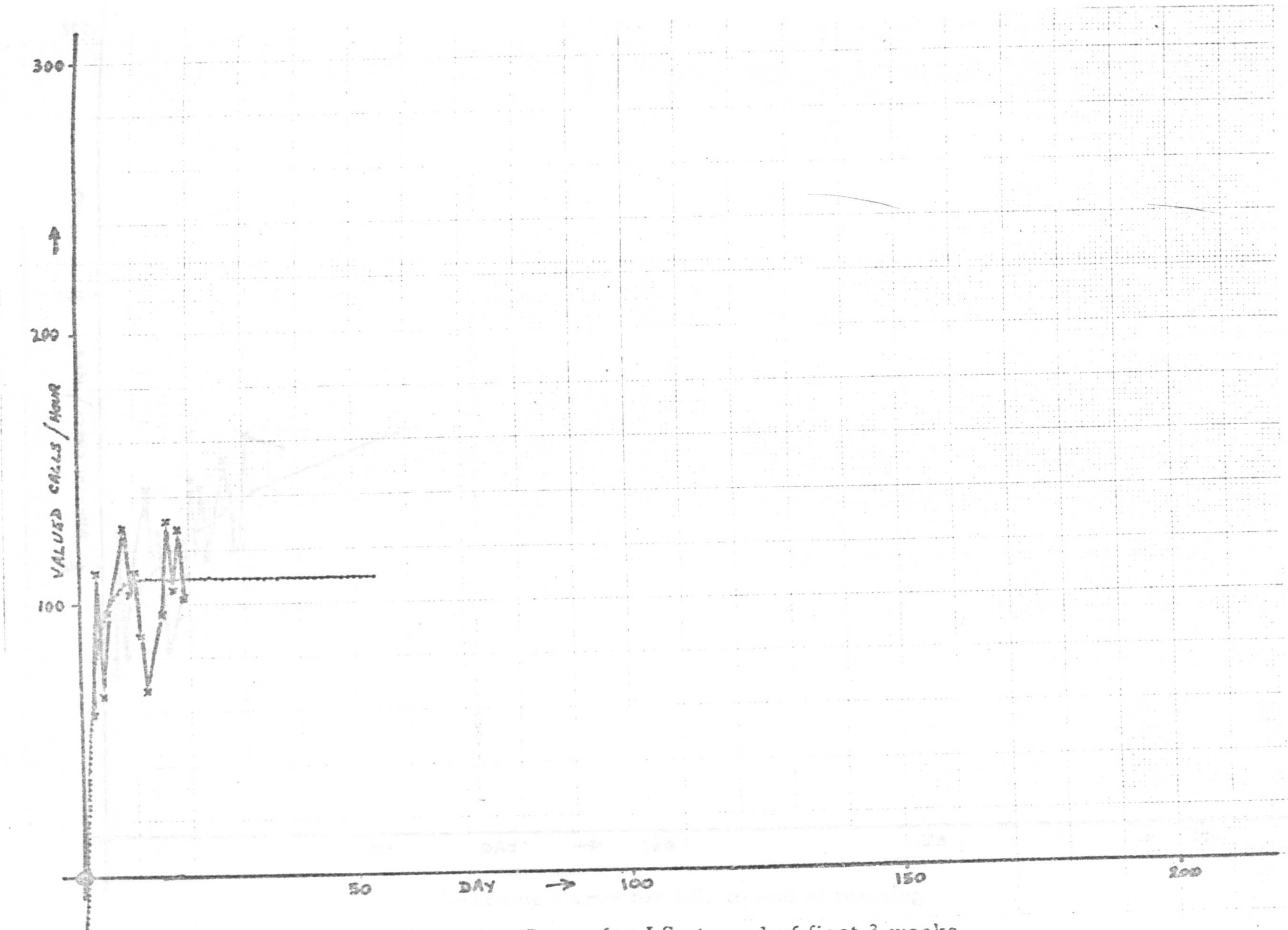
Learning Curve for EB, all observations

Diagram 11

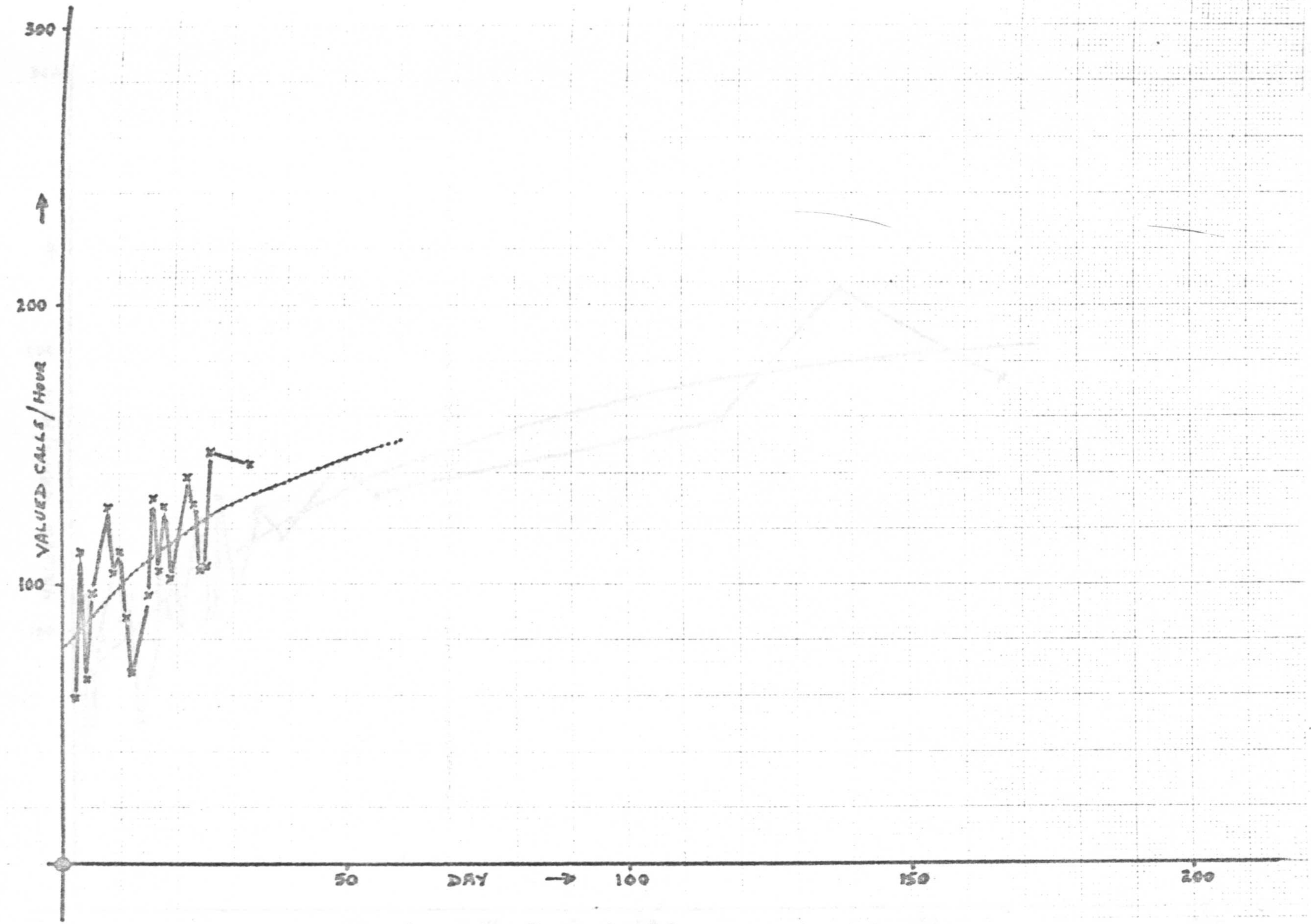


Learning Curve for EB, experience data only

Diagram 12

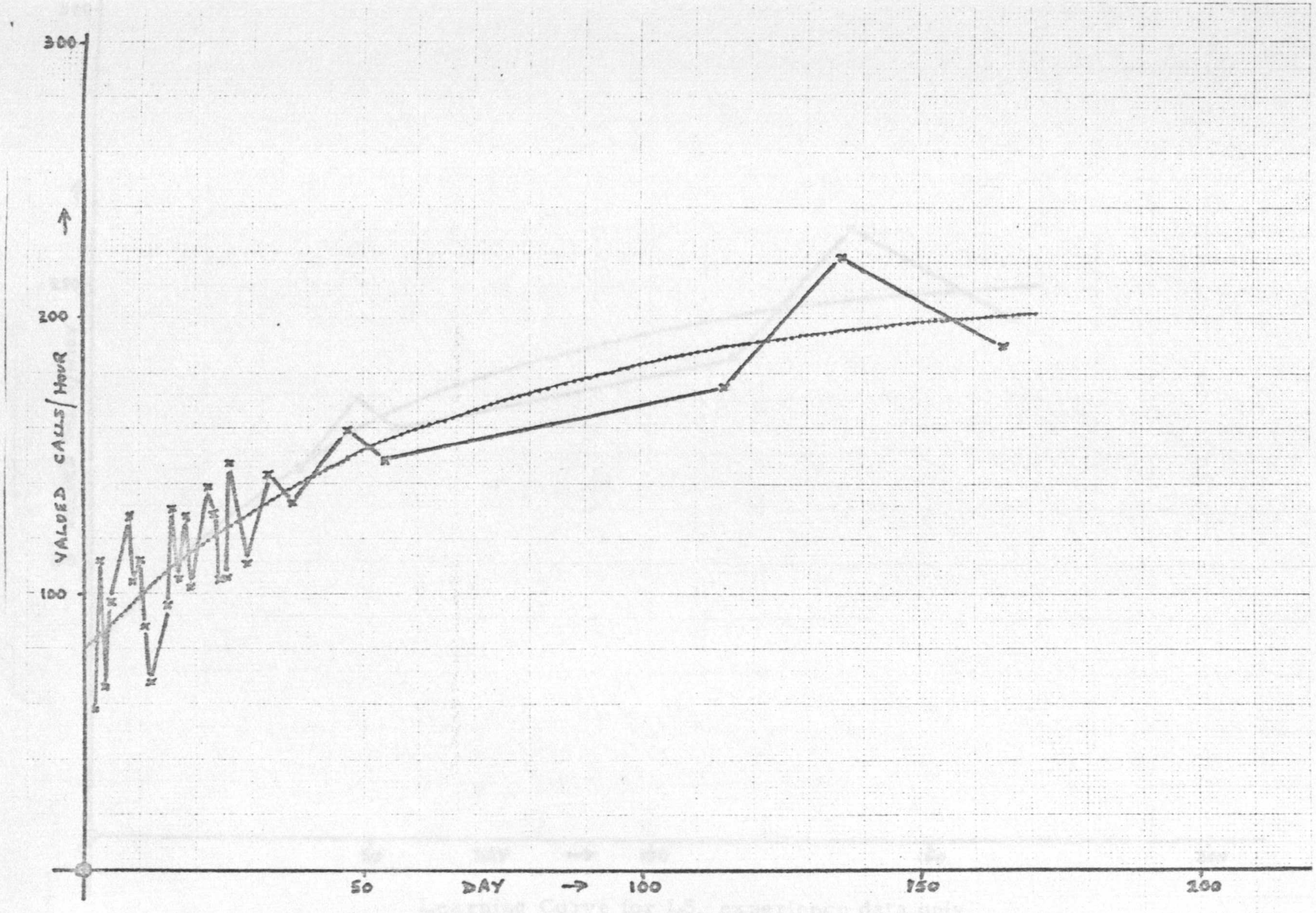


Learning Curve for LS, to end of first 3 weeks



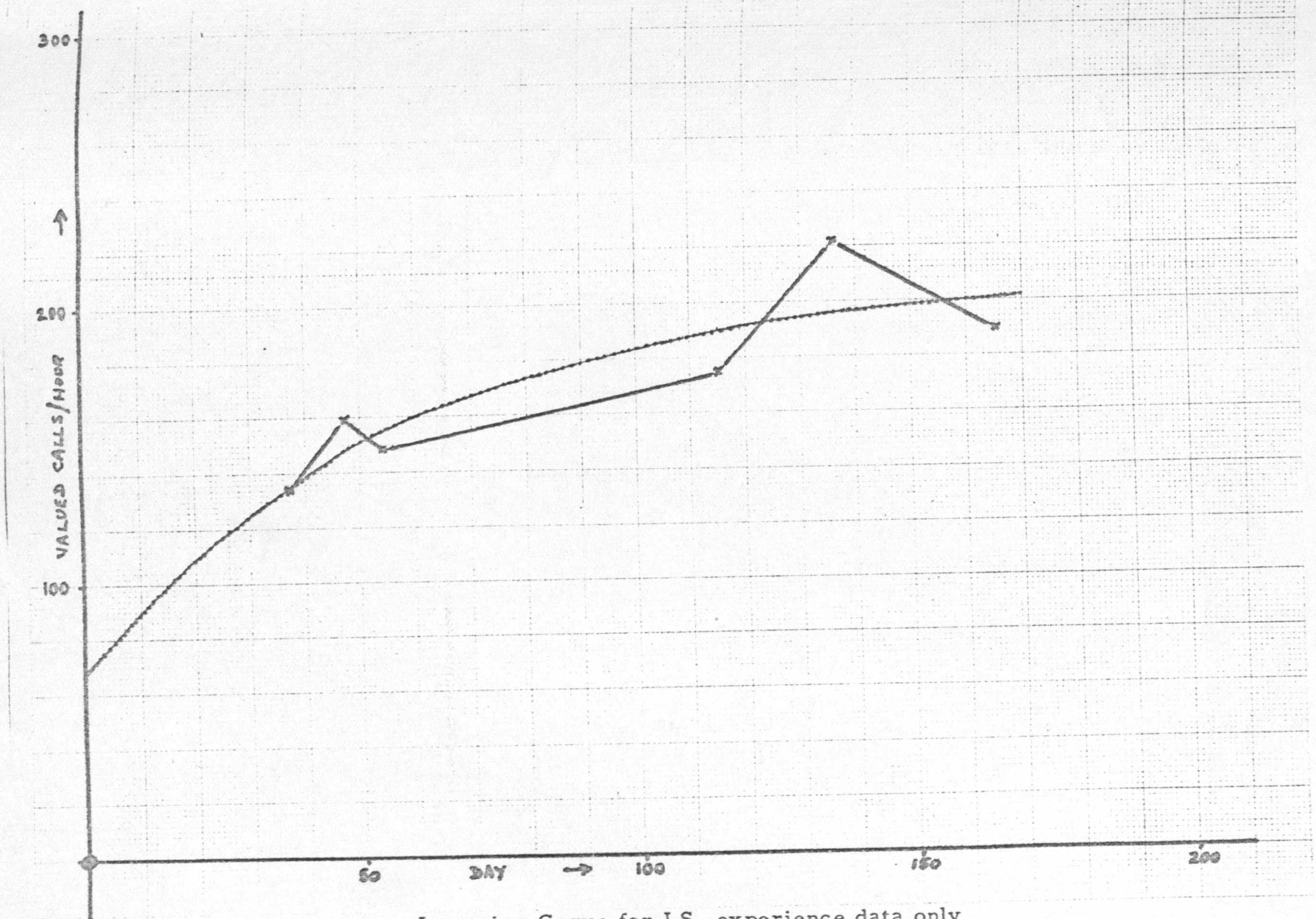
Learning Curve for LS, to end of training

Diagram 14



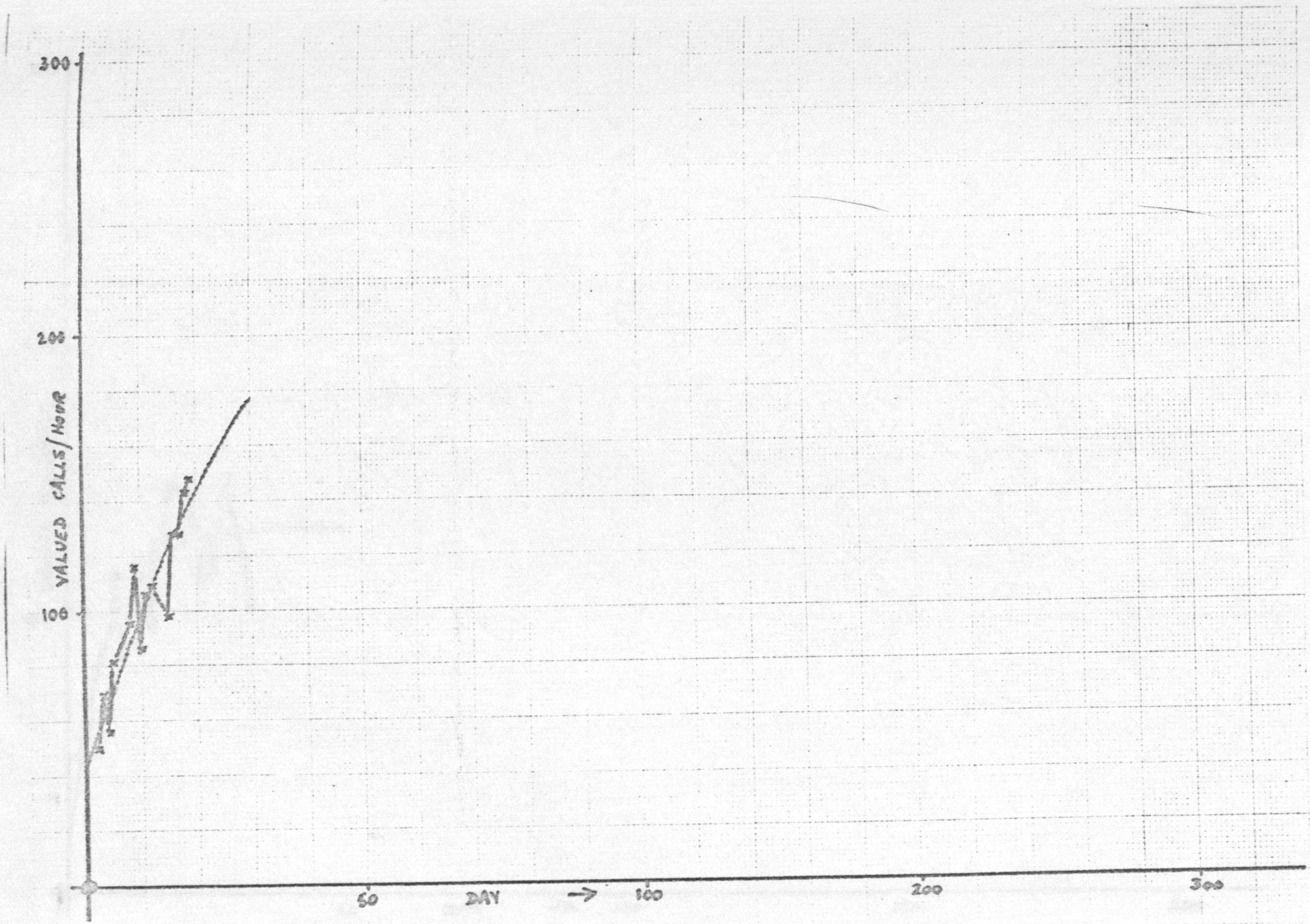
Learning Curve for LS, all observations

Diagram 15



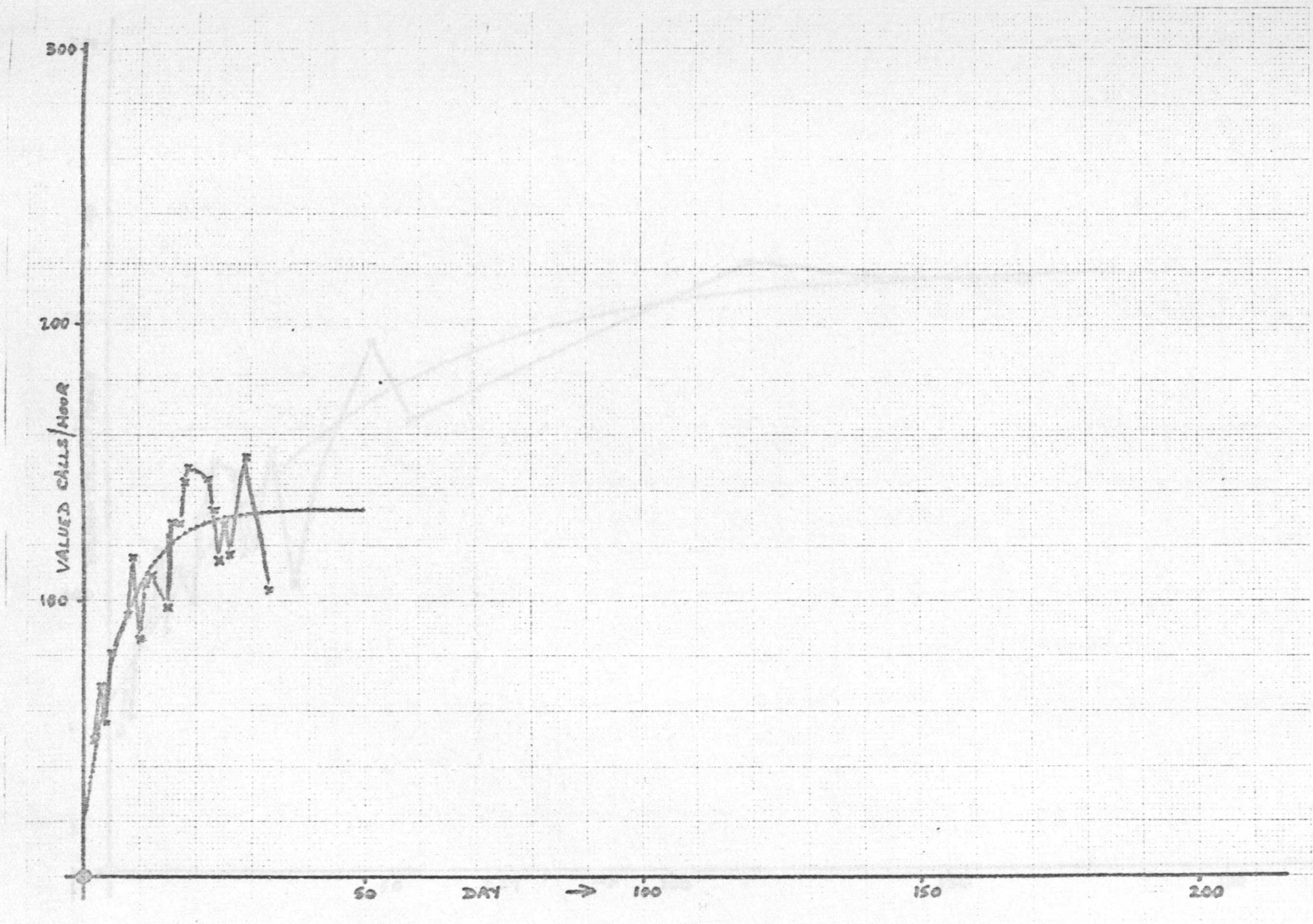
Learning Curve for LS, experience data only

Diagram 16



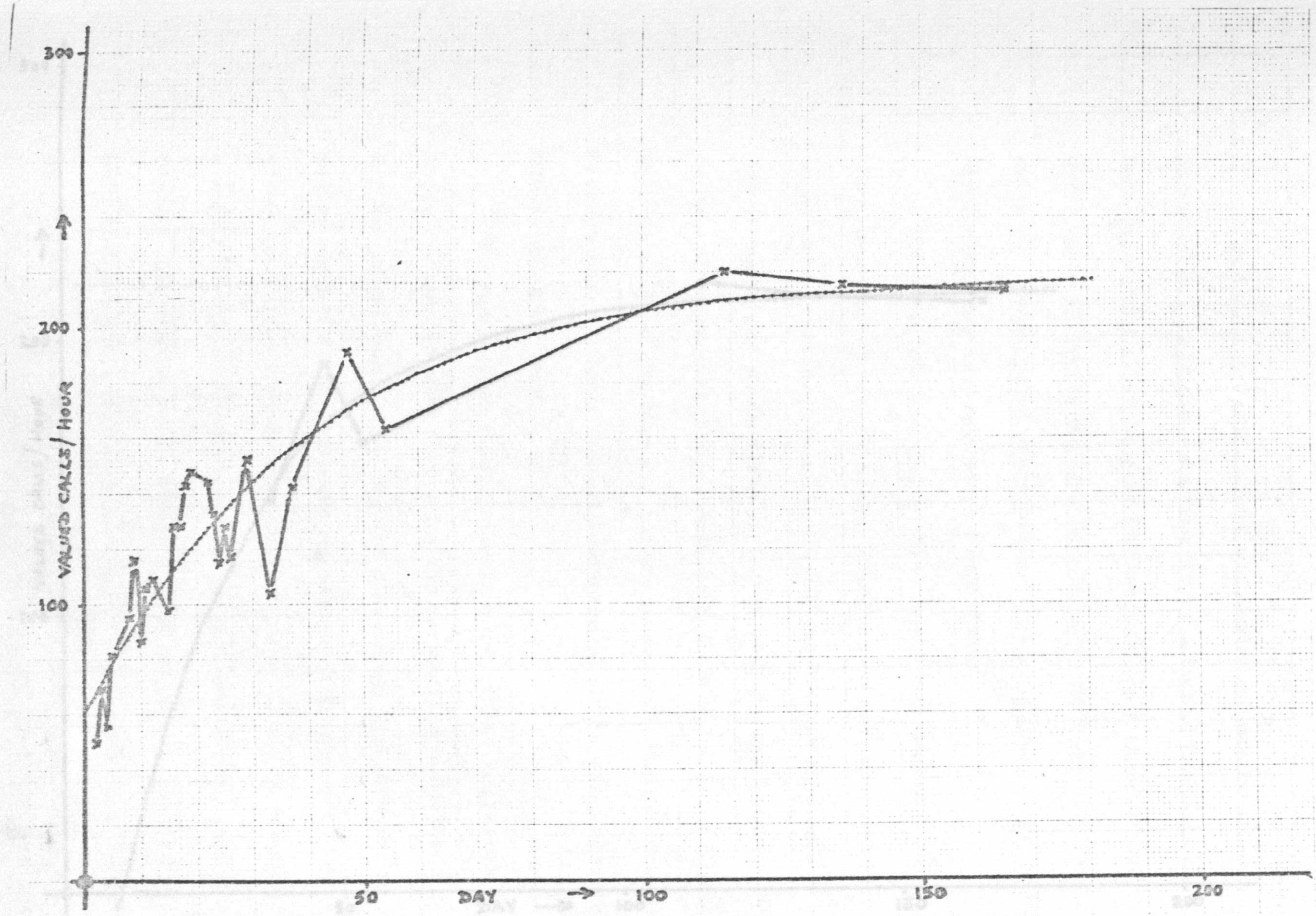
Learning Curve for KF, to end of first 3 weeks

Diagram 17



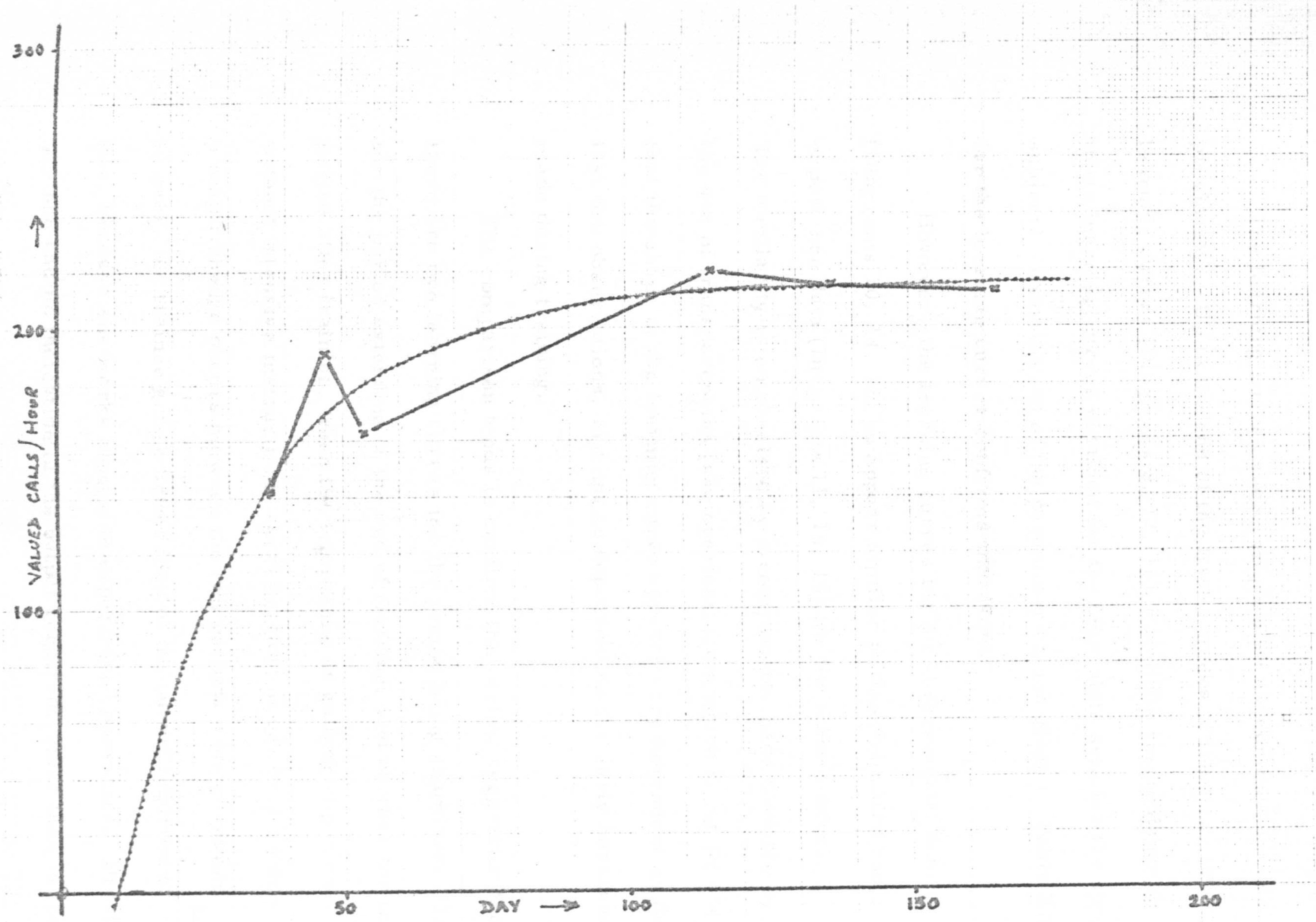
Learning Curve for KF, to end of training

Diagram 18



Learning Curve for KF, all observations

Diagram 19



Learning Curve for KF, experience data only

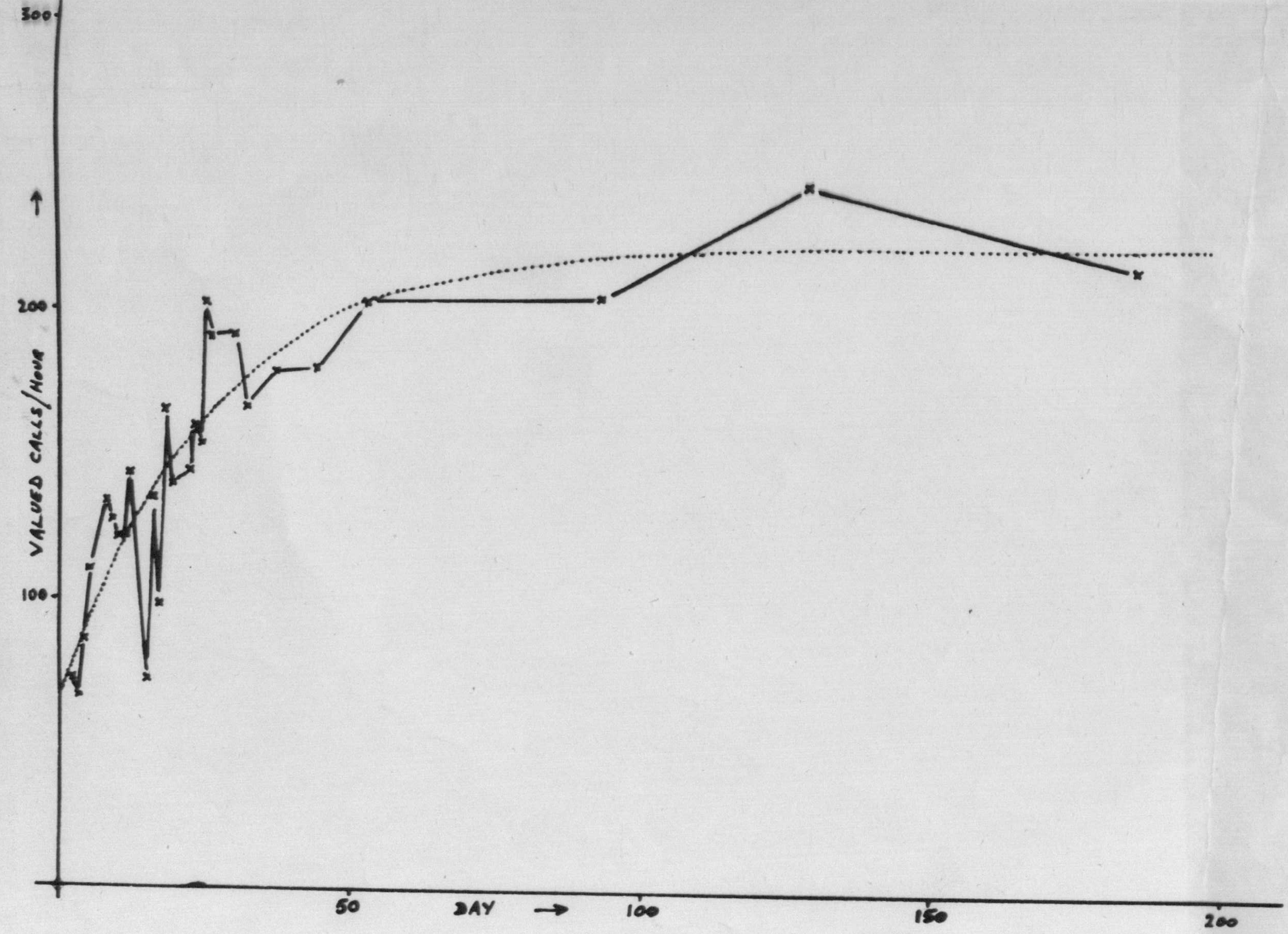
8.5. Discussion

The learning curves shown emphasise the comments made previously. Up to the end of the first 3 weeks of training, the learning curves (Diagrams 8, 12, 16) bear little resemblance to those learning curves obtained for the other data sets for the same subject. Examination of the parameter values given in Table XII for the learning curves confirms this point.

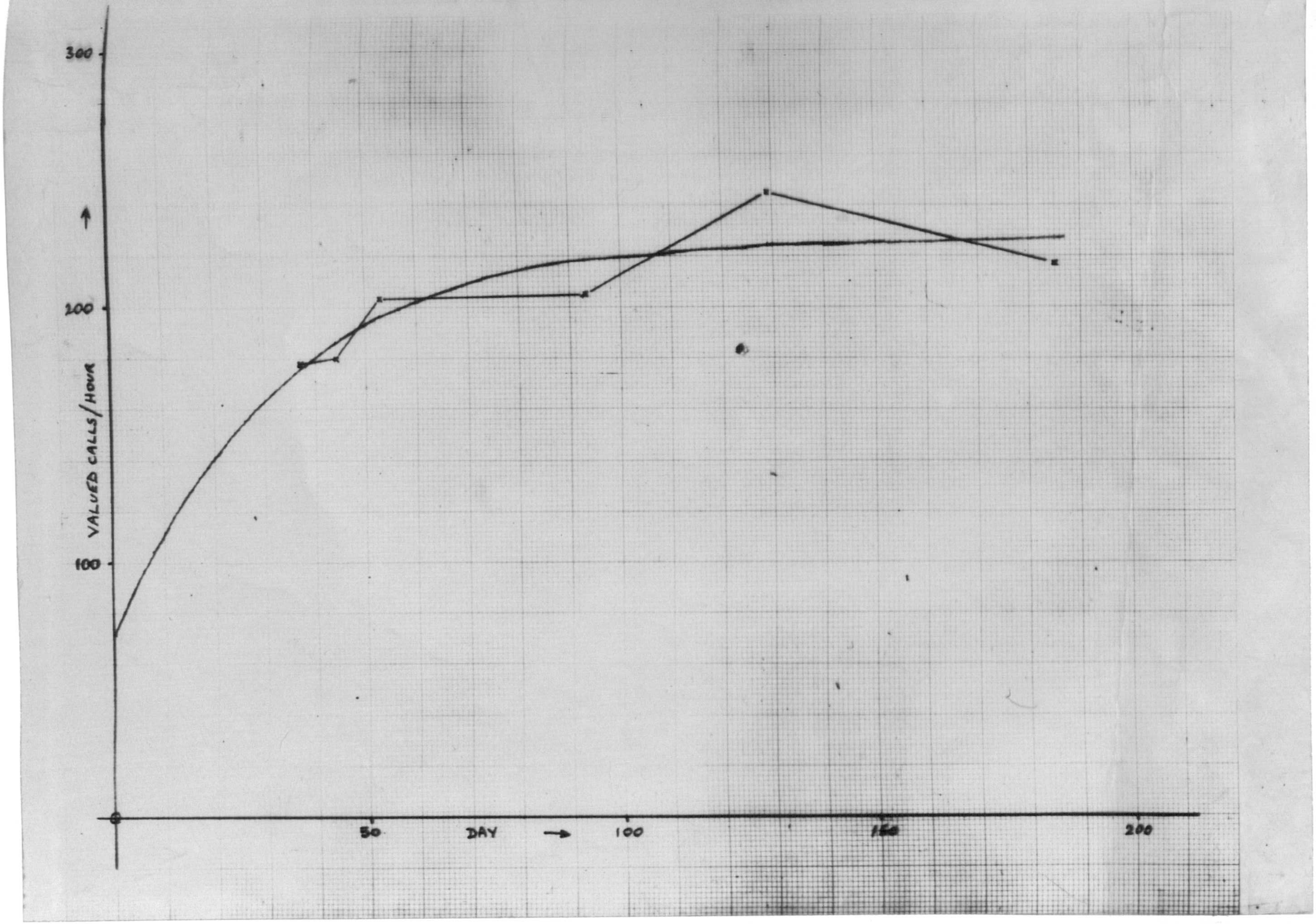
However, the learning curves for all observational data (Diagrams 10, 14, 18) do appear similar to those obtained for experience data (Diagrams 11, 15, 19) for the same subject. The similarity is emphasised by a comparison made possible by the use of transparencies (see overleaf) from which it can be seen that the shape of the learning curve appears very dependent on the last few observations, and not so dependent on the early observations made during training.

The comparison tends to confirm the earlier suggestion that there are two learning curves for the period being examined. One for the period extending to the end of training, and another for the period after training, when the telephonist is gaining experience without intensive instruction. On reflection, it can be seen that a major change occurs between the two periods - once training is over, the trainee gets a better idea of the output required of her, because she works alongside experienced operators. Previously, she had had no indication of the output required, other than in terms

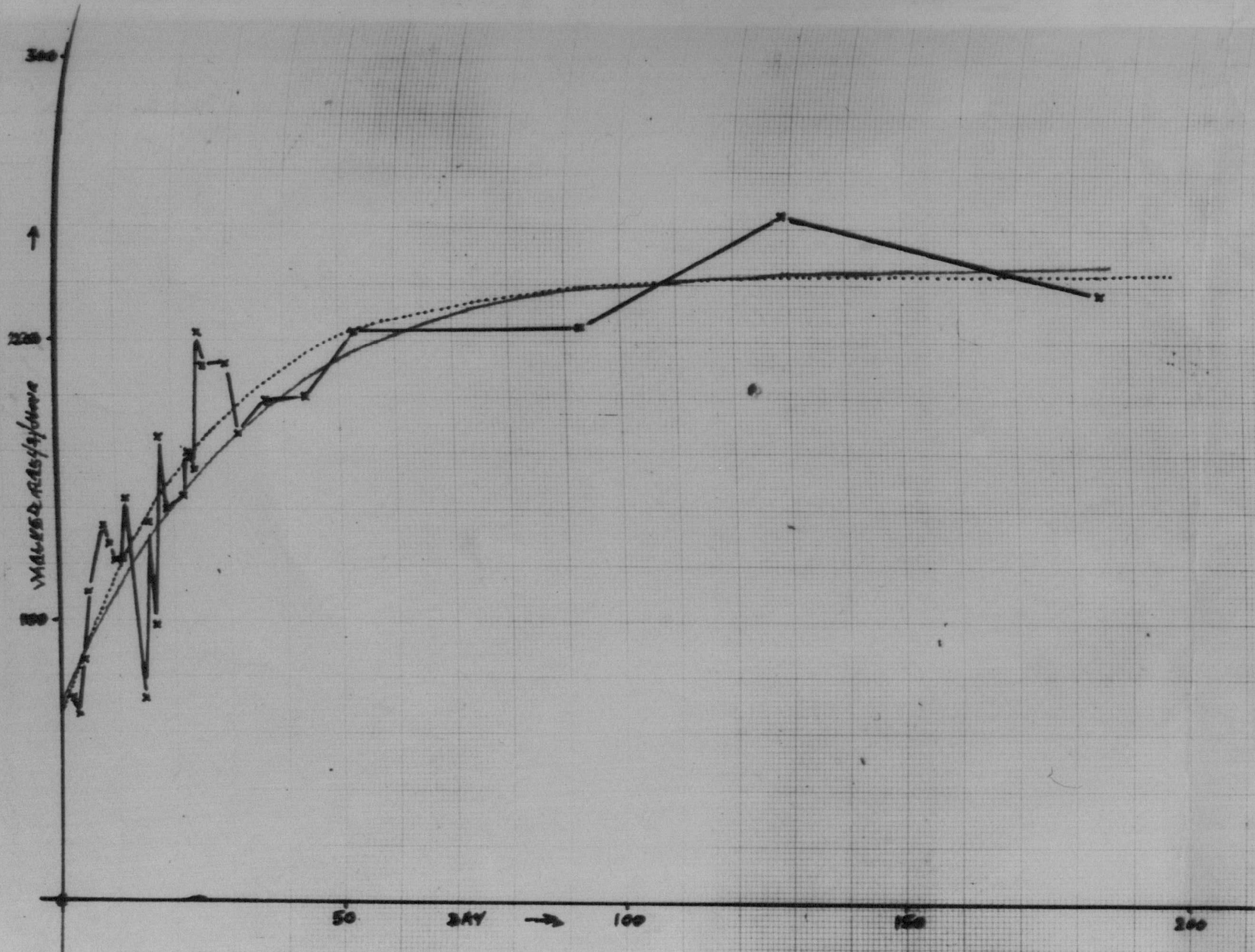
TRANSPARENCY 1 LEARNING CURVE FOR EB, ALL OBSERVATIONS
TRANSPARENCY 2 LEARNING CURVE FOR EB, EXPERIENCE DATA ONLY



300
200
100
VALUED CALLS/HOUR ↑
50
DAY →
100
150
200

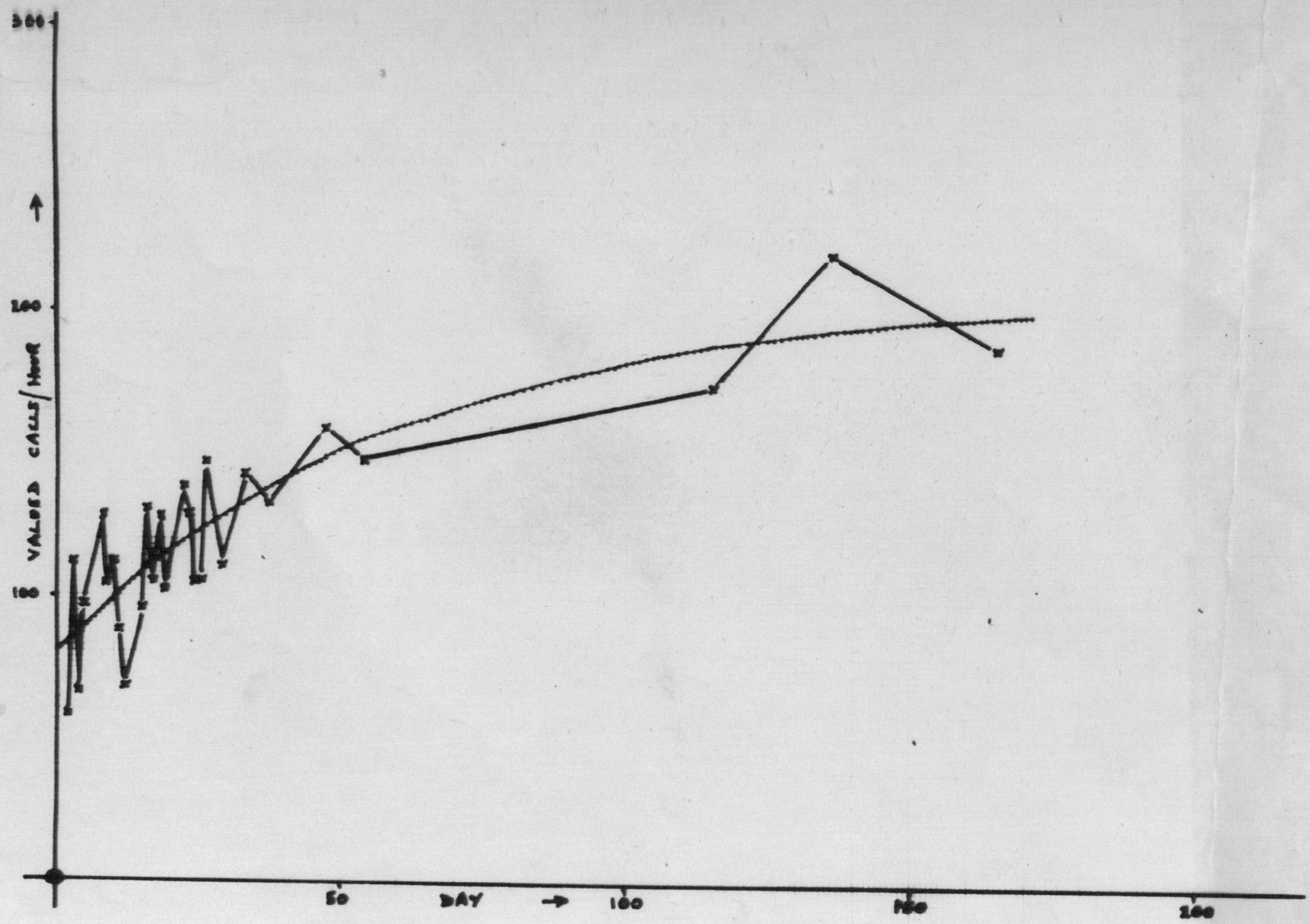


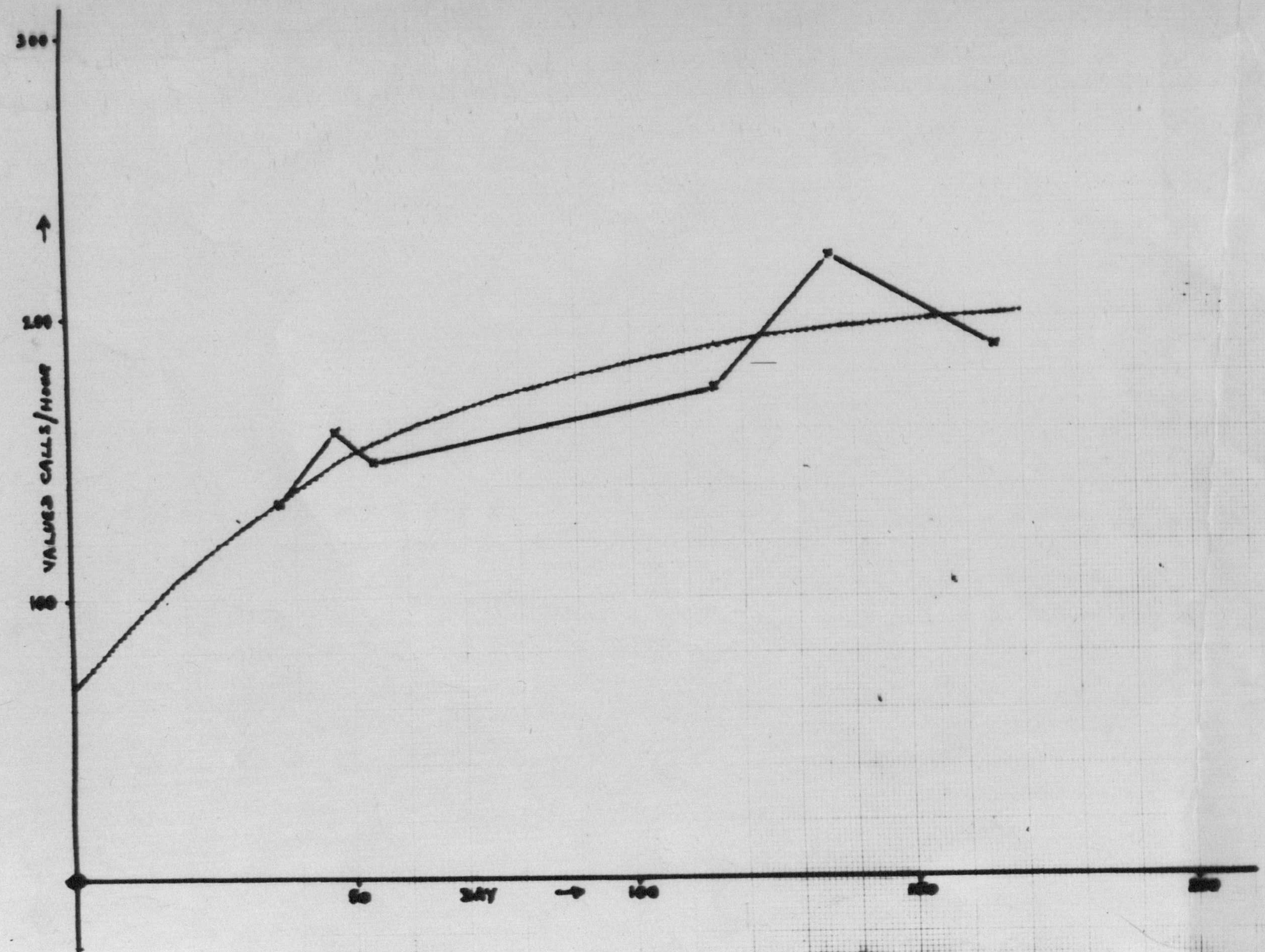
300
200
100
VALUED CALLS/HOUR
50
DAY
100
150
200


300
200
100
50
DAY →
100
150
200

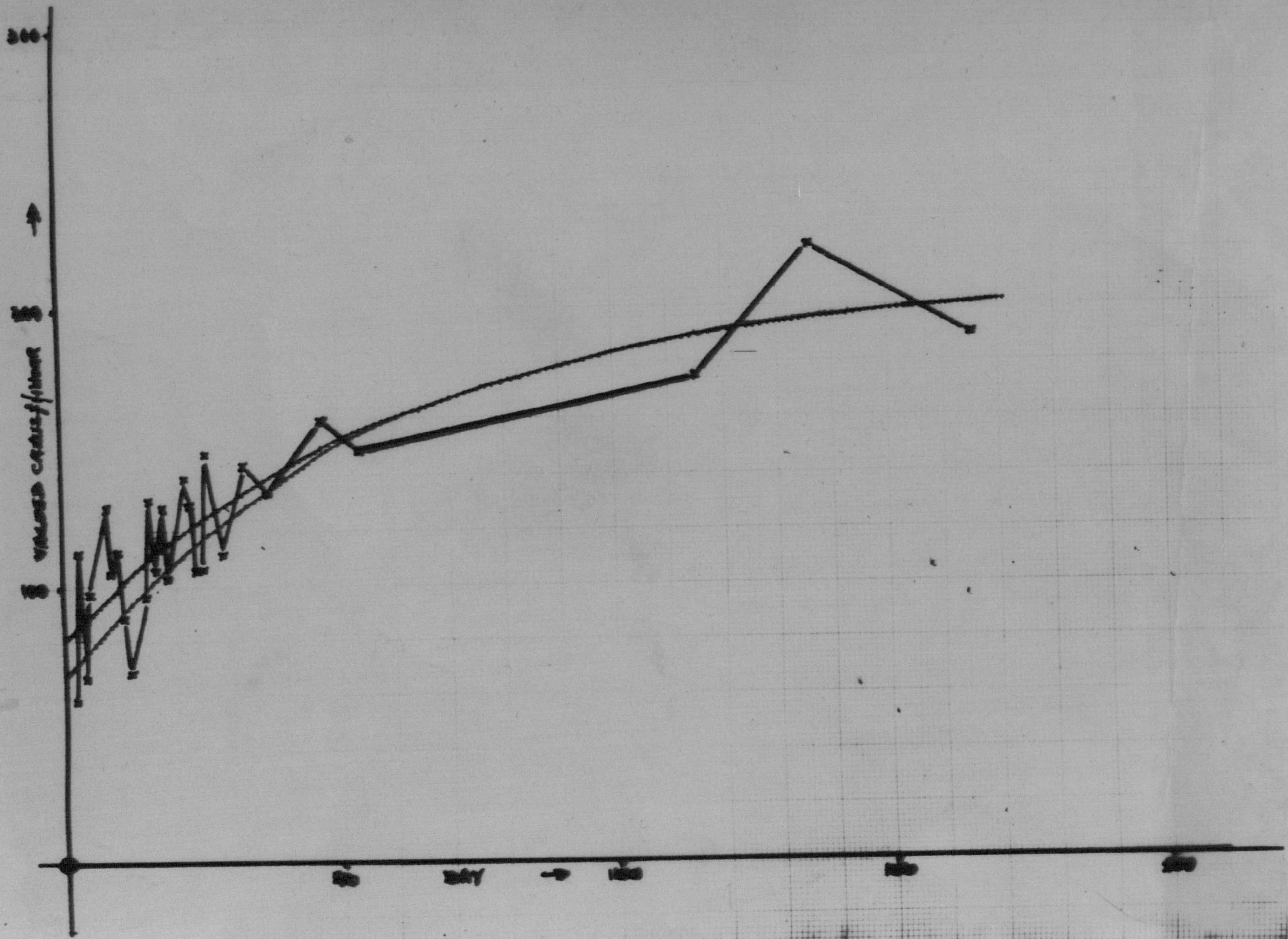
TRANSPARENCY 3 LEARNING CURVE FOR LS, ALL OBSERVATIONS

TRANSPARENCY 4 LEARNING CURVE FOR LS, EXPERIENCE DATA ONLY


300
200
100
VALUED CALLS/HOUR →
50
DAY → 100
150
200

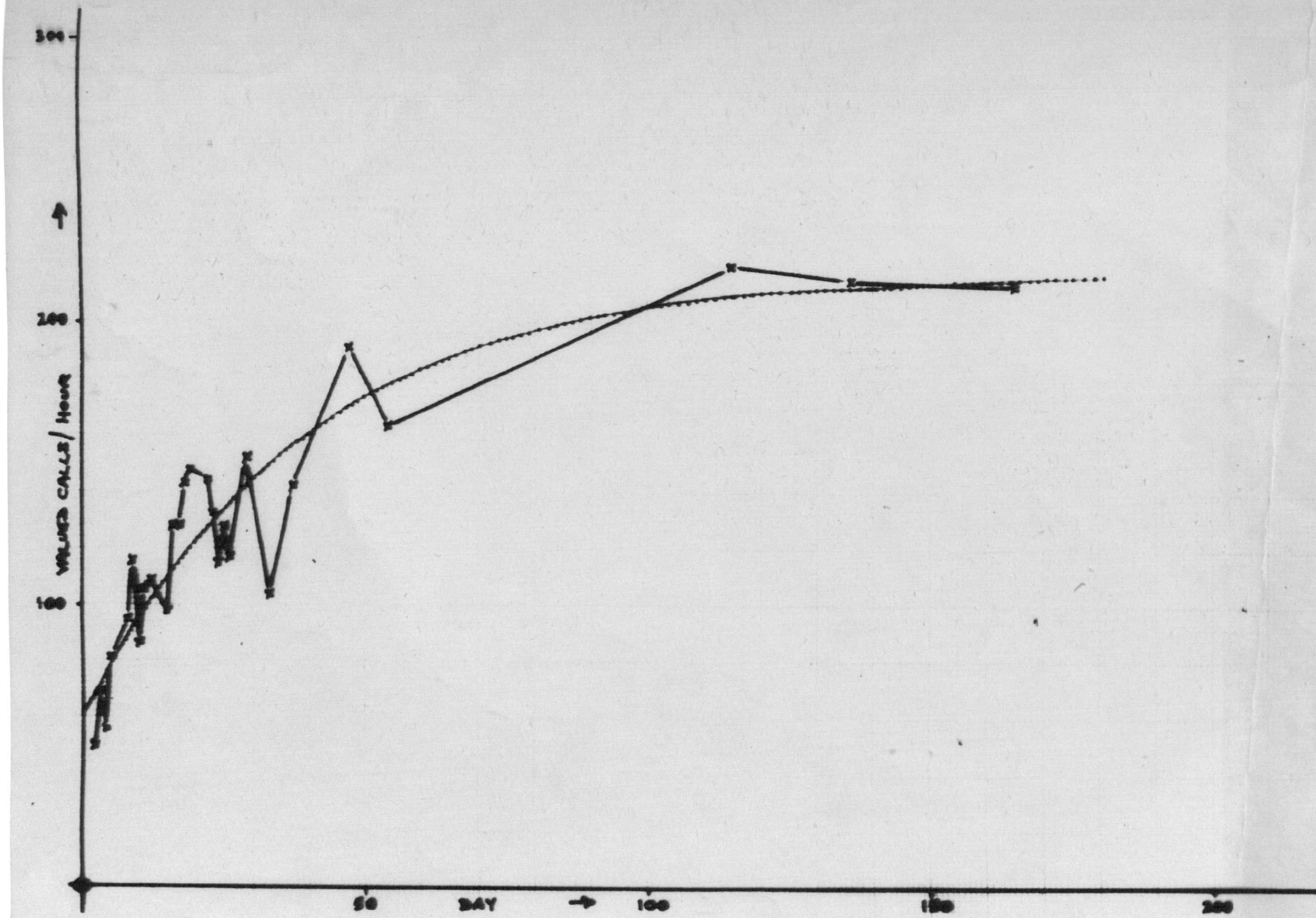


300
200
100
VALUED CALLS/HOUR
50
DAY →
100
150
200

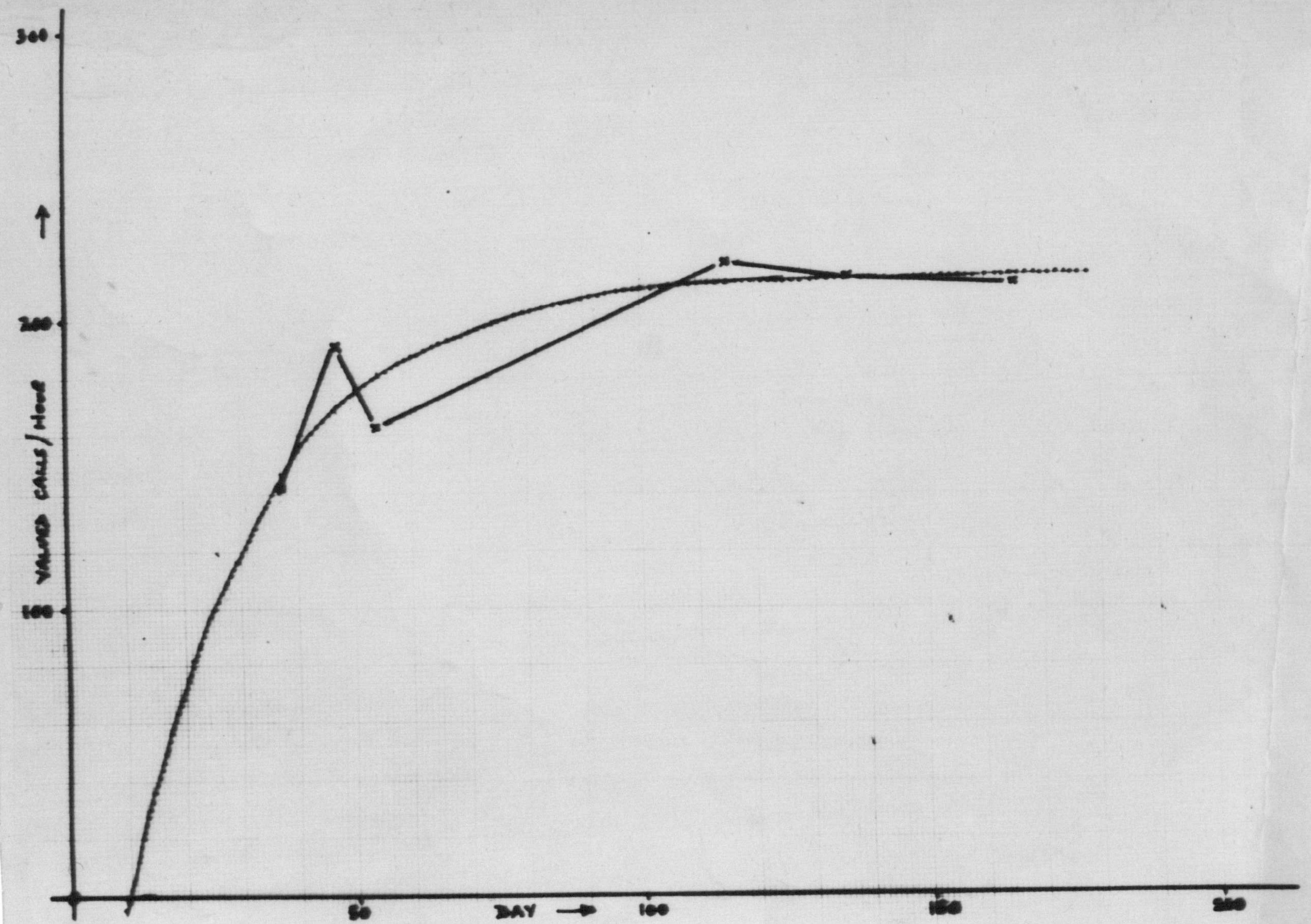

300
DAY

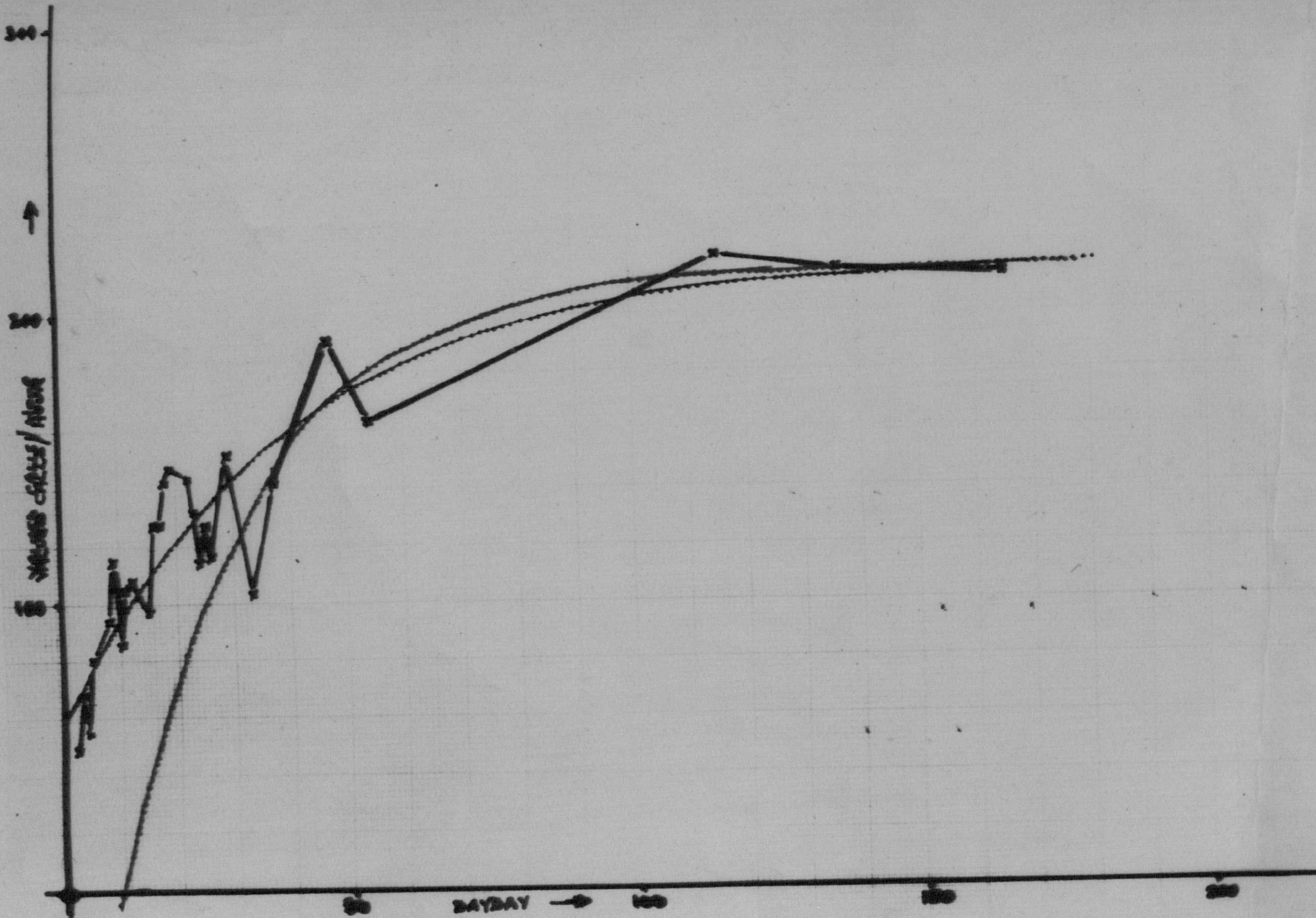
TRANSPARENCY 5 LEARNING CURVE FOR KF, ALL OBSERVATIONS

TRANSPARENCY 6 LEARNING CURVE FOR KF, EXPERIENCE DATA ONLY



300
200
100
VALUED CALLS/ HOUR
50
DAY →
100
150
200


300
200
100
VALUED CALLS/HOUR
50
DAY
100
150
200


300
200
100
50
DAYDAY →
100
150
200

of "valued calls". Unfortunately, while "valued calls" are a useful yardstick for the trainer to assess progress, the trainees have no concept of the scoring system and hence, during tests, work flat out. Under such pressures the trainee will make mistakes, but more importantly from the learning curve aspect, is likely to attain much higher scores than might reasonably be expected when compared with the normal workrate of 200 valued calls/hour expected of experience telephonists. However, it should not be forgotten that the 200 valued calls/hour standard is that work rate which has been estimated to be reasonable for an experienced operator to work at for an 8 hour day, not the possible performance when working flat out.

The conclusion is drawn that the best fit parameter estimates which predict "final" performance figures of up to 400 valued calls/hour are not unreliable, and that the data presented contains two learning curves related to (i) the training period and (ii) the experience gaining period.

Lamb[47], using a method based on activity sampling, has shown that trainee telephonists require between 4 and 6 months before they perform the elements of the task in sensibly the same time as that observed in experienced operators, thus the experience gaining period is the major part of the trainee telephonist's training.

The presence of two learning stages, however, poses problems when it comes to comparing different methods of training, because the implication is that only the training period should be considered, rather than including information relevant to the post training period. Given successful curve fitting it might then be possible to compare the times taken by a control group of trainees to reach a suitable performance standard with those times taken by an experimental group to reach the same standard. Such learning times could be calculated from the best fit curves for each trainee's performance figure.

A suitable statistical test appears to be the Mann-Whitney U-test[48], which could be used to do this comparison. During the tape recorded tests described by Lamb[49] further records of the control and experimental groups performance were taken and used in an attempt to confirm the possibility of using this test, but curve-fitting was successful in only a small number of cases, too small to be used in the statistical tests with any reliability.

The method obviously demands accurate data, and an accurate measurement system. Hackett[50] has shown that while valued calls may be used within any one exchange to provide a useful guide to trainee performance, inter-exchange comparisons of trainee performance are not valid because of high variability between exchanges. The use of the scoring system of "valued calls" thus causes problems if training methods are compared.

8.6. Alternative Reasons for the Inaccuracy of the Data Obtained From Post Office Sources.

At this point it should be noted that other data obtained by direct observation was not successfully curve fitted. Data relating to the first three weeks of training for JJ and KN, when curve-fitted, resulted in negative Y_f and τ values. Was the data inaccurate?

It is possible that inaccurate data is caused by the inherent variability in the method of giving practice to and testing the trainees. Trainee telephonists practise and take their progress checks at the switchboard, handling live traffic generated by the public. Calls are received at random, and the type of call received by the trainee may vary from a simple long distance connection to a personal call. Calls may go 'wrong' at any time, not only for reasons within the control of the telephonist. For example, the telephonist may misdial - a fair mistake to make at an early stage in training. But the equipment she uses may also be faulty, so that she gets fault indications at some stage in the call (number unobtainable tone, say). Alternatively, the switching equipment the call is routed through may develop faults. What it amounts to is that the task is not repetitive in the absolute sense. It is true that over a very long period, a telephonist will repeat the various types of call she may handle until she becomes fully versed in the necessary operating techniques, but in the training period, the trainee is only starting to build up this experience, and all calls are likely to be regarded as different rather than the same. Comparison between trainees using learning curves then becomes difficult because the weighting system developed to score the performance of the trainee relies on a large quantity of the types of call being handled. The poor results obtained for data sets relating to training in the Oxford and North West Areas which were discussed earlier, and also the failure to curve fit the more detailed observations could very well have been caused by this effect. Reference to Appendix H will show that only about 20 of the 87 data sets were successfully curve fitted.

8.7. Repetitive and Quasi-Repetitive Tasks.

Lamb[51] has coined the term "quasi-repetitive task" as descriptive of a telephonist's work. This seems a most apt description of the type of work load received by the telephonist, for over a long period, it is repetitive, yet it is not repetitive in the short term (in the sense that the assembly of components would be regarded as repetitive in the short term). In such a situation, where a learning curve approach is to be attempted, either a scoring system must be developed which allows for the varying difficulty of the type of call received, or, during tests, standardised calls must be presented to the trainee. For the first case, it seems that the scoring system developed would need to be very complex, as the 'difficulty' of a Personal call would be great for a first day trainee, not so great for a second week trainee, and less difficult still once the trainee had received the necessary tuition to allow her to handle the call in the correct manner. To apply such a scoring system in the correct manner might very well imply a complete record of all calls handled and is regarded at this stage as too complex and costly to apply. It is probable that all quasi-repetitive tasks encounter this difficulty in scoring.

The simpler method which allows an extra score for difficult calls and difficulties encountered but does not vary the score according to the point in training the call is received, has been shown to be inaccurate,[52] thus the conclusion is drawn that an entirely different approach is needed. For example, the use of tape recorded telephone calls to present problems to each trainee in the same manner might allow comparison of training methods, but would be unlikely to provide data suitable for a learning curve approach. Lamb[53] discusses this approach in detail.

8.8. Possible Application of Learning Curves to other Tasks within the Post Office.

The previous discussion has suggested that scoring the task performed is a difficulty where quasi-repetitive tasks are encountered. Certainly the work of a telephonist poses this problem. Some other examples may also be suggested, such as fault-finding, because there is the obvious difficulty of deciding whether one fault is twice or four times more difficult to solve than a second (different) fault, and also the installation of telephones, because no two installations will be the same.

Other tasks within the Post Office are more representative of repetitive tasks. The use of the Machine Jointing No. 4 demands a manipulative skill to successfully joint two wires at a reasonable pace, and there is a knack to the older method of hand twist jointing which is gained with repetition. Such skills could probably be measured and depicted quite well by the Learning Curve models discussed previously.

In other tasks not related to Engineering such as Clerical work, it is also likely that a repetitive rather than quasi-repetitive nature will be found.

On the other hand, skills required in the Research, Development and Managerial fields are much more difficult to define and also to measure, so that it is unlikely that learning curve theory could be applied to those fields in the near future. A much more promising approach is that of Lamb, using tape-recorded tests as discussed previously. This, at least, allows for the presentation of the same problem to each of the trainees, without the possible bias that might be introduced, say, by the variations in tone of voice that could be found when several questioners were used.

Further research into the use of four parameter models could also be useful, for if a model could be defined which was rather more successful than those tried to date, statistical tests might then be possible which would allow an objective comparison of training methods to be made.

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APPENDIX A

PO STUDY ON CRITERIA FOR THE EVALUATION OF TRAINING.

Investigation Objectives

We propose to determine procedures to provide measures of the effectiveness of training and of field performance against the costs involved. We would also endeavour to state the costs associated with subsequent performance, to define appropriate criteria to measure the progress of Post Office trainees, initially those undergoing training; to specify ranges of acceptable performance on their training courses, using the criteria developed and to develop some adequate measures of the effectiveness of current and future training procedures. Two Post Office men are working on the project, initially studying the training of telephonists in both Rodwell House and exchanges and then engineering training in a Maintenance Area.

The proposal requires 2 years of investigation, divided approximately into 15 months for preparation of the measurement scheme and 9 months for validation in both training centre and work area training situations. During the first 3 to 4 months data is being gathered, either from existing records, by observation or via special records in the training centres. This data will be subjected to trial analyses to identify the most appropriate factors for subsequent study. The following year will be devoted to analyzing material for the chosen models of learning performance to be developed and dimensioned. In the final 9 months' period, models will be subjected to pilot and full-scale testing, both in the training centre and with appropriate training groups in work areas.

Probable Methods

Several approaches are possible to provide scales against which training performance may be judged. These could include studies of the current activities of operators in the exchanges chosen for study, to establish some validity for the criteria proposed. These studies may take the form of analysis of activities and decisions of operators for various types of call, of determination from questionnaires, of the operators' view of job difficulty both as a trainee and as an experienced operator.

In view of the telephonists' training objectives involving their development of "accuracy, courtesy and speed", it may be appropriate to introduce into the testing situation a taped sequence of calls with known difficulties run at a traffic rate which would be experienced during the busy period.

Comparable approaches would be adopted in the engineering field.

APPENDIX B

FORMULAE FOR THE ESTIMATION OF THE PARAMETER VALUES FOR THREE PARAMETER LEARNING CURVE MODELS

(a) The Bevis Model. $y_i = Y_c + Y_f(1-e^{-t_i z})$ (where $z = 1/\tau$).

When $t_i = 0$ (start) $\quad y_o = Y_c$

When $t_i = \infty$ (final) $\quad y_\infty = Y_c + Y_f$

Given estimates of the "final" and "start" values

$$\text{Final} = Y_c + Y_f \qquad \text{B.(a).1}$$

$$\text{Start} = Y_c$$

$$Y_f = \text{Final - Start.} \qquad \text{B.(a).2.}$$

Given value t_n, y_n

$$\text{Then } Y(N) = Y_c + Y_f(1-e^{-t(N).z}) \qquad \text{B.(a).3.}$$

$$= \text{Start} + (\text{final - start})(1-e^{-t(N).z}) \qquad \text{B.(a).4}$$

$$\frac{(Y(N) - \text{start})}{(\text{final-start})} = (1-e^{-t(N).z}) \qquad \text{B.(a).5}$$

$$e^{-t(N).z} = 1 - \frac{Y(N) - \text{start}}{\text{final - start}} \qquad \text{B.(a).6}$$

$$= \frac{(\text{final-start} - Y(N) + \text{start})}{(\text{final - start})} \qquad \text{B.(a).7}$$

$$= \frac{(\text{final} - Y(N))}{(\text{final - start})} \qquad \text{B.(a).8}$$

$$e^{tN.z} = \frac{(\text{final - start})}{\text{final} - Y(N)} \qquad \text{B.(a).9}$$

$$t(N).\ Z = \ln\left(\frac{\text{final - start}}{\text{final} - Y(N)}\right) \qquad \text{B. a. (10)}$$

$$Z = \frac{1}{t(N)} \cdot \ln\left(\frac{\text{final - start}}{\text{final} - Y(N)}\right) \qquad \text{B. a. (11)}$$

(b) The Gompertz Model has been analysed previously in Section 5. 8.

(c) The Mathematical Model $y_i = b - \dfrac{1}{c + gx_i}$ B. c. (1)

when $x_i = 0$ (start) $y_i = b - \dfrac{1}{c}$ B. c. (2)

when $x_i = \infty$ (final) $y_\infty = b$ B. c. (3)

Given estimates of "final" and "start" values

$$\text{final} = b$$

$$\text{start} = b - \frac{1}{c}$$

$$\frac{1}{c} = b - \text{start} = \text{final - start} \qquad \text{B. c. (4)}$$

$$c = \frac{1}{\text{final-start}} \qquad \text{B. c. (5)}$$

And given value X(N), Y(N)

$$Y(N) = b - \frac{1}{c + gx(N)} \qquad \text{B. c. (6)}$$

$$b - Y(N) = \frac{1}{c + g.x(N)} \qquad \text{B. c. (7)}$$

$$c + g.X(N) = \frac{1}{b - Y(N)} \qquad \text{B. c. (8)}$$

$$g.\ X(N) = \frac{1}{b - Y(N)} - c \qquad \text{B. c. (9)}$$

$$g = \left(\frac{1}{b - Y(N)} - c\right) . \quad \frac{1}{X(N)} \qquad \text{B. c. (10)}$$

$$= \left(\frac{1}{(\text{final} - Y(N))} - \frac{1}{(\text{final-start})} \right) \cdot \frac{1}{X(N)} \qquad \text{B.c.(11)}$$

(d) The Accumulative Model

$$y_i = \frac{b + \theta .\ a.\ (N_i - 1)}{1 + \theta\ (N_i - 1)} \qquad \text{B.d.(1)}$$

final = a

start = b

and given a value N(N), Y(N)

$$Y(N) = \frac{b + \theta a\ (N(N) - 1)}{1 + \theta\ (N(N) - 1)} \qquad \text{B.d.(2)}$$

$$Y(N)\ (1 + \theta\ (N(N)-1)\ ) = b + \theta a\ (N(N) - 1) \qquad \text{B.d.(3)}$$

$$Y(N) + Y(N).\theta.(N(N) - 1) = b + \theta a\ N(N) - \theta a \qquad \text{B.d.(4)}$$

$$Y(N) - b = \theta a\ (N(N) - 1) - Y(N).\theta.(N(N)-1) \qquad \text{B.d.(5)}$$

$$= \theta\ (N(N) - 1)\ (a - Y(N)) \qquad \text{B.d.(6)}$$

$$\theta = \frac{Y(N) - b}{(a - Y(N))(N(N)-1)} \qquad \text{B.d.(7)}$$

$$= \frac{(Y(N) - \text{start})}{(\text{final} - Y(N))\ (N(N)-1)} \qquad \text{B.d.(8)}$$

(e) The Replacement Model $y_i = a - (a-b)(1-\theta)^{N_i - 1}$ B.e.(1)

final = a

start = b

And given a value N(N), Y(N)

$$y(N) = a - (a-b)\ (1-\theta)^{N(N)-1} \qquad \text{B.e.(2)}$$

$$\frac{(a - y(N))}{(a-b)} = (1-\theta)^{N(N)-1} \qquad \text{B. e. (3)}$$

$$1 - \theta = \sqrt[(N(N)-1)]{\frac{(a-y(N))}{(a-b)}} \qquad \text{B. e. (4)}$$

$$\theta = 1 - \sqrt[(N(N)-1)]{\frac{(a-y(N))}{(a-b)}} \qquad \text{B. e. (5)}$$

$$= 1 - \sqrt[(N(N)-1)]{\frac{(final-Y(N))}{(final-start)}} \qquad \text{B. e. (6)}$$

(f) The de Jong Model $y_i = t_1 M - t_1 (1-M) x_i^{-n}$

It has been shown previously that this equation is the equivalent

of $y_i = B - Ax_i^{-n}$ B. f. (1)

At $x_i = 1$, $y_i = B - A = start$

at $x_i = \infty$, $y_\infty = B = final$

final = B

start = B - A

A = final - start. B. f. (2)

And given a value X(N), Y(N)

$$Y(N) = B - A. \left[X(N)\right]^{-n} \qquad \text{B. f. (3)}$$

$$A(X(N))^{-n} = B - Y(N) \qquad \text{B. f. (4)}$$

$$X(N)^{-n} = \frac{(B - Y(N))}{A} \qquad \text{B. f. (5)}$$

$$X(N)^{n} = \frac{A}{(B - Y(N))} \qquad \text{B. f. (6)}$$

$$n \ln(X(N)) = \ln\left[\frac{A}{(B - Y(N))}\right] \qquad \text{B. f. (7)}$$

$$n = \frac{\ln\left[\frac{A}{(B - Y(N))}\right]}{\ln(X(N))} \qquad \text{B. f. (8)}$$

$$= \frac{\ln\left[\frac{(\text{final} - \text{start})}{(\text{final} - Y(N))}\right]}{\ln(X(N))} \qquad \text{B. f. (9)}$$

(g) The Logmathematical Model, $\log y_i = b - \frac{1}{c + gx_i}$ B. g. (1)

This model is dealt with in the same way as the mathematical model, which means that (using log (y_i) values)

$$\text{final} = b$$

$$\text{start} = b - \frac{1}{c}$$

$$c = \frac{1}{\text{final-start}} \qquad \text{B. g. (2)}$$

$$g = \left\{\frac{1}{(\text{final} - Y(N)} - \frac{1}{(\text{final-start})}\right\} \cdot \frac{1}{X(N)} \qquad \text{B. g. (3)}$$

However, to ensure a fair comparison of the sum of errors squared for this model with that of the other models, once b, c and g have been calculated for best fit, the sum of error squared should be calculated for the fit of the equation

$$y_i = e^{\left(b - \frac{1}{c + g.x_i}\right)} \qquad \text{B. g. (4)}$$

which ensures that a comparison is made for the same scale of output.

(h) The Second Order Model $y_i = Y_c + Y_f\left(1-(1 + \omega t_i)e^{-\omega t_i}\right)$

B. h. (1)

at $t_i = 0 \quad y_o = Y_c = \text{start}$

$t_i = \infty \quad y_\infty = Y_c + Y_f = \text{final}$

Now consider the values for two points, (t_1, y_1) and (t_n, y_n)

$$y_1 = Y_c + Y_f (1-(1+1\omega) e^{-1\omega}) \qquad \text{B.h.(2)}$$

$$y_N = Y_c + Y_f (1-(1+\omega t_n) e^{-\omega t_n}) \qquad \text{B.h.(3)}$$

From B.h.(2) $\dfrac{y_1 - Y_c}{Y_f} = 1 - (1+\omega) e^{-\omega}$ B.h.(4)

From B.h.(3) $\dfrac{y_n - Y_c}{Y_f} = 1 - (1+\omega t_n) e^{-\omega t_n}$ B.h.(5)

and from B.h.(4) $1 - \dfrac{y_1 - Y_c}{Y_f} = \dfrac{Y_f - y_1 + Y_c}{Y_f} = \text{final} - Y_1 = (1+\omega) e^{-\omega}$ B.h.(6)

and from B.h.(5) $1 - \dfrac{y_n - Y_c}{Y_f} = \dfrac{Y_f - y_n + Y_c}{Y_f} = \text{final} - Y_n = (1+\omega t_n) e^{-\omega t_n}$

B.h.(7)

Dividing B.h.(7) by B.h.(6)

$$\frac{(1+\omega t_n)}{(1+\omega)} e^{-\omega (t_n - 1)} = \frac{\text{final} - Y_n}{\text{final} - Y_1} \qquad \text{B.h.(8)}$$

Now if t_n is large $\dfrac{1+\omega t_n}{1+\omega} \simeq t_n$ B.h.(9)

$$t_n e^{-\omega (t_n - 1)} \simeq \frac{\text{final} - Y_n}{\text{final} - Y_1} \qquad \text{B.h.(10)}$$

$$e^{\omega (t_n - 1)} \simeq t_n \cdot \frac{(\text{final} - Y_1)}{(\text{final} - Y_n)} \qquad \text{B.h.(11)}$$

$$\omega (t_n - 1) \simeq \ln \left\{ \frac{t_n (\text{final} - Y_1)}{(\text{final} - Y_n)} \right\} \qquad \text{B.h.(12)}$$

$$\omega \simeq \frac{1}{(t_n - 1)} \ln \left\{ \frac{t_n (\text{final} - Y_1)}{(\text{final} - Y_n)} \right\} \qquad \text{B.h.(13)}$$

And as $\omega = \frac{1}{\tau}$, an approximate value of τ may be found.

APPENDIX C

LEARNING CURVE DATA FROM VARIOUS SOURCES

```
T0100     BLACKBURN MEAN SCORE OF 7 SUBJECTS.
OPERATION:- CARD SORTING
 20
 1.0 26.4 2.0 26.2 3.0 28.2 4.0 30.4
 5.0 34.3 6.0 33.9 7.0 36.6 8.0 40.5
 9.0 43.5 10.0 43.0 11.0 46.1 12.0 46.9
 13.0 47.3 14.0 52.5 15.0 52.2 16.0 51.6
 17.0 55.4 18.0 57.3 19.0 56.9 20.0 59.0
```

```
T0101     BLACKBURN MEAN SCORE OF 4 SUBJECTS.
OPERATION - CARD SORTING
 30
 1.0 25.8 2.0 26.4 3.0 28.0 4.0 29.3
 5.0 32.7 6.0 34.8 7.0 34.0 8.0 39.1
 9.0 38.8 10.0 36.9 11.0 41.8 12.0 40.8
 13.0 40.0 14.0 44.4 15.0 48.3 16.0 43.0
 17.0 46.9 18.0 50.3 19.0 47.1 20.0 50.7
 21.0 48.2 22.0 52.0 23.0 53.4 24.0 57.9
 25.0 59.2 26.0 52.8 27.0 56.6 28.0 55.8
 29.0 58.5 30.0 62.7
```

```
T0102     BLACKBURN MEAN SCORE OF 4 SUBJECTS.
OPERATION:- CROSSING OUT 'E'S
 23
 1.0 128.4 2.0 137.0 3.0 151.9 4.0 163.6
 5.0 166.2 6.0 164.6 7.0 168.5 8.0 171.1
 9.0 179.3 10.0 181.9 11.0 178.7 12.0 180.2
 13.0 180.0 14.0 187.2 15.0 180.7 16.0 184.4
 17.0 190.5 18.0 186.7 19.0 183.3 20.0 190.9
 21.0 191.8 22.0 186.1 23.0 186.1
```

```
T0103     BLACKBURN MEAN SCORE OF 7 SUBJECTS.
OPERATION:- CODE SUBSTITUTION
 23
 1.0 18.8 2.0 22.0 3.0 24.8 4.0 27.8
 5.0 28.6 6.0 30.4 7.0 33.6 8.0 33.6
 9.0 35.3 10.0 36.7 11.0 41.1 12.0 42.1
 13.0 42.2 14.0 40.7 15.0 45.1 16.0 46.5
 17.0 50.2 18.0 48.4 19.0 48.4 20.0 48.1
 21.0 52.9 22.0 54.2 23.0 54.8
```

```
T0104     BLACKBURN MEAN SCORE OF 4 SUBJECTS.
OPERATION:- CODE SUBSTITUTION
 32
 1.0 17.5 2.0 21.1 3.0 22.4 4.0 24.4
 5.0 25.9 6.0 26.8 7.0 30.5 8.0 30.3
 9.0 30.5 10.0 31.4 11.0 34.3 12.0 33.5
 13.0 35.0 14.0 37.1 15.0 38.6 16.0 39.6
 17.0 40.0 18.0 39.0 19.0 39.5 20.0 39.8
 21.0 43.5 22.0 46.6 23.0 47.2 24.0 47.3
 25.0 44.9 26.0 47.4 27.0 52.1 28.0 52.2
 29.0 51.7 30.0 56.4 31.0 52.8 32.0 55.2
```

T0105   BLACKBURN MEAN SCORE OF 6 SUBJECTS.
OPERATION:- ADDITION
18
1.0 71.7 2.0 67.4 3.0 84.0 4.0 94.9
5.0 98.5 6.0 100.7 7.0 101.8 8.0 114.4
9.0 110.9 10.0 115.3 11.0 119.1 12.0 117.6
13.0 118.9 14.0 120.6 15.0 121.8 16.0 123.5
17.0 123.9 18.0 127.4

T0106   BLACKBURN MEAN SCORE OF 4 SUBJECTS.
OPERATION:- ADDITION
27
1.0 73.3 2.0 74.2 3.0 90.7 4.0 97.8
5.0 102.7 6.0 109.9 7.0 113.2 8.0 117.9
9.0 114.7 10.0 118.4 11.0 120.9 12.0 121.8
13.0 128.1 14.0 123.7 15.0 124.7 16.0 125.6
17.0 127.9 18.0 130.9 19.0 130.9 20.0 130.2
21.0 131.2 22.0 134.3 23.0 135.6 24.0 134.9
25.0 135.3 26.0 130.7 27.0 139.9

T0107   BLACKBURN MEAN SCORE OF 6 SUBJECTS.
OPERATION:- MAZE LEARNING
23
1.0 5.6 2.0 6.6 3.0 10.3 4.0 12.5
5.0 15.8 6.0 20.5 7.0 20.2 8.0 32.3
9.0 27.1 10.0 22.5 11.0 34.7 12.0 35.5
13.0 51.1 14.0 47.6 15.0 48.0 16.0 53.1
17.0 67.7 18.0 63.6 19.0 48.0 20.0 81.6
21.0 71.7 22.0 59.1 23.0 67.9

T0108   BLACKBURN MEAN SCORE OF 4 SUBJECTS.
OPERATION:- MAZE LEARNING
32
1.0 5.6 2.0 6.2 3.0 7.9 4.0 11.6
5.0 18.8 6.0 18.9 7.0 16.1 8.0 26.4
9.0 26.6 10.0 23.6 11.0 35.1 12.0 33.3
13.0 47.4 14.0 41.1 15.0 38.3 16.0 42.7
17.0 57.3 18.0 58.0 19.0 53.0 20.0 72.0
21.0 63.1 22.0 62.8 23.0 58.4 24.0 74.6
25.0 62.1 26.0 71.3 27.0 82.6 28.0 64.6
29.0 67.1 30.0 77.7 31.0 86.3 32.0 63.2

T0109   MORCUMBE   MEAN SCORE OF 37 SUBJECTS.
OPERATION:- COVERING
20
2.5 16.0 5.0 27.0 7.5 37.5 10.0 41.0
12.5 44.0 15.0 47.0 17.5 49.5 20.0 52.5
22.5 55.5 25.0 57.5 27.5 59.5 30.0 61.0
32.5 62.5 35.0 63.0 37.5 63.5 40.0 63.8
42.5 64.0 45.0 64.0 47.5 64.0 50.0 64.0

```
TU110     MORCOMBE   MEAN SCORE OF 27 SUBJECTS.
OPERATION:- TRIMMING
 16
 2.5 20.5 5.0 34.0 7.5 47.5 10.0 52.5
 12.5 57.5 15.0 60.0 17.5 62.5 20.0 63.5
 22.5 64.5 25.0 65.3 27.5 66.0 30.0 66.5
 32.5 67.0 35.0 67.0 37.5 67.0 40.0 67.0

TU111    MORCOMBE   MEAN SCORE OF 23 SUBJECTS.
OPERATION:- HEMMING
 20
 2.5 18.5 5.0 37.0 7.5 44.0 10.0 51.0
 12.5 54.0 15.0 57.0 17.5 57.5 20.0 58.0
 22.5 59.0 25.0 60.0 27.5 62.0 30.0 64.0
 32.5 65.5 35.0 67.0 37.5 68.5 40.0 70.0
 42.5 70.0 45.0 70.0 47.5 70.0 50.0 70.0

TU112    MORCOMBE   MEAN SCORE OF 6 SUBJECTS.
OPERATION:- SIMULATED ASSEMBLY
 20
 1.0 2.8 2.0 5.8 3.0 7.0 4.0 7.5
 5.0 8.3 6.0 8.2 7.0 8.3 8.0 8.4
 9.0 8.4 10.0 8.3 11.0 8.1 12.0 8.4
 13.0 8.4 14.0 8.8 15.0 8.8 16.0 8.8
 17.0 8.7 18.0 9.0 19.0 9.0 20.0 9.3

TU113    BEVIS   MEAN SCORE OF 4 SUBJECTS.
OPERATION:- "B"
 8
 1.0 28.0 2.0 43.1 3.0 54.5 4.0 67.5
 5.0 81.1 6.0 86.1 7.0 88.0 8.0 92.0

TU114    BEVIS   MEAN SCORE OF 8 SUBJECTS.
OPERATION:- "C"
 11
 1.0 34.4 2.0 48.0 3.0 54.4 4.0 61.0
 5.0 69.2 6.0 73.2 7.0 75.3 8.0 82.7
 9.0 86.1 10.0 88.2 11.0 91.6

TU115    BEVIS   MEAN SCORE OF 15 SUBJECTS.
OPERATION:- ROLLING,(PLANT A)
 8
 1.0 1670.0 5.0 2314.0 10.0 2574.0 15.0 3314.0
 20.0 3889.0 25.0 4055.0 30.0 4205.0 35.0 4243.0

TU116    BEVIS   MEAN SCORE OF 15 SUBJECTS.
OPERATION:- BUNCHING,(PLANT A)
 10
 1.0 1800.0 2.0 2015.0 4.0 2321.0 6.0 2829.0
 8.0 3085.0 10.0 3703.0 12.0 4084.0 14.0 4225.0
 16.0 4515.0 18.0 4617.0
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T0117 BEVIS MEAN SCORE OF 6 SUBJECTS.
OPERATION:- ROLLING,(PLANT B)
11
1.0 1450.0 5.0 1642.0 10.0 1950.0 15.0 2117.0
20.0 2383.0 25.0 2950.0 30.0 3525.0 35.0 3533.0
40.0 4200.0 45.0 4375.0 50.0 4700.0

T0118 BEVIS MEAN SCORE OF 6 SUBJECTS.
OPERATION:- BUNCHING,(PLANT B)
8
1.0 1400.0 5.0 1560.0 10.0 2120.0 15.0 2360.0
20.0 2820.0 25.0 3400.0 30.0 4060.0 35.0 4480.0

T0119 BEVIS MEAN SCORE OF 1 SUBJECT.
OPERATION:- INDUSTRIAL STUDY 3
27
1.0 16.00 6.0 19.61 11.00 22.63 16.00 30.08
21.0 32.27 26.0 32.63 31.0 37.59 36.00 37.59
41.0 37.59 46.0 37.50 51.0 36.69 56.00 39.46
61.0 36.11 66.0 36.90 71.0 37.61 76.00 41.75
81.0 45.00 86.0 41.51 91.0 42.14 96.00 43.99
101.0 46.25 106.0 46.72 111.0 47.50 116.0 49.00
121.0 47.50 126.0 48.75 131.0 50.0

T0120 HACKETT AND LAMB MEAN SCORE OF 10 SUBJECTS.
OPERATION:- TELEPHONIST
19
5.0 143.5 7.0 168.5 9.0 180.5 10.0 194.3
13.0 188.8 15.0 199.0 16.0 195.0 17.0 197.0
18.0 197.0 20.0 265.5 22.0 229.5 106.0 259.0
112.0 310.0 118.0 210.0 121.0 265.0 155.0 284.0
183.0 248.0 198.0 241.0 288.0 237.0

T0121 HACKETT AND LAMB MEAN SCORE OF 9 SUBJECTS.
OPERATION:- TELEPHONIST
20
5.0 111.3 7.0 136.4 8.0 188.0 9.0 177.2
10.0 172.3 13.0 156.4 15.0 181.8 16.0 240.0
17.0 195.6 18.0 205.0 20.0 254.0 22.0 285.0
85.0 224.0 100.0 215.0 104.0 247.0 107.0 249.0
109.0 218.0 156.0 225.0 158.0 262.0 198.0 220.0

T0122 HACKETT AND LAMB MEAN SCORE OF 10 SUBJECTS.
OPERATION:- TELEPHONIST
25
4.0 85.0 5.0 116.2 6.0 110.0 7.0 136.6
8.0 115.0 9.0 149.2 10.0 143.8 11.0 125.0
12.0 171.0 13.0 146.1 14.0 163.5 15.0 154.2
16.0 176.0 17.0 185.2 18.0 179.3 19.0 175.0
20.0 151.0 22.0 204.0 109.0 256.0 123.0 235.5
127.0 214.0 134.0 279.0 169.0 257.0 305.0 254.0
318.0 226.0

T0123 HACKETT AND LAMB MEAN SCORE OF 9 SUBJECTS.
OPERATION:- TELEPHONIST
24
4.0 94.0 5.0 107.9 7.0 134.4 8.0 143.0
9.0 156.8 10.0 134.3 12.0 157.0 13.0 131.4
14.0 141.0 15.0 140.8 16.0 140.0 17.0 174.7
18.0 208.0 20.0 169.0 22.0 187.5 77.0 279.0
108.0 232.0 113.0 207.0 122.0 270.0 126.0 210.0
133.0 216.0 171.0 251.0 172.0 247.0 207.0 237.0

T0124 HACKETT AND LAMB MEAN SCORE OF 10 SUBJECTS.
OPERATION:- TELEPHONIST
24
4.0 109.0 5.0 108.4 7.0 112.8 8.0 88.0
9.0 160.0 10.0 129.3 12.0 144.6 13.0 133.4
14.0 161.0 15.0 153.0 16.0 154.0 17.0 153.3
18.0 164.8 19.0 157.5 20.0 171.5 21.0 189.0
22.0 170.5 106.0 288.0 112.0 212.0 122.0 229.0
134.0 257.0 137.0 204.0 154.0 220.0 202.0 184.0

T0125 HACKETT AND LAMB MEAN SCORE OF 10 SUBJECTS.
OPERATION:- TELEPHONIST
24
5.0 69.5 7.0 107.2 8.0 122.0 9.0 140.5
10.0 112.0 11.0 123.0 12.0 136.0 13.0 124.9
15.0 144.5 16.0 147.0 17.0 164.3 18.0 158.8
19.0 176.5 21.0 152.0 22.0 158.0 23.0 153.0
111.0 214.0 113.0 277.0 114.0 342.0 121.0 216.0
126.0 257.0 151.0 228.0 158.0 249.0 180.0 242.0

T0126 HACKETT AND LAMB MEAN SCORE OF 10 SUBJECTS.
OPERATION:- TELEPHONIST
20
5.0 69.1 7.0 97.9 9.0 115.8 10.0 117.0
11.0 125.5 13.0 117.9 15.0 134.4 17.0 163.0
18.0 138.6 19.0 156.3 20.0 150.6 21.0 170.0
22.0 157.0 109.0 265.0 114.0 254.0 126.0 218.0
133.0 229.0 140.0 221.0 150.0 206.0 224.0 260.0

T0127 HACKETT AND LAMB MEAN SCORE OF 9 SUBJECTS.
OPERATION:- TELEPHONIST
24
5.0 59.0 7.0 76.1 8.0 88.0 9.0 91.4
10.0 111.3 11.0 90.0 12.0 110.0 13.0 115.0
15.0 119.8 16.0 151.0 17.0 146.0 18.0 164.0
19.0 159.7 20.0 157.6 21.0 183.0 22.0 168.5
23.0 194.0 27.0 195.0 102.0 335.0 106.0 231.0
107.0 253.5 120.0 211.0 121.0 231.0 134.0 220.0

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T0128 HACKETT AND LAMB MEAN SCORE OF  9 SUBJECTS.
OPERATION:- TELEPHONIST
 21
 5.0 60.0 7.0 82.8 9.0 92.8 10.0 104.2
 11.0 71.0 13.0 87.4 15.0 91.1 17.0 105.6
 18.0 148.3 19.0 116.0 20.0 147.1 21.0 160.3
 22.0 168.0 30.0 199.0 32.0 237.0 109.0 214.0
 122.0 289.0 128.0 306.0 129.0 252.0 131.0 226.0
 135.0 324.5

T0129 BLACKBURN AVERAGE SCORE OF S1
OPERATION:- CARD SORTING
 35
1.0 37.8 2.0 33.6 3.0 40.0 4.0 42.4
5.0 48.8 6.0 47.7 7.0 49.4 8.0 49.4
9.0 52.5 10.0 56.8 11.0 64.6 12.0 60.9
13.0 57.5 14.0 62.7 15.0 68.9 16.0 66.7
17.0 71.2 18.0 80.8 19.0 76.4 20.0 76.4
21.0 68.9 22.0 77.8 23.0 84.0 24.0 87.5
25.0 84.0 26.0 64.6 27.0 79.2 28.0 84.0
29.0 85.7 30.0 87.5 31.0 85.7 32.0 91.3
33.0 95.5 34.0 102.5 35.0 105.0

T0130 BLACKBURN AVERAGE SCORE OF S2
OPERATION:- CARD SORTING
 35
1.0 15.6 2.0 17.8 3.0 18.3 4.0 18.9
5.0 21.6 6.0 25.8 7.0 22.8 8.0 26.2
9.0 26.2 10.0 18.3 11.0 25.1 12.0 25.1
13.0 21.1 14.0 25.4 15.0 27.5 16.0 26.2
17.0 28.8 18.0 35.0 19.0 30.2 20.0 29.4
21.0 32.8 22.0 35.9 23.0 37.2 24.0 35.6
25.0 36.2 26.0 35.3 27.0 33.1 28.0 37.8
29.0 34.4 30.0 31.6 31.0 40.4 32.0 42.0
33.0 42.4 34.0 42.0 35.0 43.7

T0131 BLACKBURN AVERAGE SCORE OF S3
OPERATION:- CARD SORTING
 32
1.0 23.9 2.0 25.9 3.0 26.8 4.0 28.4
5.0 25.1 6.0 29.6 7.0 30.0 8.0 37.2
9.0 35.0 10.0 34.1 11.0 30.7 12.0 31.3
13.0 38.5 14.0 37.5 15.0 41.6 16.0 32.3
17.0 40.8 18.0 35.9 19.0 39.3 20.0 41.6
21.0 40.4 22.0 42.4 23.0 42.4 24.0 52.5
25.0 51.9 26.0 44.7 27.0 51.2 28.0 48.8
29.0 54.5 30.0 55.3 31.0 50.0 32.0 49.4
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TU132 BLACKBURN AVERAGE SCORE OF S4
OPERATION:- CARD SORTING
23
1.0 28.2 2.0 24.1 3.0 27.5 4.0 27.1
5.0 33.1 6.0 26.6 7.0 33.9 8.0 29.4
9.0 40.4 10.0 37.5 11.0 46.7 12.0 44.2
13.0 43.7 14.0 47.2 15.0 44.2 16.0 48.8
17.0 47.7 18.0 41.2 19.0 51.2 20.0 48.3
21.0 54.5 22.0 41.6 23.0 48.8

TU133 BLACKBURN AVERAGE SCORE OF S5
OPERATION:- CARD SORTING
30
1.0 25.8 2.0 28.2 3.0 26.8 4.0 27.3
5.0 35.3 6.0 35.9 7.0 33.6 8.0 43.7
9.0 41.6 10.0 38.5 11.0 46.7 12.0 45.7
13.0 42.9 14.0 51.9 15.0 55.3 16.0 46.7
17.0 46.7 18.0 49.4 19.0 42.4 20.0 55.3
21.0 50.6 22.0 51.9 23.0 50.0 24.0 56.0
25.0 64.6 26.0 66.7 27.0 62.7 28.0 52.5
29.0 59.2 30.0 76.4

TU134 BLACKBURN AVERAGE SCORE OF S6
OPERATION:-CARD SORTING
27
1.0 30.4 2.0 28.0 3.0 32.3 4.0 37.8
5.0 41.6 6.0 42.4 7.0 47.2 8.0 47.2
9.0 50.6 10.0 60.0 11.0 59.2 12.0 58.3
13.0 64.6 14.0 68.9 15.0 60.9 16.0 65.6
17.0 68.9 18.0 82.4 19.0 76.4 20.0 85.7
21.0 84.0 22.0 63.6 23.0 77.8 24.0 97.7
25.0 91.3 26.0 91.3 27.0 105.0

TU135 BLACKBURN AVERAGE SCORE OF S7
OPERATION:- CARD SORTING
20
1.0 22.8 2.0 25.8 3.0 25.9 4.0 30.7
5.0 34.4 6.0 29.4 7.0 39.3 8.0 50.6
9.0 53.8 10.0 56.0 11.0 49.4 12.0 62.7
13.0 62.7 14.0 73.7 15.0 66.7 16.0 75.0
17.0 84.0 18.0 76.4 19.0 82.3 20.0 76.4

TU136 BLACKBURN AVERAGE SCORE OF S1
OPERATION:- CROSSING OUT "E"'S
35
1.0 133.3 2.0 154.4 3.0 172.2 4.0 195.6
5.0 202.0 6.0 197.2 7.0 201.7 8.0 211.7
9.0 217.8 10.0 221.1 11.0 203.9 12.0 212.8
13.0 213.3 14.0 222.3 15.0 223.0 16.0 219.8
17.0 230.6 18.0 222.9 19.0 233.3 20.0 225.4
21.0 237.4 22.0 234.1 23.0 218.9 24.0 257.4
25.0 252.9 26.0 253.8 27.0 257.8 28.0 270.1
29.0 268.9 30.0 270.7 31.0 260.1 32.0 260.1
33.0 271.2 34.0 275.0 35.0 279.0

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TU137 BLACKBURN AVERAGE SCORE OF S2
OPERATION:- CROSSING OUT"E"'S
 55
1.0 101.1 2.0 111.1 3.0 124.4 4.0 134.4
5.0 130.0 6.0 135.6 7.0 140.0 8.0 139.4
9.0 157.8 10.0 153.3 11.0 148.3 12.0 148.3
13.0 149.4 14.0 160.0 15.0 156.7 16.0 157.8
17.0 157.2 18.0 156.1 19.0 145.6 20.0 162.8
21.0 155.6 22.0 150.0 23.0 151.7 24.0 149.4
25.0 159.4 26.0 145.6 27.0 150.0 28.0 145.0
29.0 138.9 30.0 142.2 31.0 127.8 32.0 150.0
33.0 144.4 34.0 153.9 35.0 158.9

TU138 BLACKBURN AVERAGE SCORE OF S3
OPERATION:- CROSSING OUT "E"'S
 32
1.0 127.0 2.0 127.0 3.0 141.0 4.0 148.0
5.0 153.0 6.0 145.0 7.0 152.0 8.0 150.0
9.0 151.0 10.0 153.0 11.0 164.0 12.0 166.0
13.0 163.0 14.0 167.0 15.0 155.0 16.0 162.0
17.0 163.0 18.0 158.0 19.0 150.0 20.0 157.0
21.0 164.0 22.0 164.0 23.0 171.0 24.0 172.0
25.0 180.0 26.0 168.0 27.0 172.0 28.0 163.0
29.0 191.0 30.0 177.0 31.0 167.0 32.0 180.0

TU139 BLACKBURN AVERAGE SCORE OF S4
OPERATION:- CROSSING OUT "E"'S
 23
1.0 152.0 2.0 155.6 3.0 170.0 4.0 176.3
5.0 179.6 6.0 180.4 7.0 180.2 8.0 183.1
9.0 190.5 10.0 200.0 11.0 198.7 12.0 193.5
13.0 194.2 14.0 199.3 15.0 188.0 16.0 198.0
17.0 211.2 18.0 209.6 19.0 204.1 20.0 218.2
21.0 210.0 22.0 196.1 23.0 202.7

TU140 BLACKBURN AVERAGE SCORE OF S1
OPERATION:- CODE SUBSTITUTION
 35
1.0 22.2 2.0 25.6 3.0 30.0 4.0 28.9
5.0 32.6 6.0 32.8 7.0 40.6 8.0 40.6
9.0 41.7 10.0 41.7 11.0 48.3 12.0 48.9
13.0 49.4 14.0 50.0 15.0 51.7 16.0 55.6
17.0 53.3 18.0 58.9 19.0 51.7 20.0 52.8
21.0 60.0 22.0 62.2 23.0 62.8 24.0 66.1
25.0 65.0 26.0 63.9 27.0 71.7 28.0 71.7
29.0 70.6 30.0 73.3 31.0 73.3 32.0 72.8
33.0 73.9 34.0 77.2 35.0 78.9
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TU141 BLACKBURN AVERAGE SCORE OF S2
OPERATION:- CODE SUBSTITUTION
35
1.0 12.2 2.0 17.2 3.0 16.1 4.0 20.6
5.0 18.9 6.0 20.6 7.0 22.8 8.0 22.2
9.0 23.9 10.0 23.3 11.0 22.8 12.0 22.8
13.0 25.0 14.0 26.1 15.0 28.3 16.0 31.1
17.0 31.1 18.0 26.7 19.0 32.8 20.0 32.8
21.0 36.1 22.0 40.0 23.0 37.2 24.0 34.4
25.0 34.4 26.0 42.2 27.0 40.6 28.0 42.2
29.0 42.8 30.0 46.7 31.0 37.8 32.0 42.2
33.0 42.8 34.0 45.0 35.0 47.2

TU142 BLACKBURN AVERAGE SCORE OF S3
OPERATION:- CODE SUBSTITUTION
32
1.0 20.0 2.0 23.3 3.0 23.3 4.0 26.1
5.0 30.0 6.0 29.4 7.0 31.1 8.0 30.6
9.0 29.4 10.0 28.9 11.0 32.2 12.0 32.2
13.0 36.1 14.0 33.9 15.0 37.2 16.0 36.1
17.0 33.3 18.0 36.1 19.0 40.0 20.0 44.4
21.0 42.8 22.0 45.6 23.0 50.0 24.0 49.4
25.0 51.7 26.0 49.4 27.0 51.1 28.0 50.0
29.0 50.0 30.0 57.2 31.0 56.7 32.0 56.1

TU143 BLACKBURN AVERAGE SCORE OF S4
OPERATION:- CODE SUBSTITUTION
23
1.0 28.3 2.0 24.4 3.0 29.4 4.0 33.3
5.0 26.7 6.0 35.8 7.0 36.6 8.0 29.6
9.0 44.7 10.0 43.5 11.0 47.1 12.0 42.7
13.0 52.6 14.0 49.0 15.0 52.0 16.0 51.0
17.0 58.6 18.0 51.5 19.0 47.7 20.0 58.4
21.0 68.4 22.0 61.2 23.0 69.3

TU144 BLACKBURN AVERAGE SCORE OF S5
OPERATION:- CODE SUBSTITUTION
33
1.0 15.6 2.0 18.3 3.0 20.2 4.0 21.8
5.0 21.9 6.0 24.3 7.0 27.3 8.0 27.8
9.0 27.1 10.0 31.8 11.0 33.7 12.0 30.2
13.0 29.3 14.0 38.2 15.0 37.0 16.0 35.5
17.0 42.2 18.0 34.3 19.0 33.3 20.0 29.3
21.0 35.2 22.0 39.4 23.0 38.8 24.0 39.1
25.0 28.6 26.0 34.2 27.0 44.8 28.0 44.8
29.0 43.5 30.0 48.2 31.0 43.5 32.0 50.0
33.0 55.1

TU145 BLACKBURN AVERAGE SCORE OF S6
OPERATION:- CODE SUBSTITUTION
30
1.0 21.1 2.0 29.8 3.0 36.0 4.0 36.4
5.0 38.1 6.0 35.5 7.0 40.4 8.0 40.4
9.0 46.3 10.0 45.2 11.0 47.5 12.0 50.7
13.0 47.6 14.0 46.3 15.0 54.3 16.0 57.6
17.0 67.9 18.0 60.3 19.0 64.4 20.0 61.3
21.0 63.3 22.0 68.5 23.0 60.9 24.0 70.4
25.0 71.7 26.0 56.7 27.0 62.0 28.0 65.5
29.0 74.5 30.0 74.5

TU146 BLACKBURN AVERAGE SCORE OF S7
OPERATION:- CODE SUBSTITUTION
30
1.0 12.2 2.0 15.1 3.0 18.6 4.0 27.5
5.0 31.7 6.0 34.7 7.0 36.1 8.0 44.3
9.0 33.7 10.0 42.6 11.0 56.2 12.0 67.5
13.0 55.1 14.0 41.5 15.0 55.1 16.0 58.7
17.0 65.0 18.0 71.0 19.0 69.2 20.0 57.8
21.0 64.3 22.0 62.8 23.0 64.3 24.0 79.4
25.0 84.4 26.0 79.4 27.0 46.5 28.0 65.9
29.0 71.0 30.0 71.0

TU147 BLACKBURN AVERAGE SCORE OF S1
OPERATION:- ADDITION
35
1.0 53.3 2.0 82.8 3.0 100.6 4.0 107.8
5.0 112.8 6.0 114.4 7.0 113.9 8.0 121.7
9.0 112.8 10.0 121.7 11.0 125.0 12.0 126.7
13.0 127.2 14.0 123.3 15.0 133.9 16.0 137.2
17.0 130.6 18.0 132.8 19.0 140.6 20.0 146.1
21.0 141.1 22.0 150.6 23.0 148.3 24.0 143.3
25.0 146.1 26.0 137.2 27.0 151.1 28.0 151.7
29.0 140.0 30.0 147.8 31.0 154.4 32.0 154.4
33.0 155.5 34.0 146.1 35.0 136.1

TU148 BLACKBURN AVERAGE SCORE OF S2
OPERATION:- ADDITION
35
1.0 50.6 2.0 54.4 3.0 58.9 4.0 61.7
5.0 72.2 6.0 72.8 7.0 76.1 8.0 76.1
9.0 81.1 10.0 79.4 11.0 83.9 12.0 89.4
13.0 94.4 14.0 91.1 15.0 83.9 16.0 91.1
17.0 93.3 18.0 96.1 19.0 93.9 20.0 92.2
21.0 99.4 22.0 94.4 23.0 106.1 24.0 100.6
25.0 97.2 26.0 98.3 27.0 109.4 28.0 103.9
29.0 100.0 30.0 101.7 31.0 101.7 32.0 91.7
33.0 100.6 34.0 105.6 35.0 107.2

TU149 BLACKBURN AVERAGE SCORE OF S3
OPERATION:- ADDITION
32
1.0 102.3 2.0 73.3 3.0 115.0 4.0 125.6
5.0 125.6 6.0 127.8 7.0 132.8 8.0 136.7
9.0 135.0 10.0 136.7 11.0 136.1 12.0 133.9
13.0 138.3 14.0 133.3 15.0 133.9 16.0 135.6
17.0 138.9 18.0 136.1 19.0 132.8 20.0 135.6
21.0 142.2 22.0 133.9 23.0 141.1 24.0 146.7
25.0 145.6 26.0 138.3 27.0 140.0 28.0 134.4
29.0 147.8 30.0 142.8 31.0 140.6 32.0 147.2

TU150 BLACKBURN AVERAGE SCORE OF S4
OPERATION:- ADDITION
18
1.0 84.4 2.0 60.0 3.0 81.7 4.0 114.3
5.0 113.4 6.0 85.9 7.0 85.9 8.0 136.9
9.0 128.8 10.0 135.7 11.0 132.2 12.0 136.9
13.0 136.9 14.0 143.4 15.0 149.0 16.0 155.1
17.0 142.1 18.0 149.0

TU151 BLACKBURN AVERAGE SCORE OF S5
OPERATION:- ADDITION
27
1.0 87.1 2.0 86.1 3.0 88.4 4.0 96.1
5.0 100.0 6.0 124.5 7.0 129.8 8.0 137.1
9.0 129.8 10.0 135.6 11.0 138.6 12.0 137.1
13.0 152.5 14.0 147.0 15.0 147.0 16.0 138.6
17.0 148.8 18.0 158.4 19.0 156.4 20.0 147.0
21.0 141.9 22.0 158.4 23.0 147.0 24.0 148.9
25.0 152.2 26.0 148.9 27.0 159.1

TU152 BLACKBURN AVERAGE SCORE OF S6
OPERATION:- ADDITION
25
1.0 52.2 2.0 47.6 3.0 59.3 4.0 63.6
5.0 66.7 6.0 78.7 7.0 72.2 8.0 77.8
9.0 77.8 10.0 82.4 11.0 98.6 12.0 81.4
13.0 64.2 14.0 85.4 15.0 83.3 16.0 83.3
17.0 89.7 18.0 92.1 19.0 94.6 20.0 100.0
21.0 97.2 22.0 89.7 23.0 120.7 24.0 109.4
25.0 116.7

TU153 BLACKBURN AVERAGE SCORE OF S1
OPERATION:- MAZE LEARNING
35
1.0 5.6 2.0 5.6 3.0 6.9 4.0 14.1
5.0 18.5 6.0 26.3 7.0 31.2 8.0 40.0
9.0 43.5 10.0 38.5 11.0 50.0 12.0 62.5
13.0 52.6 14.0 47.6 15.0 37.0 16.0 76.9
17.0 90.9 18.0 125.0 19.0 111.1 20.0 125.0
21.0 142.9 22.0 90.9 23.0 100.0 24.0 142.9
25.0 90.9 26.0 100.0 27.0 125.0 28.0 71.4
29.0 111.1 30.0 100.0 31.0 142.9 32.0 90.9
33.0 76.9 34.0 25.6 35.0 125.0

TU154 BLACKBURN AVERAGE SCORE OF S2
OPERATION:- MAZE LEARNING
35
1,0 5.6 2,0 7.9 3,0 5.6 4.0 14.1
5,0 16.4 6,0 10.2 7.0 18.2 8.0 35.7
9,0 41.7 10.0 31.2 11.0 62.5 12.0 41.7
13.0 71.4 14.0 58.8 15.0 62.5 16.0 62.5
17.0 83.3 18.0 58.8 19.0 66.7 20.0 111.1
21.0 45.5 22.0 20.8 23.0 55.6 24.0 83.3
25.0 100.0 26.0 71.4 27.0 90.9 28.0 100.0
29.0 71.4 30.0 125.0 31.0 100.0 32.0 40.0
33.0 125,0 34.0 125.0 35.0 111.1

TU155 BLACKBURN AVERAGE SCORE OF S3
OPERATION:- MAZE LEARNING
32
1,0 5.6 2,0 5.6 3.0 13.5 4.0 11.8
5,0 34.5 6.0 33.3 7.0 9.5 8.0 24.4
9,0 15.4 10,0 19.2 11.0 22.2 12,0 23.3
13.0 55.6 14.0 43.5 15.0 43.5 16.0 18.2
17.0 45.5 18.0 37.0 19.0 21.3 20.0 41.7
21.0 37.0 22.0 52.6 23.0 47.6 24.0 41.7
25.0 26.3 26.0 76.9 27.0 55.6 28.0 50.0
29.0 52.6 30.0 52.6 31.0 66.7 32.0 83.3

TU156 BLACKBURN AVERAGE SCORE OF S4
OPERATION:- MAZE LEARNING
23
1,0 5.6 2,0 9.1 3.0 18.5 4.0 18.9
5,0 5.6 6.0 5.6 7.0 6.8 8.0 25.6
9.0 16.1 10.0 10.4 11.0 17.9 12.0 20.8
13.0 45.5 14.0 30.3 15.0 23.8 16.0 47.6
17.0 79.9 18.0 66.7 19.0 26.3 20.0 90.9
21.0 66.7 22.0 66.7 23.0 62.5

TU157 BLACKBURN AVERAGE SCORE OF S5
OPERATION:- MAZE LEARNING
33
1,0 5.6 2,0 5.6 3.0 5.6 4.0 6.4
5.0 5.6 6,0 5.6 7.0 5,6 8.0 5.6
9,0 5,6 10.0 5.6 11,0 5.6 12.0 5.6
13.0 9.8 14.0 14.5 15,0 10.1 16.0 13.3
17.0 9.3 18,0 11.0 19,0 12.7 20.0 10.1
21.0 27.0 22.0 23,0 23.0 30.3 24.0 30.3
25.0 31.2 26.0 37.0 27.0 58.8 28.0 37.0
29.0 33.3 30.0 33,3 31.0 35.7 32.0 38.5
33.0 22.2

T0158 BLACKBURN AVERAGE SCORE OF S6
OPERATION:- MAZE LEARNING
30
1.0 5.6 2.0 5.6 3.0 11.4 4.0 9.4
5.0 14.3 6.0 41.7 7.0 50.0 8.0 62.5
9.0 40.0 10.0 30.3 11.0 50.0 12.0 58.8
13.0 71.4 14.0 90.9 15.0 111.1 16.0 100.0
17.0 100.0 18.0 83.3 19.0 50.0 20.0 111.1
21.0 111.1 22.0 100.0 23.0 111.1 24.0 111.1
25.0 142.9 26.0 142.9 27.0 62.5 28.0 111.1
29.0 83.3 30.0 90.9

T0159 MORCOMBE CYCLE TIME (SECONDS)/TRIAL FOR GA.
OPERATION:- SIMULATED ASSEMBLY
20 MOD
1.0 175.0 2.0 155.0 3.0 165.0 4.0 145.0
5.0 135.0 6.0 130.0 7.0 127.0 8.0 128.0
9.0 127.0 10.0 127.0 11.0 127.0 12.0 127.0
13.0 128.0 14.0 126.0 15.0 127.0 16.0 127.0
17.0 128.0 18.0 126.0 19.0 127.0 20.0 127.0

T0160 MORCOMBE CYCLE TIME (SECONDS)/TRIAL FOR MS.
OPERATION:- SIMULATED ASSEMBLY
20 MOD
1.0 228.0 2.0 150.0 3.0 138.0 4.0 154.0
5.0 127.0 6.0 130.0 7.0 128.0 8.0 111.0
9.0 122.0 10.0 123.0 11.0 151.0 12.0 143.0
13.0 120.0 14.0 119.0 15.0 106.0 16.0 113.0
17.0 122.0 18.0 111.0 19.0 123.0 20.0 115.0

T0161 MORCOMBE CYCLE TIME (SECONDS)/TRIAL FOR PD.
OPERATION:- SIMULATED ASSEMBLY
20 MOD
1.0 255.0 2.0 155.0 3.0 115.0 4.0 125.0
5.0 120.0 6.0 113.0 7.0 112.0 8.0 113.0
9.0 111.0 10.0 105.0 11.0 118.0 12.0 103.0
13.0 106.0 14.0 95.0 15.0 96.0 16.0 99.0
17.0 107.0 18.0 107.0 19.0 100.0 20.0 102.0

T0162 MORCOMBE CYCLE TIME (SECONDS)/TRIAL FOR BC.
OPERATION:- SIMULATED ASSEMBLY
20 MOD
1.0 1145.0 2.0 293.0 3.0 200.0 4.0 157.0
5.0 152.0 6.0 166.0 7.0 152.0 8.0 143.0
9.0 142.0 10.0 143.0 11.0 145.0 12.0 137.0
13.0 148.0 14.0 147.0 15.0 144.0 16.0 156.0
17.0 138.0 18.0 126.0 19.0 124.0 20.0 125.0

T0163 MORCOMBE CYCLE TIME (SECONDS)/TRIAL FOR GG.
OPERATION:- SIMULATED ASSEMBLY
20 MOD
1.0 190.0 2.0 167.0 3.0 121.0 4.0 119.0
5.0 101.0 6.0 102.0 7.0 112.0 8.0 119.0
9.0 118.0 10.0 119.0 11.0 107.0 12.0 109.0
13.0 111.0 14.0 95.0 15.0 108.0 16.0 100.0
17.0 100.0 18.0 110.0 19.0 99.0 20.0 92.0

T0164 MORCOMBE CYCLE TIME (SECONDS)/TRIAL FOR JS.
OPERATION:- SIMULATED ASSEMBLY
20 MOD
1.0 118.0 2.0 114.0 3.0 111.0 4.0 105.0
5.0 94.0 6.0 91.0 7.0 93.0 8.0 102.0
9.0 94.0 10.0 101.0 11.0 92.0 12.0 95.0
13.0 101.0 14.0 97.0 15.0 101.0 16.0 89.0
17.0 94.0 18.0 87.0 19.0 92.0 20.0 88.0

T0165 HACKETT PERCENT OWN INITIATIVE.5-DAY SUM FOR EB.
OPERATION:- PLUGGING IN
18
1.0 84.00 2.0 90.42 3.0 93.26 4.0 94.59
5.0 96.36 6.0 98.11 7.0 97.50 8.0 97.43
9.0 99.11 10.0 99.06 11.0 99.19 12.0 100.0
13.0 100.0 14.0 100.0 15.0 100.0 16.0 100.0
17.0 100.0 18.0 100.0

T0166 HACKETT PERCENT OWN INITIATIVE.5-DAY SUM FOR EB.
OPERATION:- OPERATING KEYS
18
1.0 84.81 2.0 93.05 3.0 97.36 4.0 98.59
5.0 98.63 6.0 98.70 7.0 98.76 8.0 98.76
9.0 98.70 10.0 98.92 11.0 99.06 12.0 99.14
13.0 99.25 14.0 99.31 15.0 100.0 16.0 100.0
17.0 100.0 18.0 100.0

T0167 HACKETT PERCENT OWN INITIAITIVE.5-DAY SUM FOR EB.
OPERATION:- DIALLING
18
1.0 90.85 2.0 95.20 3.0 99.25 4.0 99.25
5.0 98.50 6.0 98.52 7.0 98.42 8.0 98.56
9.0 97.91 10.0 98.83 11.0 99.08 12.0 99.21
13.0 99.31 14.0 100.0 15.0 100.0 16.0 100.0
17.0 100.0 18.0 100.0

T0168 HACKETT PERCENT OWN INITIATIVE 5-DAY SUM FOR EB.
OPERATION:- USE V.I.F.
18
1.0 87.35 2.0 89.34 3.0 95.16 4.0 95.16
5.0 94.44 6.0 95.85 7.0 95.58 8.0 96.08
9.0 96.42 10.0 98.01 11.0 99.43 12.0 99.52
13.0 99.57 14.0 99.63 15.0 100.0 16.0 100.0
17.0 100.0 18.0 100.0

T0169 HACKETT PERCENT OWN INITIATIVE 5-DAY SUM FOR EB.
OPERATION:- TICKET WORK
18
1.0 84.71 2.0 91.03 3.0 95.67 4.0 97.40
5.0 98.05 6.0 94.24 7.0 94.97 8.0 92.79
9.0 92.22 10.0 90.94 11.0 95.86 12.0 95.47
13.0 97.03 14.0 98.13 15.0 99.56 16.0 99.48
17.0 100.0 18.0 100.0

```
T0170 HACKETT PERCENT OWN INITIATIVE 5-DAY SUM FOR EB.
OPERATION:- SPEAKING
 19
 1.0 40.21 2.0 50.52 3.0 60.21 4.0 65.00
 5.0 84.04 6.0 88.54 7.0 89.13 8.0 85.71
 9.0 85.71 10.0 80.95 11.0 88.0 12.0 89.83
 13.0 94.90 14.0 96.98 15.0 96.27 16.0 96.73
 17.0 96.76 18.0 96.64 19.0 96.93

T0171 HACKETT PERCENT OWN INITIATIVE 5-DAY SUM FOR EB.
OPERATION:- LISTENING
 19
 1.0 81.69 2.0 83.53 3.0 89.77 4.0 90.32
 5.0 93.15 6.0 94.5 7.0 95.52 8.0 96.11
 9.0 96.13 10.0 96.62 11.00 100.0 12.00 100.0
 13.0 100.0 14.0 100.0 15.0 99.77 16.0 99.77
 17.0 99.76 18.0 99.73 19.0 99.74

T0172 HACKETT PERCENT OWN INITIATIVE 5-DAY SUM FOR EB.
OPERATION:- FILING
 18
 1.0 83.87 2.0 96.29 3.0 92.30 4.0 93.10
 5.0 93.33 6.0 89.28 7.0 88.88 8.0 96.15
 9.0 92.85 10.0 87.50 11.0 93.54 12.0 95.74
 13.0 96.61 14.0 98.36 15.0 100.0 16.0 100.0
 17.0 100.0 18.0 100.0

T0173 HACKETT PERCENT OWN INITIATIVE 5-DAY SUM FOR EB.
OPERATION:- TIMING
 18
 1.0 63.46 2.0 79.54 3.0 81.39 4.0 79.48
 5.0 76.92 6.0 78.57 7.0 86.95 8.0 84.61
9.0 83.33 10.0 85.71 11.0 96.15 12.0 96.66
 13.0 100.0 14.0 100.0 15.0 100.0 16.0 100.0
 17.0 100.0 18.0 100.0

T0174 HACKETT PERCENT OWN INITIATIVE 5-DAY SUM FOR EB.
OPERATION:- CLEARING DOWN
 18
 1.0 85.0 2.0 86.04 3.0 86.95 4.0 93.75
5.0 94.33 6.0 95.91 7.0 96.00 8.00 97.91
 9.0 97.87 10.0 97.82 11.0 98.18 12.0 98.46
 13.0 98.64 14.0 98.75 15.0 100.0 16.0 100.0
 17.0 100.0 18.0 100.0

T0175 LAMB  VALUED CALLS COUNT FOR J DURING TRAINING.
OPERATION:- TELEPHONIST
 18
 2.0 101.5 3.0 100.0 4.0 118.5 5.0 102.5
8.0 117.0 9.0 135.5 10.0 131.0 11.0 147.0
 12.0 152.0 15.0 150.75 16.0 177.0 17.0 180.25
18.0 154.0 19.0 99.5 22.0 152.75 23.0 165.5
25.0 180.0 26.0 169.0
```

T0176 LAMB VALUED CALLS COUNT FOR J,ALL OBSERVATIONS
OPERATION:- TELEPHONIST
27
2.0 101.5 3.0 100.0 4.0 118.5 5.0 102.5
8.0 117.0 9.0 135.5 10.0 131.0 11.0 147.0
12.0 152.0 15.0 150.75 16.0 177.0 17.0 180.25
18.0 154.0 19.0 99.5 22.0 152.75 23.0 165.5
25.0 180.0 26.0 169.0 30.0 168.75 33.0 187.25
40.0 264.5 50.0 165.5 58.0 214.0 75.0 289.0
108.0 239.75 129.0 241.25 163.0 254.25

T0177 LAMB TEST SCORES FOR J.
OPERATION:- TELEPHONIST
5
5.0 100.0 9.0 139.0 17.0 163.5 19.0 186.0
121.0 244.0

T0178 LAMB VALUED CALLS COUNT FOR K. DURING TRAINING
OPERATION:- TELEPHONIST
19
2.0 49.5 3.0 68.5 4.0 56.0 5.0 87.25
8.0 95.0 9.0 115.75 10.0 85.5 11.0 105.75
12.0 109.0 15.0 98.25 16.0 128.25 17.0 136.25
18.0 143.0 19.0 147.5 22.0 144.0 23.0 132.75
24.0 114.5 25.0 127.5 26.0 117.0

T0179 LAMB VALUED CALLS COUNT FOR K,ALL OBSERVATIONS
OPERATION:- TELEPHONIST
27
2.0 49.5 3.0 68.5 4.0 56.0 5.0 81.25
8.0 95.0 9.0 115.75 10.0 85.5 11.0 105.75
12.0 109.0 15.0 98.25 16.0 128.25 17.0 136.35
18.0 143.0 19.0 147.5 22.0 144.0 23.0 132.75
24.0 114.5 25.0 127.5 26.0 117.0 29.0 152.0
33.0 104.25 37.0 141.5 47.0 190.5 54.0 162.5
115.0 219.5 136.0 214.75 165.0 213.25

T0180 LAMB TEST SCORES FOR K.
OPERATION:- TELEPHONIST
6
5.0 134.0 9.0 93.5 17.0 136.25 19.0 157.5
22.0 187.75 121.0 215.5

T0181 LAMB VALUED CALLS COUNT FOR L. DURING TRAINING
OPERATION:-TELEPHONIST
19
2.0 59.0 3.0 112.0 4.0 65.5 5.0 76.5
8.0 128.0 9.0 104.0 10.0 112.0 11.0 58.25
12.0 68.0 15.0 96.0 16.0 130.75 17.0 105.0
18.0 127.75 19.0 101.5 22.0 139.0 23.0 128.75
24.0 104.75 25.0 105.75 26.0 146.5

```
T0182 LAMB VALUED CALLS COUNT FOR L,ALL OBSERVATIONS
OPERATION:- TELEPHONIST
 27
2.0 59.0 3.0 112.0 4.0 65.5 5.0 76.5
8.0 128.0 9.0 104.0 10.0 112.0 11.0 58.25
12.0 68.0 15.0 96.0 16.0 130.75 17.0 105.0
18.0 127.75 19.0 101.5 22.0 139.0 23.0 128.75
24.0 104.75 25.0 105.75 26.0 146.5 29.0 110.5
33.0 143.0 37.0 133.0 47.0 158.75 54.0 147.5
115.0 174.5 136.0 222.5 165.0 190.0

T0183 LAMB  TEST SCORES FOR L.
OPERATION:-TELEPHONIST
  6
5.0 127.0 9.0 80.5 17.0 151.75 19.0 157.5
22.0 187.75 120.0 281.75

T0184  LAMB VALUED CALLS COUNT FOR S.DURING TRAINING
OPERATION:- TELEPHONIST
 17
2.0 94.0 3.0 126.0 4.0 93.0 5.0 125.0
8.0 140.25 9.0 117.0 10.0 192.5 11.0 175.0
12.0 205.5 15.0 166.75 16.0 181.5 17.0 154.0
18.0 167.5 22.0 179.25 23.0 182.0 25.0 161.5
26.0 155.0

T0185 LAMB VALUED CALLS COUNT FOR S,ALL OBSERVATIONS
OPERATION:- TELEPHONIST
 26
2.0 94.0 3.0 126.0 4.0 93.0 5.0 125.0
8.0 140.25 9.0 117.0 10.0 192.5 11.0 175.0
12.0 205.5 15.0 166.75 16.0 181.5 17.0 154.0
18.0 167.5 22.0 179.25 23.0 182.0 25.0 161.5
26.0 155.0 29.0 191.25 33.0 222.5 40.0 287.0
47.0 222.5 54.0 228.25 75.0 289.0 108.0 230.5
128.0 236.0 158.0 263.5

T0186 LAMB  TEST SCORES FOR S
OPERATION:- TELEPHONIST
  5
5.0 97.0 9.0 151.0 17.0 178.5 19.0 173.5
127.0 215.0

T0187 MINTER (CORRESPONDENCE)
 13
 0.0 30.0 4.0 49.49 8.0 55.55 12.0 68.41
 16.0 77.90 20.0 84.68 24.0 89.38 28.00 92.27
 32.0 88.12 36.0 98.63 40.0 103.51 44.0 103.11
 48.0 101.52
```

APPENDIX D

DETAILS CONCERNING BLACKBURN'S EXPERIMENTS.

In order to illustrate points in the arguments developed in the preceding pages frequent reference has been made to the experiments performed by me. Details of these experiments are presented here in order that the statements can be verified, if necessary, by reference to the actual figures.

The nature of the Experiments.

Five experiments were performed : - Card sorting, Maze learning, Code substitution, Crossing out e's and Addition.

(1) In the card-sorting experiment the observer had to sort a pack of 42 cards into their appropriate compartments on a table in front of him. The compartments were marked in random order and the pack of cards was also arranged in a random order for the first trial, although the same order was used in successive trials and with all observers. The observer was given the pack face upwards, and one sorting constituted a trial, his time being noted. The arrangement of the compartments is shown in Fig. 25, and the order of the cards was as follows: 4d, As, 5h, 2c, Qs, Js, Kc, 6d, Ah, Ks, 4s, 3c, 10s, 2s, Kh, 5d, 7c, Jc, Jh, 7h, Qd, 6h, 8d, Qh, 10c, 3d, Qc, 4h, 7d, 8h, 5s, 9s, 3h, 2d, 6c, 9c, 10h, 8s, 9d, Ac, 7s, Ad (where d, s, c, h, stand for diamonds, spades, clubs and hearts respectively).

8H	4S	3H	7C	AH	3D
KH	5S	10H	AC	QS	JC
KS	9D	6D	4H	9S	7D
4D	JS	2C	8D	QD	QH
JH	9C	AS	8S	5D	2D
7S	QC	7H	2S	3C	6H
KC	10S	AD	6C	5H	10C

FIG. 25

ARRANGEMENT OF COMPARTMENTS IN CARD SORTING EXPERIMENT; H, HEARTS; S, SPADES; C, CLUBS; D, DIAMONDS.

(2) In the maze-learning experiment the observer had to learn a stylus maze which was placed on the far side of a black cloth screen through which he put his hand. The observer was thus unable to see what he was doing, and he had to learn the maze by means of either visual images or kinaesthetic sensations, or a combination of both. The score was the time taken to get the pencil from the entrance to the exit. One run through the maze constituted a trial. The design of the maze is shown in Fig. 26.

(3) In the code-substitution experiment a rather complicated code was used in which the letters of the alphabet were represented by different combinations of the figures "1" and "2", and the figures "1" and "2" had to be represented by a stroke to the left (for "1")

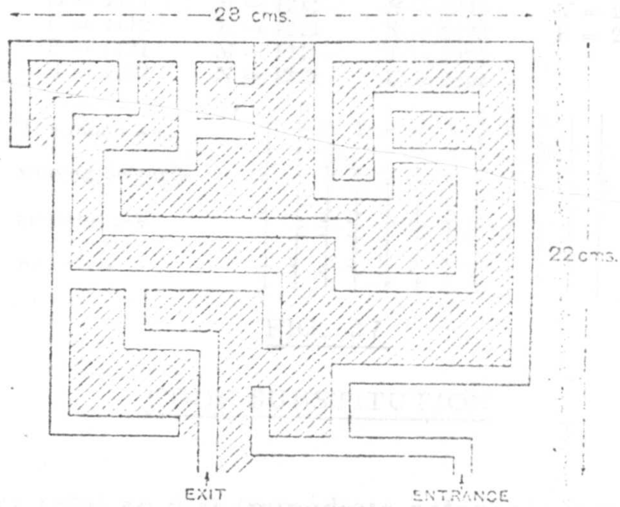

FIG. 26

PLAN OF THE MAZE.

or to the right (for "2") of a series of vertical lines on the form provided for the purpose. The arrangement of the code is shown in Fig. 27. This key was kept constantly in front of the observer

CODE SUBSTITUTION

1 = One mark to the LEFT. 2 = One mark to the RIGHT.

A = 11	H = 12	O = 21	V = 22
B = 111	I = 121	P = 211	W = 221
C = 112	J = 122	Q = 212	X = 222
D = 1111	K = 1211	R = 2111	Y = 2211
E = 1112	L = 1212	S = 2112	Z = 2212
F = 1121	M = 1221	T = 2121	
G = 1122	n = 1222	U = 2122	

1 = One mark to the *LEFT*. 2 = One mark to the *RIGHT*.

A = 11	H = 12	O = 21	V = 22
B = 111	I = 121	P = 211	W = 221
C = 112	J = 122	Q = 212	X = 222
D = 1111	K = 1211	R = 2111	Y = 2211
E = 1112	L = 1212	S = 2112	Z = 2212
F = 1121	M = 1221	T = 2121	
G = 1122	N = 1222	U = 2122	

The moon was
shining brightly
and the sky
was clear.

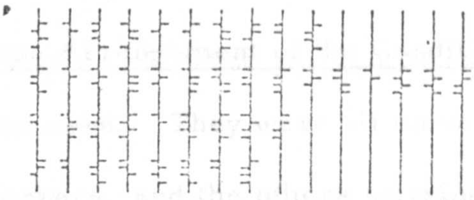

FIG. 27

CODE SUBSTITUTION

at every trial so that immediate reference could be made to it if required. The same passage containing about 100 words of prose was put before the observers on every trial, but only a portion of this was translated each time. Details concerning the practice periods will be found in the next section.

(4) In the addition experiment a page of Kraepelin's Rechenhefte was put before the observers, and they had to add successive pairs of the figures. The first two figures were added and the unit figure of the sum (if the result were greater than 9) was written at the side of the second figure, then the second and third figures were added and the unit figure of the sum was written by the side of the third figure, and so on, until the observer reached the bottom of the first column, after which he proceeded to the second column, and so on. Details concerning the practice periods will be found in the next section.

(5) Crossing out e's. This consisted of crossing out all the e's in a page of French words arranged in an order not making sense (see Fig. 28). There were 10 e's on each line - although this was

not noticed by any of the observers - and the position in which they occurred differed in every line. Details concerning the practice periods will be found in the next section.

Observers and the Arrangement of the Conditions of Learning.

7 observers were used. They were all university students, two being research workers, and the others working for their final degree is psychology.

Observer 1 did one trial a day on each of the tests for 6 days a week, and the order in which he did the tests remained the same on successive days. This order was - Card sorting, Addition, Code substitution, Crossing out e's, and Maze learning. Each trial in card sorting and maze learning consisted of one distribution of the cards or of one run through the maze each day. Each practice period in the addition, code substitution, and crossing out e's experiments consisted of three minutes work. This remained the same throughout the whole experiment, with the exception that in the e's test this observer managed to complete the whole page in under three minutes after the 14th trial. After this trial his record was scored by the time he took to do the page of e's.

Observer 2 did the experiments under exactly the same conditions, and with exactly the same arrangement, as Observer 1, except that in the e's test he never managed to complete the page in the three minutes allotted.

Observer 3 also did the experiment under the same conditions, with the exception that the order in which he did the tests varied from

routes voir premier pas mieux dire et le mouchoir emmener
soleil de marier le demeurer de bonnes des froid front
lui de se cinq le lendemain trouver minutes retard cellier
la virile de moyens jamais rarement sauvage perte les bleu
laissa le splendeur les or magnificences reve de sa beaux
nulle sur collier de cette rois certainement resistance on la
ces charmants plantureuse au prix de querelle est large ne
sans une des sires les plus de vegetation reporter esprit
passee son rivages de chart retour des la capricieuses ses
peur montrent des paysages faits des yeux etre nulle arret
part dans premiere rois les yeux la lumiere matelots meme
pour le aux dernieres implorent comme tant de terre pieux
souriait cruel beau rue ce des aspects ce radieux priere
plus grandirent avec et de pres de viendrait pelerinages la
de trois parvenait a cette la precieuse mirage que appelait

FIG. 28

MATERIAL USED IN THE E'S EXPERIMENT.

day to day.

Observer 4 did the same as Observers 1 and 2 in regard to the maze and the card distributing tests, except that his trials were not quite so regular - one or two days being occasionally missed between trials (this, however, had no discernible effect on his results). In the addition, code substitution, and e's tests the conditions were different for this observer. His record was scored by the time in successive trials that he took to do a fixed quantity of the work - this fixed quantity being the amount he did in three minutes on his first trial.

Observers 5, 6, and 7 did all their trials of one task on one afternoon, i.e., their trials were massed. They performed one test on the same day each week, and between each trial they gave their introspections before proceeding to the next trial. One longer interval of about 10 minutes was permitted after about the 14th trial. Apart from the fact that their trials were massed the constitution of the trials in the different tests was the same for them as for Observer 4, that is to say, one trial in the maze consisted of one run, and one trial in the card sorting test of one distribution of the cards: in the addition, crossing out e's, and code substitution tests their trials consisted of doing exactly the same amount in subsequent trials as they did in their first three minute trial, their scores being the time they took to do it.

The conditions and arrangements of the trials were deliberately altered for the different observers by me because I was primarily interested in discovering whether the different processes could be represented by typical learning curves. Consequently as many of the different factors as possible were altered, so that if the particular process did have any predominant characteristics they would become apparent.

The scores of the different observers in the tests are given below in Tables 1-V. The observers were not specially penalised if they made errors. The fact that they had made an error was regarded as sufficient penalty, hindering, as it did, the formation of the final adjustments required for a perfect knowledge of the problem. The scores are all given in the achievement form, this being the form on which the curves given in the preceding chapters have been based. In the maze test this has been obtained by simply taking the reciprocals of the original times: while the scores in the other tests are based on the average performance in the arbitrary time of 100 seconds. In order to check the figures given below with the figures given in the graphs it must also be remembered that in some cases the graphs have been based on a "moving average" in order to eliminate the day to day fluctuations. This moving average had a base of <u>three trials</u>, i.e. the first point was obtained by summing the performance scores in trials 1-3, the second by summing the performance scores in trials 2-4, the third in trials 3-5, etc. In every case it has been stated on the graphs when the moving average system was used.

NOTE: - The tables mentioned in the text are not included as the relevant data is in Appendix C.

APPENDIX E

TELEPHONIST TRAINING DATA OBTAINED FROM RECORDS IN OXFORD AND NORTH WEST TELEPHONE AREA

```
T0201 TELEPHONIST TRAINING DATA    T1, EXCHANGE A
  7
5.0 58.0 7.0 84.0 10.0 105.0 13.0 104.0
18.0 133.0 20.0 147.0 21.0 163.0
T0202 TELEPHONIST TRAINING DATA    T2, EXCHANGE A
  7
5.0 118.0 7.0 122.0 10.0 128.0 13.0 147.0
15.0 166.0 17.0 216.0 169.0 257.0
T0203 TELEPHONIST TRAINING DATA    T3, EXCHANGE A
  7
5.0 116.0 7.0 153.0 10.0 137.0 13.0 172.0
15.0 144.0 17.0 165.0 18.0 161.0
T0204 TELEPHONIST TRAINING DATA    T4, EXCHANGE A
  7
5.0 74.0 7.0 107.0 10.0 140.0 13.0 116.0
15.0 124.0 17.0 157.0 20.0 174.0
T0205 TELEPHONIST TRAINING DATA    T5, EXCHANGE A
  7
5.0 85.0 8.0 88.0 12.0 110.0 15.0 126.0
17.0 155.0 18.0 157.0 19.0 151.0
T0206 TELEPHONIST TRAINING DATA    T6, EXCHANGE A
  6
5.0 104.0 7.0 104.0 10.0 104.0 13.0 123.0
15.0 134.0 18.0 156.0
T0207 TELEPHONIST TRAINING DATA    T7, EXCHANGE A
  7
5.0 65.0 7.0 92.0 10.0 138.0 13.0 78.0
15.0 108.0 18.0 111.0 20.0 166.0
T0208 TELEPHONIST TRAINING DATA    T8, EXCHANGE A
  5
5.0 123.0 7.0 147.0 10.0 170.0 13.0 174.0
15.0 212.0
T0209 TELEPHONIST TRAINING DATA    T9, EXCHANGE A
  8
5.0 28.0 7.0 44.0 10.0 87.0 13.0 118.0
15.0 130.0 17.0 171.0 19.0 175.0 20.0 159.0
T0210 TELEPHONIST TRAINING DATA    T10, EXCHANGE A
  6
5.0 93.0 8.0 122.0 12.0 136.0 15.0 165.0
17.0 152.0 19.0 162.0
T0211 TELEPHONIST TRAINING DATA    T11, EXCHANGE A
  7
5.0 140.0 7.0 113.0 10.0 113.0 13.0 130.0
15.0 126.0 18.0 144.0 19.0 153.0
T0212 TELEPHONIST TRAINING DATA    T12, EXCHANGE A
  6
5.0 56.0 7.0 90.0 10.0 120.0 13.0 146.0
15.0 159.0 17.0 168.0
T0213 TELEPHONIST TRAINING DATA    T13, EXCHANGE A
  6
5.0 97.0 7.0 131.0 10.0 123.0 13.0 141.0
15.0 157.0 17.0 163.0
T0214 TELEPHONIST TRAINING DATA    T14, EXCHANGE A
  7
5.0 55.0 7.0 111.0 10.0 90.0 13.0 166.0
15.0 154.0 18.0 163.0 151.0 228.0
```

```
T0219 TELEPHONIST TRAINING DATA   T19, EXCHANGE A
  7
5.0 49.0 7.0 78.0 10.0 143.0 13.0 140.0
18.0 162.0 22.0 163.0 180.0 252.0
T0221 TELEPHONIST TRAINING DATA   T21, EXCHANGE A
  6
5.0 42.0 7.0 80.0 10.0 104.0 18.0 145.0
19.0 164.0 224.0 260.0
T0222 TELEPHONIST TRAINING DATA   T22, EXCHANGE B
  7
5.0 142.0 7.0 134.0 10.0 89.0 13.0 134.0
15.0 139.0 17.0 161.0 172.0 247.0
T0223 TELEPHONIST TRAINING DATA   T23, EXCHANGE A
  8
4.0 94.0 8.0 143.0 12.0 157.0 14.0 141.0
16.0 140.0 17.0 136.0 18.0 208.0 133.0 216.0
T0224 TELEPHONIST TRAINING DATA   T24, EXCHANGE B
  6
5.0 111.0 7.0 148.0 9.0 156.0 13.0 126.0
15.0 170.0 127.0 214.0
T0225 TELEPHONIST TRAINING DATA   T25, EXCHANGE A
  6
5.0 230.0 7.0 235.0 10.0 258.0 13.0 262.0
15.0 230.0 198.0 241.0
T0226 TELEPHONIST TRAINING DATA   T26, EXCHANGE C
  7
5.0 95.0 7.0 119.0 10.0 138.0 12.0 142.0
15.0 171.0 17.0 167.0 154.0 220.0
T0227 TELEPHONIST TRAINING DATA   T27, EXCHANGE A
  9
5.0 32.0 7.0 122.0 10.0 63.0 13.0 135.0
15.0 141.0 16.0 165.0 17.0 158.0 19.0 206.0
186.0 232.0
T0229 TELEPHONIST TRAINING DATA   T29, EXCHANGE A
 10
5.0 58.0 7.0 85.0 10.0 121.0 13.0 121.0
15.0 129.0 16.0 151.0 17.0 152.0 18.0 159.0
19.0 159.0 120.0 211.0
T0230 TELEPHONIST TRAINING DATA   T30, EXCHANGE A
  7
5.0 126.0 8.0 188.0 10.0 209.0 13.0 141.0
15.0 203.0 16.0 240.0 158.0 262.0
T0231 TELEPHONIST TRAINING DATA   T31, EXCHANGE A
  7
5.0 98.0 7.0 108.0 10.0 155.0 13.0 165.0
15.0 168.0 18.0 205.0 156.0 225.0
T0232 TELEPHONIST TRAINING DATA   T32, EXCHANGE A
  7
5.0 87.0 7.0 112.0 10.0 157.0 13.0 162.0
15.0 142.0 17.0 212.0 171.0 251.0
T0233 TELEPHONIST TRAINING DATA   T33, EXCHANGE A
  7
4.0 85.0 6.0 109.0 9.0 119.0 12.0 171.0
14.0 183.0 16.0 204.0 134.0 279.0
```

T0234 TELEPHONIST TRAINING DATA   T34, EXCHANGE B
6
6.0 111.0 8.0 115.0 10.0 143.0 14.0 144.0
17.0 180.0 305.0 254.0
T0235 TELEPHONIST TRAINING DATA   T35, EXCHANGE C
8
5.0 101.0 7.0 116.0 10.0 115.0 13.0 153.0
16.0 154.0 18.0 146.0 20.0 166.0 202.0 164.0
T0236 TELEPHONIST TRAINING DATA   T36, EXCHANGE A
8
4.0 109.0 8.0 88.0 12.0 146.0 14.0 161.0
16.0 154.0 17.0 142.0 18.0 176.0 137.0 203.0
T0238 TELEPHONIST TRAINING DATA   T38, EXCHANGE A
8
5.0 129.0 7.0 105.0 10.0 123.0 13.0 124.0
15.0 140.0 17.0 158.0 18.0 193.0 137.0 205.0
T0239 TELEPHONIST TRAINING DATA   T39, EXCHANGE A
7
5.0 93.0 7.0 116.0 10.0 178.0 13.0 148.0
15.0 156.0 17.0 188.0 123.0 262.0
T0240 TELEPHONIST TRAINING DATA   T40, EXCHANGE A
8
5.0 67.0 7.0 100.0 10.0 126.0 13.0 98.0
15.0 120.0 19.0 127.0 20.0 170.0 134.0 220.0
T0241 TELEPHONIST TRAINING DATA   T41, EXCHANGE A
7
5.0 128.0 7.0 161.0 10.0 153.0 13.0 126.0
15.0 149.0 17.0 208.0 198.0 220.0
T0242 TELEPHONIST TRAINING DATA   T42, EXCHANGE A
8
5.0 36.0 7.0 115.0 10.0 85.0 13.0 103.0
15.0 98.0 17.0 110.0 19.0 169.0 131.0 226.0
T0243 TELEPHONIST TRAINING DATA   T43, EXCHANGE B
4
5.0 142.0 7.0 126.0 10.0 153.0 13.0 191.0
T0244 TELEPHONIST TRAINING DATA   T44, EXCHANGE A
10
5.0 31.0 7.0 109.0 10.0 124.0 13.0 100.0
15.0 153.0 16.0 141.0 17.0 166.0 18.0 166.0
19.0 182.0 121.0 216.0
T0246 TELEPHONIST TRAINING DATA   T46, EXCHANGE A
8
5.0 106.0 7.0 140.0 10.0 140.0 13.0 110.0
15.0 128.0 17.0 154.0 19.0 156.0 126.0 257.0
T0247 TELEPHONIST TRAINING DATA   T47, EXCHANGE A
6
5.0 94.0 7.0 131.0 10.0 173.0 13.0 156.0
15.0 201.0 85.0 224.0
T0248 TELEPHONIST TRAINING DATA   T48, EXCHANGE A
5
5.0 146.0 7.0 128.0 10.0 128.0 16.0 148.0
18.0 192.0
T0249 TELEPHONIST TRAINING DATA   T49, EXCHANGE B
5
5.0 117.0 7.0 170.0 10.0 196.0 13.0 171.0
288.0 237.0

T0251 TELEPHONIST TRAINING DATA T51, EXCHANGE F
7
5,0 111,0 7,0 122,0 9,0 135,0 13,0 137,0
15,0 143,0 17,0 189,0 318,0 226,0
T0252 TELEPHONIST TRAINING DATA T52, EXCHANGE F
7
5,0 137,0 7,0 113,0 9,0 157,0 13,0 116,0
20,0 145,0 22,0 167,0 122,0 229,0
T0253 TELEPHONIST TRAINING DATA T53, EXCHANGE G
10
5,0 60,0 7,0 90,0 9,0 106,0 11,0 110,0
13,0 143,0 15,0 153,0 17,0 137,0 19,0 161,0
21,0 170,0 109,0 265,0
T0254 TELEPHONIST TRAINING DATA T54, EXCHANGE F
7
5,0 115,0 7,0 113,0 9,0 120,0 13,0 132,0
15,0 153,0 17,0 170,0 122,0 270,0
T0255 TELEPHONIST TRAINING DATA T55, EXCHANGE F
7
5,0 88,0 7,0 121,0 9,0 177,0 13,0 106,0
15,0 102,0 17,0 201,0 113,0 277,0
T0256 TELEPHONIST TRAINING DATA T56, EXCHANGE F
7
5,0 115,0 7,0 149,0 9,0 176,0 13,0 196,0
20,0 347,0 22,0 256,0 118,0 210,0
T0257 TELEPHONIST TRAINING DATA T57, EXCHANGE G
8
5,0 62,0 7,0 88,0 9,0 163,0 13,0 137,0
15,0 171,0 17,0 131,0 20,0 193,0 106,0 288,0
T0258 TELEPHONIST TRAINING DATA T58, EXCHANGE G
9
5,0 54,0 7,0 73,0 9,0 79,0 11,0 70,0
13,0 82,0 15,0 91,0 17,0 120,0 18,0 201,0
135,0 347,0
T0259 TELEPHONIST TRAINING DATA T59, EXCHANGE F
7
5,0 113,0 7,0 137,0 9,0 148,0 13,0 111,0
15,0 126,0 17,0 159,0 126,0 210,0
T0260 TELEPHONIST TRAINING DATA T60, EXCHANGE F
7
5,0 75,0 7,0 95,0 9,0 135,0 13,0 116,0
15,0 170,0 17,0 153,0 158,0 249,0
T0261 TELEPHONIST TRAINING DATA T61, EXCHANGE F
7
5,0 95,0 7,0 123,0 9,0 173,0 13,0 197,0
20,0 254,0 22,0 285,0 104,0 247,0
T0262 TELEPHONIST TRAINING DATA T62, EXCHANGE G
9
5,0 62,0 7,0 101,0 9,0 87,0 13,0 120,0
15,0 105,0 17,0 81,0 22,0 166,0 27,0 195,0
121,0 231,0

T0263 TELEPHONIST TRAINING DATA T63, EXCHANGE F
7
5,0 113,0 7,0 138,0 9,0 200,0 13,0 173,0
15,0 192,6 17,0 190.0 107,0 249,0
T0264 TELEPHONIST TRAINING DATA T64, EXCHANGE F
7
5,0 105,0 10,0 134.0 12,0 146,0 17,0 161,0
19,0 162,0 21,0 189.0 134,0 257,0
T0265 TELEPHONIST TRAINING DATA T65, EXCHANGE G
7
5,0 50,0 7,0 80.0 9.0 90,0 17,0 153.0
18,0 161,0 19,0 160.0 107,0 245,0
T0266 TELEPHONIST TRAINING DATA T66, EXCHANGE G
10
5,0 54,0 7,0 62,0 10.0 73,0 13,0 75,0
15,0 90,0 17,0 97,0 19,0 107,0 20,0 133,0
22,0 185,0 128,0 306,0
T0267 TELEPHONIST TRAINING DATA T67, EXCHANGE F
7
5,0 80,0 7,0 95,0 9.0 133,0 13,0 127,0
20,0 144,0 22,0 171.0 106,0 231,0
T0268 TELEPHONIST TRAINING DATA T68, EXCHANGE G
8
5,0 123,0 7,0 117,0 9,0 136,0 13,0 163,0
16,0 195,0 17,0 189,0 18,0 197,0 112,0 310,0
T0269 TELEPHONIST TRAINING DATA T69, EXCHANGE F
7
5,0 75,0 7,0 82.0 9.0 136,0 13,0 126,0
15,0 145,0 17,0 166.0 111,0 214,0
T0270 TELEPHONIST TRAINING DATA T70, EXCHANGE G
10
5,0 61,0 7,0 72,0 9.0 89,0 11,0 76,0
13,0 106,0 15,0 109.0 17,0 155,0 18.0 179,0
19,0 188,0 102,0 335,0
T0271 TELEPHONIST TRAINING DATA T71, EXCHANGE F
7
5,0 113,0 7,0 112,0 9,0 122,0 13,0 106,0
15,0 113,0 17,0 159.0 126,0 218,0
T0272 TELEPHONIST TRAINING DATA T72, EXCHANGE F
7
5,0 110,0 7,0 138,0 9.0 162,0 13,0 137,0
15,0 192,0 17,0 202,0 107,0 249,0
T0273 TELEPHONIST TRAINING DATA T73, EXCHANGE F
7
5,0 144,0 7,0 232,0 9,0 234,0 13,0 203,0
15,0 197,0 17,0 217.0 183,0 248,0
T0274 TELEPHONIST TRAINING DATA T74, EXCHANGE G
10
5,0 74,0 7,0 89,0 9.0 117,0 11,0 76,0
13,0 86,0 15,0 104,0 17,0 137,0 19,0 116,0
20,0 141,0 32,0 237,0
T0275 TELEPHONIST TRAINING DATA T75, EXCHANGE F
7
5,0 82,0 7,0 170,0 9,0 195,0 13,0 156,0
20,0 178,0 22,0 192,0 108,0 252,0

```
T0276 TELEPHONIST TRAINING DATA   T76, EXCHANGE G
 10
5.0 59.0 7.0 79.0 9.0 110.0 11.0 141.0
13.0 123.0 15.0 106.0 17.0 166.0 19.0 155.0
20.0 154.0 114.0 254.0
T0277 TELEPHONIST TRAINING DATA   T77, EXCHANGE F
  7
5.0 204.0 7.0 214.0 9.0 218.0 13.0 181.0
15.0 178.0 17.0 189.0 121.0 265.0
T0278 TELEPHONIST TRAINING DATA   T78, EXCHANGE F
  9
5.0 45.0 7.0 137.0 9.0 130.0 13.0 106.0
15.0 137.0 17.0 122.0 18.0 115.0 119.0 147.0
140.0 221.0
T0279 TELEPHONIST TRAINING DATA   T79, EXCHANGE F
  7
5.0 78.6 7.0 87.0 9.0 120.0 13.0 87.0
20.0 124.0 22.0 157.0 133.0 229.0
T0280 TELEPHONIST TRAINING DATA   T80, EXCHANGE F
  7
5.0 85.0 7.0 82.0 9.0 117.0 13.0 106.0
20.0 144.0 22.0 151.0 109.0 214.0
T0281 TELEPHONIST TRAINING DATA   T81, EXCHANGE F
  7
5.0 75.0 7.0 141.0 9.0 154.0 13.0 129.0
15.0 153.0 17.0 189.0 207.0 237.0
T0282 TELEPHONIST TRAINING DATA   T82, EXCHANGE G
  9
5.0 40.0 7.0 32.0 9.0 58.0 11.0 104.0
17.0 155.0 19.0 158.0 21.0 183.0 23.0 194.0
107.0 262.0
T0283 TELEPHONIST TRAINING DATA   T83, EXCHANGE F
  7
5.0 110.0 7.0 117.0 10.0 159.0 13.0 133.0
20.0 182.0 22.0 174.0 112.0 212.0
T0284 TELEPHONIST TRAINING DATA   T84, EXCHANGE G
  9
5.0 43.0 7.0 57.0 13.0 70.0 15.0 89.0
17.0 88.0 19.0 115.0 20.0 131.0 21.0 174.0
129.0 252.0
T0285 TELEPHONIST TRAINING DATA   T85, EXCHANGE F
  7
5.0 132.0 7.0 155.0 9.0 167.0 13.0 112.0
15.0 132.0 17.0 106.0 77.0 279.0
T0286 TELEPHONIST TRAINING DATA   T86, EXCHANGE F
  7
5.0 60.0 7.0 93.0 9.0 107.0 13.0 111.0
15.0 149.0 17.0 232.0 150.0 206.0
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T0287 TELEPHONIST TRAINING DATA   T87, EXCHANGE F
  7
5.0 117.0 7.0 113.0 10.0 157.0 13.0 115.0
20.0 160.0 22.0 183.0 113.0 207.0
T0288 TELEPHONIST TRAINING DATA   T88, EXCHANGE F
  7
5.0 105.0 7.0 57.0 13.0 70.0 15.0 89.0
20.0 151.0 22.0 204.0 123.0 205.0
T0289 TELEPHONIST TRAINING DATA   T89, EXCHANGE F
  7
5.0 122.0 7.0 142.0 9.0 162.0 13.0 161.0
20.0 184.0 22.0 203.0 106.0 259.0
T0290 TELEPHONIST TRAINING DATA   T90, EXCHANGE G
 10
5.0 58.0 7.0 99.0 9.0 58.0 11.0 67.0
13.0 79.0 15.0 50.0 17.0 61.0 19.0 55.0
20.0 168.0 135.0 302.0
T0291 TELEPHONIST TRAINING DATA   T91, EXCHANGE F
  7
5.0 123.0 7.0 157.0 9.0 192.0 13.0 161.0
15.0 163.0 17.0 208.0 109.0 218.0
T0292 TELEPHONIST TRAINING DATA   T92, EXCHANGE F
  7
5.0 115.0 7.0 135.0 9.0 159.0 13.0 152.0
15.0 186.0 17.0 170.0 100.0 215.0
T0293 TELEPHONIST TRAINING DATA   T93, EXCHANGE G
 10
5.0 73.0 7.0 75.0 10.0 120.0 13.0 91.0
15.0 99.0 17.0 126.0 19.0 134.0 21.0 144.0
30.0 199.0 122.0 289.0
T0294 TELEPHONIST TRAINING DATA   T94, EXCHANGE G
 10
5.0 91.0 7.0 107.0 9.0 114.0 11.0 123.0
16.0 135.0 18.0 144.0 21.0 152.0 22.0 153.0
23.0 153.0 114.0 342.0
T0295 TELEPHONIST TRAINING DATA   T95, EXCHANGE F
  7
5.0 115.0 7.0 153.0 9.0 155.0 13.0 186.0
15.0 178.0 17.0 193.0 155.0 284.0
T0296 TELEPHONIST TRAINING DATA   T96, EXCHANGE G
 10
5.0 129.0 7.0 146.0 9.0 147.0 11.0 125.0
13.0 161.0 15.0 146.0 17.0 175.0 18.0 185.0
19.0 175.0 109.0 256.0
```

APPENDIX F

BEST FIT PARAMETER VALUES FOR DATA IN APPENDIX C

BEST FIT PARAMETER VALUES BEVIS MODEL

DATA SET	Y_c	Y_f	TAN	FINAL	START
100	22.0	80.32	32.21	102.32	22.00
101	24.16	72.86	44.69	97.01	24.16
102	115.26	72.64	4.68	187.90	115.26
103	17.38	58.79	23.60	76.17	17.38
104	18.68	109.09	79.22	127.77	18.68
105	55.77	73.31	6.12	129.07	55.77
106	61.00	73.91	6.29	134.91	61.00
107	-1.41	290.24	77.70	288.83	-1.41
108	-3.63	123.42	29.22	119.78	-3.63
109	-2.29	69.28	6.36	65.67	-2.29
110	-2.29	69.28	6.36	136.27	-2.29
111	10.21	58.48	9.83	68.69	10.21
112	-1.87	10.51	1.64	8.64	-1.87
113	4.96	105.18	4.33	110.14	4.96
114	24.34	82.33	6.63	106.68	24.34
115	1460.26	3502.32	19.79	4962.59	1460.26
116	1475.11	5862.90	22.25	7338.01	1475.11
117					

BEST FIT PARAMETER VALUES BEVIS MODEL

DATA SET	Y_c	Y_f	TAN	FINAL	START
118					
119	181.26	31.47	49.21	49.49	181.26
120	99.64	157.46	13.52	257.10	99.64
121	-22.78	259.24	6.68	236.46	-22.78
122	65.33	180.59	18.91	245.92	65.33
123	74.73	164.11	20.13	238.84	74.73
124	71.35	156.42	20.14	227.77	71.35
125	58.49	195.46	26.33	253.95	58.49
126	38.95	197.37	21.50	236.32	38.95
127	-13.82	261.49	17.33	247.68	-13.82
128	1.09	272.12	25.94	273.21	1.09
129	33.82	130.69	53.74	164.52	33.82
130					
131					
132	20.47	37.08	14.21	20.47	20.47
133	24.99	90.35	52.98	115.34	24.99
134	26.00	324.98	111.41	350.97	26.00
135	14.71	178.98	40.84	193.68	14.71
136	157.26	155.08	26.40	312.34	157.26
137	80.42	70.57	3.21	150.99	80.42
138	129.15	49.79	14.03	178.94	129.15
139	144.42	64.82	7.28	209.24	144.42
140	21.70	93.90	39.15	115.60	21.70

BEST FIT PARAMETER VALUES BEVIS MODEL

DATA SET	Y_c	Y_f	TAN	FINAL	START
141	13.64	152.14	144.71	165.79	13.64
142					
143					
144	17.98	82.66	75.77	100.64	17.98
145	22.59	62.97	20.63	85.56	22.59
146	3.18	72.56	10.59	104.35	3.18
147	61.66	86.55	7.71	148.22	61.66
148	44.24	60.48	9.95	104.73	44.24
149	72.88	67.06	3.64	139.94	72.88
150	61.42	117.75	12.01	179.17	61.42
151	63.27	92.06	6.58	155.32	63.27
152	53.80	932.60	401.27	986.41	53.80
153	-24.09	139.68	11.82	115.59	-24.09
154	-2.66	170.77	35.63	168.11	-2.66
155					
156					
157					
158	-18.74	142.99	12.52	124.25	-18.74
159	16.95	11.66	3.11	28.61	16.95
160	10.43	19.52	2.35	29.95	10.43
161	3.44	31.07	2.08	34.51	3.44
162	-12.18	38.03	1.92	25.86	-12.18
163	8.20	26.18	2.09	34.39	8.20

BEST FIT PARAMETER VALUES BEVIS MODEL

DATA SET	Y_c	Y_f	TAN	FINAL	START
164	26.60	12.10	3.15	38.70	26.60
165	77.72	22.12	2.65	99.84	77.72
166	63.68	35.64	1.12	99.31	63.68
167	76.43	22.82	1.02	99.25	76.43
168	85.63	14.82	5.11	100.44	85.63
169	63.08	33.38	0.98	96.46	63.08
170	21.37	74.75	3.75	96.12	21.37
171	76.30	24.24	4.42	100.54	76.30
172					
173	67.33	61.93	21.22	129.26	67.33
174	79.31	20.88	4.26	100.19	79.31
175	82.11	94.50	11.72	176.62	82.11
176	92.28	165.03	36.31	257.31	92.28
177	57.07	186.90	17.58	243.98	57.07
178	24.21	112.15	7.67	136.36	24.21
179	60.89	156.50	39.17	217.39	60.89
180	86.82	132.13	25.54	218.95	86.82
181					
182	73.70	133.43	55.27	207.13	73.70
183	72.09	219.27	36.90	291.36	72.09
184	44.69	129.58	4.63	174.27	44.69
185	95.23	160.28	25.09	255.52	95.23
186	32.76	176.94	9.89	209.71	32.76
187	30.90	77.30	17.85	108.20	30.90

BEST FIT PARAMETER VALUES GOMPERTZ MODEL

DATA SET	K	A	B	FINAL	START
100	76.18	0.30	0.93	76.18	22.56
101	76.96	0.32	0.95	76.96	24.80
102	187.48	0.63	0.79	187.48	117.78
103	64.07	0.29	0.92	64.07	18.45
104	79.43	0.25	0.96	79.43	19.48
105	126.94	0.46	0.81	126.94	58.21
106	133.92	0.48	0.83	133.92	64.34
107	87.28	0.040	0.89	87.28	3.51
108	82.34	0.043	0.89	82.34	3.54
109	64.52	0.22	0.90	64.52	14.05
110	66.33	0.15	0.81	66.33	9.74
111	68.00	0.26	0.88	68.00	17.78
112	8.61	0.098	0.45	8.61	0.85
113	99.25	0.14	0.66	99.25	13.97
114	98.93	0.28	0.79	98.93	28.10
115	4628.92	0.33	0.92	4628.92	1515.32
116	5733.79	0.27	0.90	5733.79	1532.85
117	10254.76	0.13	0.98	10254.76	1329.12
118	25148.37	0.052	0.98	25148.37	1307.36
119	48.72	0.40	0.97	48.72	19.70
120	257.11	0.43	0.92	257.11	111.66
121	236.58	0.19	0.84	236.58	45.40
122	245.71	0.32	0.93	245.71	77.98

BEST FIT PARAMETER VALUES GOMPERTZ MODEL

DATA SET	K	A	B	FINAL	START
123	238.72	0.36	0.94	238.72	84.78
124	227.72	0.35	0.94	227.72	80.38
125	253.18	0.29	0.95	253.18	73.39
126	236.01	0.24	0.93	236.01	57.12
127	247.07	0.11	0.92	247.07	28.01
128	270.54	0.10	0.94	270.54	27.65
129	124.51	0.28	0.96	124.51	35.23
130	87.55	0.19	0.98	87.55	17.16
131	129.82	0.19	0.98	129.82	24.29
132	53.58	0.39	0.89	53.58	20.76
133	88.27	0.29	0.96	88.27	25.99
134	146.46	0.19	0.95	146.46	27.72
135	106.18	0.15	0.90	106.18	16.44
136	305.13	0.53	0.95	305.13	160.36
137	150.95	0.56	0.71	150.95	84.57
138	178.31	0.73	0.92	178.31	129.95
139	208.54	0.70	0.86	208.54	145.62
140	93.14	0.25	0.96	93.14	23.52
141	69.12	0.20	0.96	69.12	14.17
142	300.40	0.073	0.98	300.40	21.99
143	134.84	0.18	0.96	134.84	24.16
144	77.37	0.24	0.97	77.37	19.13
145	77.72	0.31	0.92	77.72	24.14

BEST FIT PARAMETER VALUES GOMPERTZ MODEL

DATA SET	K	A	B	FINAL	START
146	71.49	0.14	0.86	71.49	9.80
147	147.77	0.46	0.86	147.77	68.20
148	103.49	0.45	0.88	103.49	46.64
149	139.64	0.53	0.72	139.64	74.61
150	166.40	0.38	0.88	166.40	63.32
151	153.71	0.43	0.83	153.71	66.37
152	282.41	0.20	0.98	282.41	55.13
153					
154	115.47	0.070	0.92	115.47	8.08
155					
156	139.97	0.016	0.92	139.97	2.18
157					
158	109.12	0.017	0.83	109.12	1.84
159	28.57	0.61	0.70	28.57	17.38
160	29.89	0.42	0.62	29.89	12.48
161	34.33	0.24	0.54	34.33	8.12
162	25.59	0.015	0.44	25.59	0.39
163	34.24	0.31	0.54	34.24	10.46
164	38.63	0.69	0.70	38.63	26.76
165	99.81	0.78	0.67	99.81	78.28
166	99.31	0.68	0.39	99.31	66.30
167	99.25	0.78	0.37	99.25	77.53
168	100.40	0.85	0.81	100.40	85.80

BEST FIT PARAMETER VALUES GOMPERTZ MODEL

DATA SET	K	A	B	FINAL	START
169	96.45	0.68	0.35	96.45	65.69
170	94.95	0.29	0.70	94.95	27.44
171	100.43	0.76	0.78	100.43	76.72
172					
173	121.66	0.56	0.93	121.66	67.60
174	100.09	0.79	0.78	100.09	79.55
175	172.11	0.49	0.89	172.11	84.10
176	253.86	0.38	0.96	253.86	97.67
177	243.78	0.29	0.93	243.78	71.62
178	134.17	0.26	0.84	134.17	35.31
179	214.64	0.32	0.96	214.64	69.02
180	218.17	0.40	0.95	218.17	88.29
181	322.67	0.23	0.98	322.67	74.73
182	201.22	0.38	0.97	201.22	76.58
183	284.60	0.26	0.96	284.60	75.46
184	173.75	0.34	0.77	173.75	58.52
185	254.54	0.40	0.95	254.54	102.88
186	209.31	0.28	0.89	209.31	59.71
187	104.03	0.32	0.92	104.03	32.80

BEST FIT PARAMETER VALUES MATHEMATICAL MODEL

DATA SET	B	C	G	FINAL	START
100	163.97	0.0070	0.00013	163.97	21.98
101	146.95	0.0081	0.00011	146.95	24.01
102	200.22	0.010	0.0035	200.22	101.28
103	111.49	0.011	0.00030	111.49	17.08
104	209.05	0.0053	0.000039	209.05	18.58
105	151.85	0.010	0.0016	151.85	52.40
106	150.22	0.010	0.0021	150.22	52.21
107					
108	201.82	0.0049	0.00010	201.82	-3.60
109	77.39	0.013	0.0015	77.39	-0.04
110	75.39	0.0092	0.0035	75.39	-33.33
111	76.78	0.011	0.0025	76.78	-11.12
112	9.21	0.0083	0.15	9.21	-111.41
113	155.11	0.0066	0.0012	155.11	2.53
114	141.88	0.0084	0.0010	141.88	22.21
115	6704.22	0.00019	0.0000069	6704.22	1449.91
116	11607.18	0.000099	0.0000026	11607.18	1474.84
117					
118					
119	57.42	0.024	0.00058	57.42	15.73
120	263.95	0.0037	0.00092	263.95	-9.62
121					
122	259.46	0.0043	0.00042	259.46	28.96

BEST FIT PARAMETER VALUES MATHEMATICAL MODEL

DATA SET	B	C	G	FINAL	START
123	256.45	0.0049	0.00038	256.45	51.96
124	243.26	0.0051	0.00041	243.26	48.72
125	281.63	0.0040	0.00022	281.63	32.02
126	257.91	0.0040	0.00030	257.91	5.88
127	274.48	0.0029	0.00030	274.48	-75.95
128	323.25	0.0029	0.00013	323.25	-12.72
129	249.34	0.0046	0.000054	249.34	33.43
130	165.71	[illegible]	[illegible]	[illegible]	[illegible]
131	[illegible]	0.0038	[illegible]	[illegible]	[illegible]
132	79.07	0.017	0.00078	79.07	20.66
133	167.26	0.0070	0.000089	167.26	24.60
134					
135	351.42	0.0030	0.000039	351.42	14.83
136	356.60	0.0048	0.00019	356.60	150.14
137	155.44	0.0078	0.0094	155.44	27.34
138	190.06	0.015	0.0014	190.06	123.55
139	226.29	0.011	0.0018	226.29	139.20
140	172.94	0.0065	0.00011	172.94	21.27
141	305.73	0.0034	0.000012	305.73	13.65
142	102.17	0.026	[illegible]	[illegible]	[illegible]
143					
144	131.43	0.0088	0.000092	131.43	17.46
145	118.17	0.010	0.00036	118.17	22.04

BEST FIT PARAMETER VALUES MATHEMATICAL MODEL

DATA SET	B	C	G	FINAL	START
146	99.59	0.010	0.00091	99.59	0.75
147	159.92	0.0080	0.0022	159.92	35.04
148	120.43	0.012	0.0014	120.43	40.27
149	148.09	0.012	0.0054	148.09	63.50
150	244.33	0.0055	0.00031	244.33	61.12
151	178.27	0.0083	0.0014	178.27	59.19
152					
153	165.71	0.0053	0.00038	165.71	-24.32
154	262.57	0.0038	0.000073	262.57	-3.99
155					
156					
157					
158	183.5	0.0049	0.00031	183.5	-19.62
159	30.16	0.063	0.038	30.16	14.15
160	31.45	0.019	0.046	31.45	-20.36
161	36.73	0.011	0.033	36.73	-54.68
162	28.52	0.0099	0.029	28.52	-72.60
163	36.92	0.024	0.030	36.92	-4.69
164	40.63	0.063	0.031	40.63	24.63
165	102.17	0.026	0.029	102.17	63.88
166					
167					
168	103.29	0.049	0.015	103.29	82.73

BEST FIT PARAMETER VALUES MATHEMATICAL MODEL

DATA SET	B	C	G	FINAL	START
169					
170	109.22	0.0099	0.0041	109.22	8.42
171	105.41	0.031	0.0097	105.41	73.03
172					
173					
174	104.86	0.036	0.011	104.86	77.45
175	213.96	0.0074	0.00054	213.96	79.72
176	300.28	0.0047	0.00013	300.28	86.66
177	262.19	0.0041	0.00042	262.19	20.05
178	162.86	0.0066	0.0011	162.86	10.89
179	249.74	0.0050	0.00015	249.74	50.88
180	247.17	0.0062	0.00024	247.17	86.10
181					
182	253.27	0.0055	0.000087	253.27	71.37
183	360.65	0.0034	0.000079	360.65	70.82
184	192.44	0.0052	0.0023	192.44	1.70
185	282.63	0.0050	0.00026	282.63	81.22
186	215.69	0.0012	0.0015	215.69	-608.14
187	135.48	0.0095	0.00047	135.48	29.94

BEST FIT PARAMETER VALUES WILTSHIRE MODEL

DATA SET	C	K	ALPHA	N	FINAL	START
100	67.73	43.33	0.023	1.40	67.73	24.41
101						
102	191.24	98.16	0.43	0.68	191.24	93.08
103						
104						
105	126.18	65.55	0.12	1.19	126.18	60.63
106	139.29	93.42	0.29	0.72	139.29	45.88
107	73.26	65.44	0.0030	2.18	73.26	7.82
108	76.93	70.35	0.0052	1.88	76.93	6.58
109	68.86	75.71	0.19	0.71	68.86	-6.85
110	67.14	71.98	0.18	0.95	67.14	-4.84
111						
112						
113	93.14	70.18	0.085	1.82	93.14	22.96
114						
115	4323.81	2589.15	0.010	1.67	4323.81	1734.66
116	4857.07	3060.84	0.017	1.74	4857.07	1796.23
117	5771.53	4265.34	0.0014	1.76	5771.53	1506.19
118						
119						
120						
121						
122	247.22	220.58	0.14	0.73	247.22	26.64

BEST FIT PARAMETER VALUES WILTSHIRE MODEL

DATA SET	C	K	ALPHA	N	FINAL	START
123						
124						
125						
126	236.96	212.85	0.071	0.88	236.96	24.12
127						
128						
129						
130						
131						
132						
133						
134						
135	83.93	60.71	0.0059	2.08	83.93	23.22
136						
137						
138						
139	214.71	80.49	0.23	0.74	214.71	134.22
140						
141	82.05	67.55	0.0097	1.17	82.05	14.50
142						
143	350.45	326.17	0.0043	1.10	350.45	24.28
144						
145	92.45	71.28	0.056	0.90	92.45	21.16

BEST FIT PARAMETER VALUES WILTSHIRE MODEL

DATA SET	C	K	ALPHA	N	FINAL	START
146	71.73	62.58	0.049	1.28	71.73	9.16
147						
148	105.34	62.39	0.11	0.95	105.34	42.95
149						
150	151.60	77.57	0.022	1.77	151.60	74.03
151	150.52	70.48	0.028	1.79	150.52	80.05
152						
153						
154						
155						
156						
157						
158						
159						
160						
161						
162						
163						
164						
165	100.94	63.64	1.33	0.42	100.94	37.30
166						
167						
168						
169						

BEST FIT PARAMETER VALUES WILTSHIRE MODEL

DATA SET	C	K	ALPHA	N	FINAL	START
170	97.13	82.18	0.34	0.88	97.13	14.95
171	100.41	23.40	0.20	1.05	100.41	77.00
172						
173						
174	99.38	16.72	0.096	1.50	99.38	82.56
175						
176	254.12	155.69	0.017	1.13	254.12	98.43
177	262.70	564.88	0.75	0.32	262.70	-302.19
178						
179						
180						
181						
182	215.50	145.88	0.027	0.88	215.50	69.63
183						
184						
185	257.05	167.71	0.053	0.91	257.05	89.33
186						
187						

BEST FIT PARAMETER VALUES ACCUMULATIVE MODEL

DATA SET	A	B	THETA	FINAL	START
100	92.31	22.73	0.0012	92.31	22.73
101	92.36	24.87	0.00079	92.36	24.36
102	197.77	106.60	0.0023	197.77	106.60
103	76.83	18.54	0.0017	76.83	18.54
104	93.72	19.68	0.00073	93.72	19.68
105	142.13	55.11	0.0022	142.13	55.11
106	145.02	57.57	0.0024	145.02	57.57
107	104.59	5.01	0.0022	104.59	5.01
108	97.75	4.95	0.0022	97.75	4.95
109	71.66	11.30	0.0077	71.66	11.30
110	72.05	2.37	0.017	72.05	2.37
111	73.57	10.97	0.0010	73.57	10.97
112	9.07	0.33	0.022	9.07	0.33
113	120.51	13.68	0.0054	120.51	13.68
114	117.50	27.45	0.0031	117.50	27.45
115	5721.09	961.92	0.000097	5721.09	961.92
116	7591.04	1304.59	0.000036	7591.04	1304.59
117	11147.35	1129.01	0.000017	11147.35	1129.01
118	18261.02	1087.11	0.000011	18261.02	1087.11
119	53.04	15.01	0.0051	53.04	15.01
120	310.94	131.77	0.00064	310.94	131.77
121	269.36	91.12	0.0014	269.36	91.12
122	867.76	99.04	0.000060	867.76	99.04

BEST FIT PARAMETER VALUES ACCUMULATIVE MODEL

DATA SET	A	B	THETA	FINAL	START
123	467.46	98.69	0.00017	467.46	98.69
124	388.23	93.74	0.00021	388.23	93.74
125	675.17	85.65	0.00011	675.17	85.65
126	447.36	75.90	0.00026	447.36	75.90
127	416.67	56.65	0.00030	416.67	56.65
128	905.35	52.16	0.00012	905.35	52.16
129	146.74	35.16	0.00049	146.74	35.16
130	94.04	17.26	0.00046	94.04	17.26
131	135.37	24.40	0.00029	135.37	24.40
132	64.27	20.94	0.0023	64.27	20.94
133	102.52	25.87	0.00069	102.52	25.87
134	169.68	28.22	0.00052	169.68	28.22
135	128.07	17.12	0.0014	128.07	17.12
136	335.53	155.19	0.00022	335.53	155.19
137	154.75	51.98	0.0081	154.75	51.98
138	187.62	124.92	0.00066	187.62	124.92
139	222.87	141.06	0.00097	222.87	141.06
140	110.66	23.66	0.00082	110.66	23.66
141	81.37	14.43	0.00082	81.37	14.43
142	200.85	22.12	0.00020	200.85	22.12
143	143.72	24.30	0.00050	143.72	24.30
144	81.63	18.67	0.00072	81.63	18.67
145	92.71	24.17	0.0013	92.71	24.17

BEST FIT PARAMETER VALUES ACCUMULATIVE MODEL

DATA SET	A	B	THETA	FINAL	START
146	82.17	9.62	0.0036	82.17	9.62
147	156.34	52.78	0.0024	156.34	52.78
148	114.23	43.77	0.0018	114.23	43.77
149	145.70	59.47	0.0052	145.70	59.47
150	196.09	61.71	0.00096	196.09	61.71
151	169.34	62.46	0.0019	169.34	62.46
152	288.37	55.26	0.00015	288.37	55.26
153	122.25	0.53	0.0031	122.25	0.53
154	128.90	9.08	0.0015	128.90	9.08
155	430.27	14.37	0.00013	430.27	14.37
156	147.99	2.15	0.0013	147.99	2.15
157					
158	126.08	3.22	0.0029	126.08	3.22
159	29.90	15.50	0.024	29.90	15.50
160	31.26	4.98	0.051	31.26	4.98
161	36.23	-0.15	0.050	36.23	-0.15
162	27.45	-0.08	0.060	27.45	-0.08
163	36.31	5.69	0.039	36.31	5.69
164	40.37	25.41	0.015	40.37	25.41
165	102.00	67.38	0.011	102.00	67.38
166					
167					
168					

BEST FIT PARAMETER VALUES ACCUMULATIVE MODEL

DATA SET	A	B	THETA	FINAL	START
169					
170	104.03	21.10	0.0068	104.03	21.10
171	104.78	73.83	0.0036	104.78	78.83
172					
173					
174	104.30	77.84	0.0034	104.30	77.84
175	197.87	84.41	0.0010	197.87	84.41
176					
177					
178	152.86	34.05	0.0027	152.86	34.05
179					
180					
181	318.08	75.48	0.00014	318.08	75.48
182	257.05	89.33		257.05	89.33
183					
184	193.87	61.10	0.0028	193.87	61.10
185	674.28	108.22	0.000077	674.28	108.22
186	248.53	32.65	0.0045	248.53	32.65
187	119.82	18.32	0.0049	119.82	18.32

BEST FIT PARAMETER VALUES REPLACEMENT MODEL

DATA SET	A	B	THETA	FINAL	START
100	68.69	23.08	0.0017	68.69	23.08
101	70.26	25.36	0.0011	70.26	25.36
102	187.25	119.52	0.0014	187.25	119.52
103	59.79	19.29	0.0021	59.79	19.29
104	68.84	20.12	0.0010	68.84	20.12
105	125.75	59.76	0.0019	125.74	59.76
106	133.40	66.57	0.0016	133.40	66.57
107	76.11	5.68	0.0026	76.11	5.68
108	76.58	6.45	0.0023	76.58	6.45
109	64.07	16.99	0.0050	64.07	16.99
110	66.17	13.17	0.0090	66.17	13.17
111	67.81	21.05	0.0054	67.81	21.05
112	8.61	1.90	0.092	8.61	1.90
113	96.22	16.90	0.0055	96.22	16.90
114	96.00	30.29	0.0032	96.00	30.29
115	4557.81	1077.64	0.00010	4557.81	1077.64
116	5484.52	1331.49	0.000050	5484.52	1331.49
117	7095.29	1134.99	0.000028	7095.29	1134.99
118	10528.10	1088.54	0.000020	10528.10	1088.54
119	48.33	19.30	0.0032	48.33	19.30
120	267.33	135.08	0.00069	267.33	135.08
121	242.36	104.77	0.0011	242.36	104.77
122	519.29	99.05	0.00011	519.29	99.05

BEST FIT PARAMETER VALUES REPLACEMENT MODEL

DATA SET	A	B	THETA	FINAL	START
123	320.64	98.51	0.00027	320.64	98.51
124	274.71	93.44	0.00033	274.71	93.44
125	412.32	85.08	0.00019	412.32	85.08
126	309.22	76.38	0.00039	309.32	76.38
127	291.45	57.17	0.00044	291.45	57.17
128	525.49	51.71	0.00021	525.49	51.71
129	112.94	36.42	0.00061	112.94	36.42
130	63.57	17.37	0.00074	63.57	17.37
131	87.60	24.44	0.00049	87.60	24.44
132	51.68	20.97	0.0029	51.68	20.97
133	80.87	26.86	0.00083	80.87	26.86
134	123.14	29.18	0.00071	123.14	29.18
135	92.70	17.67	0.0018	92.70	17.67
136	299.06	162.30	0.00020	299.06	162.30
137	150.93	86.82	0.0024	150.93	86.82
138	177.91	130.59	0.00048	177.91	130.59
139	208.10	146.58	0.00079	208.10	146.58
140	85.86	24.87	0.00098	85.86	24.87
141	57.07	14.59	0.0012	57.07	14.59
142	119.85	22.13	0.00036	119.85	22.13
143	97.84	24.55	0.00078	97.84	24.55
144	71.72	19.96	0.00069	71.72	19.96
145	74.28	25.21	0.0015	74.28	25.21

BEST FIT PARAMETER VALUES REPLACEMENT MODEL

DATA SET	A	B	THETA	FINAL	START
146	70.15	12.98	0.0029	70.15	12.98
147	147.39	71.50	0.0011	147.39	71.50
148	102.79	48.38	0.0014	102.79	48.38
149	139.31	72.66	0.0027	139.31	72.66
150	159.39	63.57	0.0011	159.39	63.57
151	152.72	68.18	0.0015	152.72	68.18
152	224.07	55.98	0.00019	224.07	55.98
153	105.48	33.28	0.0025	105.48	33.28
154	110.20	14.27	0.0011	110.20	14.27
155	287.75	14.57	0.00019	287.75	14.57
156	91.76	1.88	0.0021	91.76	1.88
157	42.97	-0.27	0.0038	42.97	-0.27
158	106.59	6.65	0.0024	106.59	6.65
159	28.55	17.68	0.013	28.55	17.68
160	29.87	13.56	0.017	29.87	13.56
161	34.35	10.24	0.018	34.35	10.24
162	25.58	2.12	0.034	25.58	2.12
163	34.20	11.25	0.019	34.20	11.25
164	38.60	26.94	0.0098	38.60	26.94
165	99.80	78.71	0.0041	99.80	78.71
166	99.30	67.02	0.0095	99.30	67.02
167	99.25	77.83	0.010	99.25	77.83
168	100.37	85.94	0.0021	100.37	85.94

BEST FIT PARAMETER VALUES REPLACEMENT MODEL

DATA SET	A	B	THETA	FINAL	START
169	96.45	66.15	0.011	96.45	66.15
170	94.50	30.23	0.0043	94.50	30.23
171	100.36	77.07	0.0025	100.36	77.07
172					
173	116.63	67.71	0.00078	116.63	67.71
174	100.01	79.73	0.0026	100.01	79.73
175	168.52	85.71	0.0012	168.52	85.71
176					
177					
178	133.60	41.25	0.0021	133.60	41.25
179					
180					
181	217.89	75.70	0.00024	217.89	75.70
182					
183					
184	206.52	34.54	0.0021	206.52	34.54
185	456.61	109.13	0.00012	456.61	109.13
186	212.85	57.41	0.0033	212.85	57.41
187	102.62	24.68	0.0039	102.62	24.68

BEST FIT PARAMETER VALUES DE JONG MODEL

DATA SET	B	A	N	FINAL	START
100					
101					
102	255.50	130.22	0.22	255.50	125.28
103					
104					
105					
106	422.88	355.14	0.066	422.88	67.75
107					
108					
109	169.17	174.71	0.14	169.17	-4.54
110	83.90	105.90	0.53	83.90	-22.00
111	100.07	109.62	0.33	100.07	-9.55
112	9.24	6.46	0.96	9.24	2.79
113					
114					
115					
116					
117					
118					
119					
120	268.27	397.21	0.70	268.27	-128.94
121	238.41	1188.32	1.35	238.41	-949.91
122	301.57	355.13	0.36	301.57	-53.56

BEST FIT PARAMETER VALUES DE JONG MODEL

DATA SET	B	A	N	FINAL	START
123	339.09	347.62	0.25	339.09	-8.52
124	310.43	321.04	0.27	310.43	-10.61
125	462.66	516.32	0.18	462.66	-53.66
126	353.52	434.36	0.26	353.52	-80.83
127	336.43	553.74	0.39	336.43	-217.31
128	2500.36	2579.23	0.030	2500.36	-78.88
129					
130					
131					
132					
133					
134					
135					
136					
137	156.49	61.15	0.75	156.49	95.34
138					
139					
140					
141					
142					
143					
144					
145					

BEST FIT PARAMETER VALUES DE JONG MODEL

DATA SET	B	A	N	FINAL	START
146					
147	230.15	172.68	0.22	230.15	57.47
148					
149	165.64	76.20	0.36	165.64	89.44
150					
151					
152					
153					
154					
155					
156					
157					
158					
159	32.21	12.25	0.44	32.21	19.96
160	32.67	16.34	0.69	32.67	16.33
161	37.79	23.48	0.76	37.79	14.31
162	29.32	26.56	0.80	29.32	2.76
163	39.31	20.98	0.57	39.31	18.32
164	46.60	16.70	0.28	46.60	29.90
165	104.36	20.50	0.58	104.36	83.86
166	99.96	15.29	1.38	99.96	84.68
167	99.72	8.95	1.33	99.72	90.78
168	133.22	45.93	0.12	133.22	87.30

BEST FIT PARAMETER VALUES DE JONG MODEL

DATA SET	B	A	N	FINAL	START
169	98.33	13.09	0.89	98.33	85.24
170	156.99	120.09	0.25	156.99	36.90
171	136.67	56.27	0.16	136.67	80.40
172					
173					
174	153.12	69.72	0.099	153.12	83.41
175					
176					
177	330.73	378.70	0.31	330.73	-47.97
178	351.10	334.41	0.13	351.10	16.69
179					
180					
181					
182					
183					
184	206.52	171.99	0.54	206.52	69.05
185	1883.3	182.47	0.024	1883.3	58.83
186	219.23	472.54	0.85	219.23	-253.31
187					

BEST FIT PARAMETER VALUES LOGARITHMIC MODEL

DATA SET	B	C	G	FINAL	START
100	5.05	0.52	0.026	156.31	22.71
101	4.77	0.62	0.028	118.16	23.77
102	5.30	1.50	0.69	200.44	102.88
103	4.54	0.58	0.052	93.81	16.86
104	4.70	0.54	0.026	100.46	17.59
105	5.04	0.97	0.22	155.22	55.27
106	5.02	0.99	0.32	150.99	54.78
107	5.52	0.23	0.025	248.68	3.47
108	5.21	0.24	0.031	182.49	3.07
109	4.32	0.32	0.13	75.54	3.26
110	4.33	0.14	0.24	76.38	0.06
111	4.31	0.12	0.24	74.40	0.02
112					
113	5.06	0.38	0.20	157.81	11.04
114	4.87	0.56	0.20	130.78	22.06
115	8.84	0.66	0.046	6898.00	1516.70
116	9.37	0.49	0.034	11788.00	1562.00
117	11.48	0.24	0.0020	97184.00	1380.34
118	12.97	0.17	0.0013	43137.00	1316.87
119	4.00	0.76	0.044	54.47	14.67
120	5.57	0.48	0.24	262.55	30.94
121					
122	5.55	0.52	0.12	257.49	37.81

BEST FIT PARAMETER VALUES LOGARITHMIC MODEL

DATA SET	B	C	G	FINAL	START
123	5.54	0.68	0.10	254.59	58.27
124	5.49	0.74	0.093	243.76	62.60
125	5.60	0.43	0.080	270.60	26.68
126				253.38	16.59
127	5.63	0.23	0.077	278.14	3.78
128	5.81	0.39	0.037	333.75	25.93
129	5.22	0.58	0.026	185.68	32.69
130	5.16	0.42	0.0079	174.58	16.58
131	5.94	0.36	0.0046	383.33	24.07
132	4.55	0.69	0.041	93.35	22.30
133	4.69	0.65	0.038	108.77	23.10
134	5.49	0.44	0.023	241.85	25.16
135	5.96	0.32	0.017	389.65	18.49
136	5.70	1.19	0.18	298.16	128.70
137				155.74	47.89
138	5.23	2.32	0.33	186.36	121.02
139	5.42	2.07	0.42	225.71	139.16
140	4.90	0.53	0.035	134.77	20.60
141	4.96	0.43	0.013	142.67	13.70
142	7.22	0.24	0.023	1366.16	21.72
143	6.09	0.34	0.078	442.80	23.91
144	4.17	0.68	0.059	64.82	14.47
145	4.65	0.63	0.063	104.34	21.33

BEST FIT PARAMETER VALUES LOGARITHMIC MODEL

DATA SET	B	C	G	FINAL	START
146	4.60	0.37	0.093	99.85	6.50
147	5.04	0.48	0.50	153.94	19.06
148	4.80	0.94	0.17	120.92	41.88
149	5.00	1.28	0.67	149.72	68.58
150	5.62	0.68	0.059	277.08	63.65
151	5.22	0.98	0.20	184.93	66.58
152	5.10	0.80	0.057	164.39	47.00
153	5.27	0.21	0.047	193.98	1.56
154	5.29	0.23	0.035	198.53	2.46
155	4.64	0.32	0.040	103.14	4.36
156					
157					
158	5.43	0.20	0.046	227.58	1.59
159	3.41	1.47	1.01	30.35	15.37
160					
161					
162					
163	3.61	0.44	1.00	37.10	3.79
164	3.71	2.10	1.19	40.68	25.24
165	4.63	2.15	2.96	102.17	64.17
166					
167					
168	4.63	4.47	1.59	103.17	82.48

BEST FIT PARAMETER VALUES LOGARITHMIC MODEL

DATA SET	B	C	G	FINAL	START
169					
170	4.71	0.55	0.40	111.06	18.26
171	4.66	2.76	0.99	105.59	73.48
172					
173	5.08	1.14	0.061	160.13	66.66
174	4.66	3.34	1.05	105.14	77.96
175	5.37	1.05	0.11	215.12	83.17
176	5.71	0.83	0.038	301.66	90.55
177	5.56	0.46	0.12	259.88	30.19
178	5.10	0.51	0.16	165.30	23.04
179	5.40	0.59	0.062	221.56	40.95
180	5.56	0.97	0.044	260.76	93.37
181	7.10	0.35	0.0035	1211.75	70.94
182	5.51	0.77	0.025	247.24	67.92
183	5.95	0.63	0.024	385.38	79.15
184	5.29	0.68	0.32	198.98	46.03
185	5.63	0.81	0.081	277.53	80.40
186					
187	4.89	0.68	0.072	132.[illegible]9	30.26

BEST FIT PARAMETER VALUES SECOND ORDER MODEL

DATA SET	Y_c	Y_F	TAU	FINAL	START
100	25.54	40.56	6.60	66.11	25.54
101	27.62	38.37	9.72	65.99	27.62
102	128.72	57.46	2.54	186.18	128.72
103	21.36	37.00	6.69	58.36	21.36
104	22.08	42.60	11.87	64.69	22.08
105	66.44	58.06	2.92	124.50	66.44
106	72.85	59.45	3.22	132.32	72.85
107	5.10	91.16	8.93	96.28	5.10
108	4.72	81.48	8.74	86.19	4.72
109	18.09	45.83	6.69	63.93	18.09
110	13.69	52.53	3.75	66.22	13.69
111	20.97	46.15	5.21	67.11	20.97
112	1.03	7.57	1.03	8.60	1.03
113	20.08	78.75	1.91	98.83	20.08
114	34.30	60.67	2.68	94.98	34.30
115	1720.16	2730.35	7.49	4450.50	1720.16
116	1776.25	3453.89	5.52	5230.15	1776.25
117	1503.34	5390.19	24.91	6893.54	1503.34
118	1459.73	7067.31	23.92	8527.05	1459.73
119	21.45	25.09	20.94	46.54	21.45
120	136.52	120.57	7.97	257.09	136.52
121	70.04	166.55	4.64	236.60	70.04
122	96.96	148.51	9.92	245.47	96.96

BEST FIT PARAMETER VALUES SECOND ORDER MODEL

DATA SET	Y_C	Y_F	TAU	FINAL	START
123	103.76	134.90	10.66	238.66	103.76
124	97.96	129.66	10.52	227.62	97.96
125	92.03	160.78	13.54	252.82	92.03
126	74.81	161.06	11.38	235.87	74.81
127	41.39	205.70	9.76	270.69	41.39
128	43.67	227.01	13.01	247.09	43.67
129	39.45	66.25	10.98	105.71	39.45
130	18.96	35.90	16.26	54.87	18.96
131	26.51	44.68	15.99	71.20	26.51
132	23.75	27.65	5.25	51.39	23.75
133	28.61	42.29	9.87	70.91	28.61
134	31.94	84.90	10.44	116.85	31.94
135	21.06	79.20	7.05	100.26	21.06
136	173.36	110.96	9.89	284.32	173.36
137	97.56	53.29	2.01	150.85	97.56
138	135.34	38.16	5.99	173.51	135.34
139	154.61	51.28	3.63	205.89	154.61
140	27.37	56.77	10.35	84.15	27.37
141	16.46	40.71	13.93	57.18	16.46
142	24.43	69.96	20.40	178.54	24.43
143	26.82	57.77	10.36	84.60	26.82
144	20.75	32.88	11.68	53.63	20.75
145	28.34	45.45	7.42	73.79	28.34

BEST FIT PARAMETER VALUES SECOND ORDER MODEL

DATA SET	Y_c	Y_F	TAU	FINAL	START
146	13.06	57.94	5.02	71.01	13.06
147	78.76	67.53	4.34	146.30	78.76
148	53.21	48.70	4.95	101.92	53.21
149	84.21	55.02	1.97	139.23	84.21
150	72.40	85.56	4.37	157.96	72.40
151	77.32	74.92	3.38	152.25	77.32
152	59.36	79.42	11.79	138.78	59.36
153	-5.00	113.84	5.85	108.83	-5.00
154	7.17	104.56	9.38	111.73	7.17
155	15.60	158.21	28.00	173.81	15.60
156	5.71	429.04	31.99	434.76	5.71
157	2.18	119.72	28.74	121.90	2.18
158	-1.35	113.64	5.64	112.28	-1.35
159	19.52	9.00	1.85	28.51	19.52
160	14.82	14.93	1.35	29.75	14.82
161	9.83	24.36	1.15	34.18	9.83
162	-2.53	28.19	1.16	25.66	-2.53
163	13.96	20.23	1.20	34.19	13.96
164	28.67	9.80	1.67	38.47	28.67
165	82.77	16.83	1.55	99.59	82.77
166	75.95	23.33	0.76	99.27	75.95
167	84.68	14.56	0.71	99.24	84.68
168	88.33	11.52	2.69	99.86	88.33

BEST FIT PARAMETER VALUES SECOND ORDER MODEL

DATA SET	Y_c	Y_F	TAU	FINAL	START
169	75.78	20.67	0.69	96.45	75.78
170	34.94	59.39	2.01	94.34	34.94
171	80.80	19.09	2.39	99.89	80.80
172					
173	71.67	38.37	6.01	110.04	71.67
174	82.91	16.63	2.24	99.55	82.91
175	94.59	72.60	5.15	167.19	94.59
176	111.95	138.07	16.72	250.01	111.95
177	90.80	152.72	9.46	243.52	90.80
178	46.93	86.28	4.27	133.21	46.93
179	80.25	130.33	18.08	210.59	80.25
180	102.42	115.89	11.97	218.31	102.42
181	79.91	86.08	13.89	166.0	79.91
182	85.66	110.19	21.81	195.85	85.66
183	91.71	191.83	14.97	283.54	91.71
184	79.52	94.26	2.95	173.77	79.52
185	119.65	133.09	13.08	252.74	119.65
186	78.40	129.57	5.98	207.97	78.40
187	36.14	64.99	7.45	101.24	36.24

APPENDIX G

A COMPARITIVE LISTING OF "START" AND "FINAL" VALUES CALCULATED FROM "BEST FIT" PARAMETERS

CURVE	FINISH	START	CURVE	FINISH	START
0100			0102		
BEV	102.32	22.00		187.90	115.26
GOM	76.18	22.56		187.48	117.78
MTH	163.97	21.98		200.22	101.28
WILT	67.73	24.41		191.24	93.08
ACC	92.31	22.73		197.77	106.60
REP	68.69	23.08		187.25	119.52
DJ				255.49	125.28
MTHL	156.31	22.71		200.44	102.88
2ORD	66.11	25.55		186.18	128.72
0101			0103		
BEV	97.01	24.16		76.17	17.38
GOM	76.96	24.80		64.07	18.45
MTH	146.95	24.01		111.49	17.08
WILT					
ACC	92.36	24.87		76.83	18.54
REP	70.26	25.36		59.79	19.29
DJ					
MTHL	118.16	23.77		93.81	16.86
2ORD	65.99	27.63		58.36	21.36

CURVE	FINISH	START	CURVE	FINISH	START
0104			0106		
BEV	127.77	18.68		134.91	61.00
GOM	79.43	19.48		133.92	64.34
MTH	209.05	18.58		150.22	52.21
WILT				139.29	45.88
ACC	93.72	19.68		145.02	57.57
REP	68.84	20.12		133.40	66.57
DJ				422.88	67.75
MTHL	110.47	17.59		150.99	54.78
2ORD	64.69	22.09		132.32	72.86
0105			0107		
BEV	129.07	55.77		288.83	-1.41
GOM	126.94	58.21		87.28	3.51
MTH	151.85	52.40			
WILT	126.18	60.63		73.26	7.82
ACC	142.13	55.11		104.59	5.01
REP	125.75	59.76		76.11	5.68
DJ					
MTHL	155.22	55.27		248.68	3.47
2ORD	124.50	66.44		96.28	5.11

CURVE	FINISH	START	CURVE	FINISH	START
0108			0110		
BEV	119.78	-3.64		136.27	-2.29
GOM	82.34	3.54		66.33	9.74
MTH	201.83	-3.60		75.39	-33.33
WILT	76.94	6.58		67.14	-4.84
ACC	97.75	4.95		72.05	2.37
REP	76.58	6.45		66.17	13.17
DJ				83.90	-22.00
MTHL	182.49	3.07		76.38	0.06
2ORD	86.19	4.72		66.22	13.69

CURVE	FINISH	START	CURVE	FINISH	START
0109			0111		
BEV	65.67	7.32		68.69	10.21
GOM	64.52	14.05		68.00	17.78
MTH	77.39	-.04		76.78	-11.12
WILT	68.86	-6.85			
ACC	71.66	11.30		73.57	10.97
REP	64.07	16.99		67.81	21.05
DJ	169.17	-4.54		100.07	-9.55
MTHL	75.54	3.26		74.40	0.02
2ORD	63.93	18.10		67.11	20.97

CURVE	FINISH	START	CURVE	FINISH	START
0112			0114		
BEV	8.64	-1.87		106.68	24.34
GOM	8.61	0.85		98.93	28.10
MTH	9.21	-111.41		141.88	22.21
WILT					
ACC	9.07	0.33		117.50	27.45
REP	8.61	1.90		96.00	30.29
DJ	9.24	2.79			
MTHL				130.78	22.06
2ORD	8.60	1.04		94.98	34.30
0113			0115		
BEV	110.14	4.96		4962.59	1460.26
GOM	99.25	13.97		4628.92	1515.32
MTH	155.11	2.53		6704.22	1449.91
WILT	93.14	22.96		4323.81	1734.66
ACC	120.51	13.68		5721.09	961.92
REP	96.22	16.90		4557.81	1077.63
DJ					
MTHL	157.81	11.04		6898.00	1516.70
2ORD	98.83	20.08		4450.50	1720.15

CURVE	FINISH	START	CURVE	FINISH	START
0116			0118		
BEV	7338.01	1475.11			
GOM	5733.79	1532.85		25148.37	1307.36
MTH	11607.18	1474.84			
WILT	4857.07	1796.23			
ACC	7591.04	1304.59		18261.02	1087.11
REP	5484.52	1331.49		10528.10	1088.54
DJ					
MTHL	11788.00	1562.00		431370.0	1316.87
2ORD	5230.15	1776.25		8527.05	1459.73

CURVE	FINISH	START	CURVE	FINISH	START
0117			0119		
BEV				49.59	18.13
GOM	10254.77	1329.12		48.72	19.70
MTH				57.42	15.73
WILT	5771.53	1506.19			
ACC	11147.35	1129.01		53.04	15.01
REP	7095.29	1135.00		48.33	19.30
DJ					
MTHL	97184.00	1380.34		54.47	14.67
2ORD	6893.54	1503.34		46.54	21.45

CURVE	FINISH	START	CURVE	FINISH	START
0120			0122		
BEV	257.10	99.64		245.92	65.33
GOM	257.11	111.66		245.71	77.98
MTH	263.95	-9.62		259.46	28.96
WILT				247.22	26.64
ACC	310.94	131.77		867.76	99.04
REP	267.33	135.08		519.29	99.05
DJ	268.27	-128.94		301.57	-53.56
MTHL	262.55	30.94		257.49	37.81
2ORD	257.09	136.52		245.47	96.96
0121			0123		
BEV	236.46	-22.78		238.84	74.73
GOM	236.58	45.40		238.72	84.78
MTH				256.45	51.96
WILT					
ACC	269.36	91.12		467.46	98.69
REP	242.36	104.77		320.64	98.51
DJ	238.41	-949.91		339.10	-8.52
MTHL				254.59	58.27
2ORD	236.60	70.04		238.66	103.76

CURVE	FINISH	START	CURVE	FINISH	START
0124			0126		
BEV	227.77	71.35		236.32	38.95
GOM	227.72	80.38		236.01	57.12
MTH	243.26	48.72		257.92	5.88
WILT				236.96	24.12
ACC	388.23	93.74		447.36	75.90
REP	274.71	93.44		309.22	76.38
DJ	310.43	-10.61		353.52	-80.83
MTHL	243.76	62.60		253.38	16.59
2ORD	227.62	97.96		235.87	74.81
0125			0127		
BEV	253.95	58.49		247.68	-13.82
GOM	253.19	73.39		247.07	28.01
MTH	281.63	32.02		274.48	-75.95
WILT					
ACC	675.17	85.65		416.67	56.65
REP	412.32	85.08		291.45	57.17
DJ	462.66	-53.66		336.43	-217.31
MTHL	270.60	26.68		278.14	3.78
2ORD	252.82	92.03		270.69	41.39

CURVE	FINISH	START	CURVE	FINISH	START
0128			0130		
BEV	273.21	1.09			
GOM	270.54	27.65		87.55	17.16
MTH	323.25	-12.72			
WILT					
ACC	905.35	52.16		94.04	17.26
REP	525.49	51.71		63.57	17.37
DJ	2500.36	-78.88			
MTHL	333.75	25.93		174.58	16.58
2ORD	247.09	43.68		54.87	18.96
0129			0131		
BEV	164.52	33.82			
GOM	124.52	35.23		129.82	24.29
MTH	249.34	33.43			
WILT					
ACC	146.74	35.15		135.37	24.40
REP	112.94	36.42		87.60	24.44
DJ					
MTHL	185.68	32.69		383.33	24.07
2ORD	105.71	39.46		71.20	26.52

CURVE	FINISH	START	CURVE	FINISH	START
0132			0134		
BEV	57.55	20.47		350.97	25.99
GOM	53.58	20.76		146.46	27.72
MTH	79.07	20.66			
WILT					
ACC	64.27	20.94		169.68	28.22
REP	51.68	20.97		123.14	29.18
DJ					
MTHL	93.35	22.30		241.85	25.16
2ORD	51.39	23.75		116.85	31.94
0133			0135		
BEV	115.34	24.99		193.68	14.71
GOM	88.27	25.99		106.17	16.44
MTH	167.26	24.60		351.42	14.83
WILT				83.93	23.22
ACC	102.52	25.87		128.07	17.12
REP	80.87	26.86		92.70	17.67
DJ					
MTHL	108.77	23.10		389.65	18.49
2ORD	70.91	28.61		100.26	21.06

CURVE	FINISH	START	CURVE	FINISH	START
0136			0138		
BEV	312.34	157.26		178.94	129.15
GOM	305.13	160.36		178.31	129.95
MTH	356.60	150.14		190.06	123.55
WILT					
ACC	335.53	155.19		187.62	124.92
REP	299.06	162.30		177.91	130.59
DJ					
MTHL	298.16	128.70		186.36	121.02
2ORD	284.32	173.36		173.51	135.34
0137			0139		
BEV	150.99	80.42		209.24	144.42
GOM	150.95	84.57		208.54	145.62
MTH	155.44	27.34		226.29	139.20
WILT				214.71	134.22
ACC	154.75	51.98		222.87	141.06
REP	150.93	86.82		208.10	146.58
DJ	156.49	95.34			
MTHL	155.74	47.89		225.71	139.16
2ORD	150.85	97.56		205.89	154.61

CURVE	FINISH	START	CURVE	FINISH	START
0140			0142		
BEV	115.60	21.70			
GOM	93.14	23.52		300.40	21.99
MTH	172.94	21.27			
WILT					
ACC	110.66	23.66		200.85	22.12
REP	85.86	24.37		119.85	22.13
DJ					
MTHL	134.77	20.60		1366.16	21.72
2ORD	84.15	27.37		178.54	24.43
0141			0143		
BEV	165.79	13.64			
GOM	69.12	14.17		134.84	24.16
MTH	305.73	13.65			
WILT	82.05	14.50		350.45	24.28
ACC	81.37	14.43		143.72	24.30
REP	57.07	14.59		97.84	24.55
DJ					
MTHL	142.67	13.70		442.80	23.91
2ORD	57.18	16.47		84.60	26.83

CURVE	FINISH	START	CURVE	FINISH	START
0144			0146		
BEV	100.64	17.98		104.35	31.79
GOM	77.37	19.13		71.50	9.80
MTH	131.43	17.46		99.60	0.75
WILT				71.73	9.16
ACC	81.63	18.87		82.17	9.62
REP	71.72	19.96		70.15	12.98
DJ					
MTHL	64.82	14.47		99.85	6.50
2ORD	53.63	20.75		71.01	13.07
0145			0147		
BEV	85.56	22.59		148.22	61.66
GOM	77.72	24.14		147.77	68.20
MTH	118.16	22.04		159.92	35.04
WILT	92.45	21.16			
ACC	92.71	24.17		156.34	52.78
REP	74.28	25.21		147.39	71.50
DJ				230.15	57.47
MTHL	104.34	21.33		153.94	19.06
2ORD	73.79	28.34		146.30	78.96

CURVE	FINISH	START	CURVE	FINISH	START
0148			0150		
BEV	104.73	44.24		179.17	61.42
GOM	103.49	46.64		166.40	63.32
MTH	120.44	40.27		244.33	61.12
WILT	105.34	42.95		151.60	74.03
ACC	114.23	43.77		196.09	61.71
REP	102.79	48.38		159.39	63.57
DJ					
MTHL	120.92	41.88		277.08	63.65
2ORD	101.92	53.21		157.96	72.40
0149			0151		
BEV	139.94	72.88		155.32	63.27
GOM	139.64	74.61		153.71	66.37
MTH	148.09	63.50		178.27	59.19
WILT				150.52	80.05
ACC	145.70	59.47		169.34	62.46
REP	139.31	72.66		152.72	68.18
DJ	165.64	89.44			
MTHL	149.72	68.58		184.93	66.58
2ORD	139.23	84.21		152.25	77.32

CURVE	FINISH	START	CURVE	FINISH	START
0152			0154		
BEV	986.41	53.80		168.11	-2.66
GOM	282.41	55.13		115.47	8.08
MTH				262.57	-3.99
WILT					
ACC	288.37	55.26		128.90	9.08
REP	224.07	55.98		110.20	14.27
DJ					
MTHL	164.39	47.00		198.53	2.46
2ORD	138.78	59.36		111.73	7.17
0153			0155		
BEV	115.59	-24.09			
GOM					
MTH	165.71	-24.32			
WILT					
ACC	122.25	0.53		430.27	14.37
REP	105.48	33.28		287.75	14.57
DJ					
MTHL	193.98	1.56		103.14	4.36
2ORD	108.83	-5.00		173.81	15.60

CURVE	FINISH	START	CURVE	FINISH	START
0156			0158		
BEV				124.25	-18.74
GOM	139.97	2.18		109.12	1.84
MTH				183.52	-19.62
WILT					
ACC	147.99	2.15		126.08	3.22
REP	91.76	1.88		106.59	6.65
DJ					
MTHL				227.58	1.59
2ORD	434.76	5.72		112.28	-1.35
0157			0159		
BEV				28.61	16.95
GOM				28.57	17.38
MTH				30.16	14.15
WILT					
ACC				29.90	15.50
REP	42.97	-.27		28.55	17.68
DJ				32.21	19.96
MTHL				30.35	15.37
2ORD	121.90	2.18		28.51	19.57

CURVE	FINISH	START	CURVE	FINISH	START
0160			0162		
BEV	29.95	10.43		25.86	-12.18
GOM	29.89	12.48		25.59	0.39
MTH	31.45	-20.36		28.52	-72.60
WILT					
ACC	31.26	4.98		27.45	-0.08
REP	29.87	13.56		25.58	2.12
DJ	32.67	16.33		29.32	2.76
MTHL					
2ORD	29.75	14.82		25.66	-2.53
0161			0163		
BEV	34.51	3.44		34.39	8.20
GOM	34.33	8.12		34.24	10.46
MTH	36.73	-54.68		36.92	-4.69
WILT					
ACC	36.23	-0.15		36.31	5.69
REP	34.35	10.24		34.20	11.25
DJ	37.79	14.31		39.31	18.32
MTHL				37.10	3.79
2ORD	34.18	9.83		34.19	13.95

CURVE	FINISH	START	CURVE	FINISH	START
0164			0166		
BEV	38.70	26.59		99.31	63.68
GOM	38.63	26.76		99.31	66.30
MTH	40.63	24.68			
WILT					
ACC	40.37	25.41			
REP	38.60	26.94		99.30	67.02
DJ	46.60	29.90		99.97	84.68
MTHL	40.68	25.24			
2ORD	38.47	28.68		99.27	75.95
0165					
BEV	99.84	77.72	0167	99.25	76.43
GOM	99.81	78.28		99.25	77.53
MTH	102.17	63.88			
WILT	100.94	63.64			
ACC	102.00	67.38			
REP	99.80	78.71		99.25	77.83
DJ	104.36	83.86		99.72	90.78
MTHL	102.17	64.17			
2ORD	99.59	82.77		99.24	84.68

CURVE	FINISH	START	CURVE	FINISH	START
0168			0170		
BEV	100.44	85.62		96.12	21.37
GOM	100.40	85.80		94.95	27.44
MTH	103.29	82.73		109.22	8.42
WILT				97.13	14.95
ACC				104.03	21.10
REP	100.37	85.94		94.50	30.23
DJ	133.23	87.30		156.99	36.90
MTHL	103.17	82.48		111.06	18.26
2ORD	99.86	88.33		94.34	34.95
0169			0171		
BEV	96.46	63.08		100.54	76.30
GOM	96.45	65.69		100.43	76.72
MTH				105.41	73.03
WILT				100.41	77.00
ACC				104.78	73.83
REP	96.45	66.15		100.36	77.07
DJ	98.33	85.24		136.67	80.40
MTHL				105.59	73.48
2ORD	96.45	75.78		99.89	80.80

CURVE	FINISH	START	CURVE	FINISH	START
0172			0174		
BEV				100.19	79.31
GOM				100.09	79.55
MTH				104.86	77.45
WILT				99.38	82.56
ACC				104.30	77.84
REP				100.01	79.73
DJ				153.13	83.41
MTHL				105.14	77.96
2ORD				99.55	82.92
0173			0175		
BEV	129.26	67.33		176.62	82.12
GOM	121.66	67.60		172.11	84.10
MTH				213.96	79.72
WILT					
ACC				197.87	84.41
REP	116.63	67.71		168.52	85.71
DJ					
MTHL	160.127	66.66		215.12	83.17
2ORD	110.04	71.67		167.19	94.59

CURVE	FINISH	START	CURVE	FINISH	START
0176			0178		
BEV	257.31	92.28		136.36	24.21
GOM	253.86	97.67		134.17	35.31
MTH	300.28	86.66		162.86	10.89
WILT	254.12	98.43			
ACC				152.86	34.05
REP				133.60	41.25
DJ				351.10	16.69
MTHL	301.66	90.55		165.30	23.04
2ORD	250.01	111.95		133.21	46.93
0177			0179		
BEV	243.98	57.07		217.39	60.89
GOM	243.77	71.62		214.64	69.02
MTH	262.19	20.05		249.74	50.88
WILT	262.70	-302.19			
ACC					
REP					
DJ	330.73	-47.97			
MTHL	259.88	30.19		221.56	40.95
2ORD	243.52	90.80		210.59	80.25

CURVE	FINISH	START	CURVE	FINISH	START
0180			0182		
BEV	218.95	86.82		207.13	73.70
GOM	218.17	88.29		201.23	76.58
MTH	247.17	86.10		253.27	71.37
WILT				215.51	69.63
ACC				257.05	89.33
REP					
DJ					
MTHL	260.76	93.37		247.24	67.92
2ORD	218.31	102.42		195.85	85.66
0181			0183		
BEV				291.36	72.09
GOM	322.67	74.73		284.60	75.46
MTH				360.65	70.82
WILT					
ACC	318.08	75.48			
REP	217.89	75.70			
DJ					
MTHL	1211.75	70.94		385.38	79.15
2ORD	166.00	79.91		283.54	91.71

CURVE	FINISH	START	CURVE	FINISH	START
0184			0186		
BEV	174.27	44.69		209.71	32.76
GOM	173.75	58.52		209.31	59.71
MTH	192.44	1.70		215.69	-608.14
WILT					
ACC	193.87	61.10		248.53	32.65
REP	206.52	34.54		212.85	57.41
DJ	175.27	69.05		219.23	-253.31
MTHL	198.98	46.03			
2ORD	173.77	79.52		207.97	78.40
0185			0187		
BEV	255.52	95.24		108.20	30.91
GOM	254.54	102.88		104.03	32.80
MTH	282.63	81.22		135.48	29.94
WILT	257.05	89.33			
ACC	674.28	108.22		119.82	18.32
REP	456.61	109.13		102.62	24.68
DJ	1883.29	58.83			
MTHL	277.53	80.40		132.59	30.26
2ORD	252.74	119.65		101.24	36.24

APPENDIX H

PARAMETER VALUES FOR TELEPHONIST TRAINING DATA

GIVEN IN APPENDIX E

DATA SET	Yc	YF	TAU
201 FE*	-	-	-
201	36.12	1105.66	183.95
202 FE	53.24	205.93	16.97
202	117.82	-0.41	-3.10
203 FE	-	-	-
203	-4304.38	4460.75	1.06
204 FE	-	-	-
204	59.34	2108.41	377.61
205 FE	-	-	-
205	66.06	-39.93	-15.68
206 FE	-	-	-
206	97.49	-2.71	-5.75
207 FE	-	-	-
207	95.25	-0.000075	-1.45
208 FE	-	-	-
208	95.40	-249.16	-41.16
209 FE	-	-	-
209	-63.26	353.03	17.92
210 FE	-	-	-
210	16.20	157.58	7.43
211 FE	-	-	-
211			
212 FE	-	-	-
212	-58.35	263.52	8.62

* 'FE' data sets include the full efficiency check.

DATA SET	Yc	YF	TAU
213 FE	-	-	-
213	76.17	290.37	48.19
214 FE	-18.64	245.87	12.22
214	-48.21	243.45	8.34
219 FE	-4.66	252.29	16.21
219	-216.25	382.88	4.36
221 FE	12.64	272.11	18.91
221	-68.66	249.95	8.26
222 FE	115.88	156.96	95.02
222			
223 FE	70.86	144.94	16.54
223	26.82	141.95	5.85
224 FE	102.24	111.78	21.40
224			
225 FE			
225			
226 FE	39.81	180.02	12.96
226	31.02	174.05	10.56
227 FE			
227			
229 FE			
229			
230 FE	76.99	184.33	12.01
230			
231 FE	1.50	225.34	9.64
231	41.54	321.62	26.68
232 FE	13.35	237.86	12.94
232	30.02	282.32	20.43
233 FE	12.87	267.61	14.00
233	53.79	-275.96	-36.41
234 FE	55.65	198.51	19.28
234	85.51	-125.48	-31.41

DATA SET	Yc	YF	TAU
235 FE	55.57	128.93	11.89
235	64.09	138.04	16.18
236 FE	61.74	143.11	15.33
236	84.22	-88.76	-27.12
238 FE	76.25	131.52	18.02
238			
239 FE	47.08	214.12	17.00
239	-173.68	348.26	3.52
240 FE	42.59	178.23	23.06
240			
241 FE	111.20	109.73	21.73
241			
242 FE	16.60	210.62	22.57
242	78.07	-0.14	-2.95
243 FE	-	-	-
243	132.39	-0.27	-2.41
244 FE	-42.56	258.59	11.14
244	-41.37	258.82	11.32
246 FE	104.13	182.66	69.36
246			
247 FE	-23.06	246.00	7.39
247	-104.96	303.31	4.65
248 FE	-	-	-
248			
249 FE	39.06	194.05	7.88
249	-5837.63	6020.58	1.10
251 FE	72.13	154.67	18.40
251			
252 FE	118.40	139.55	77.14
252			
253 FE	15.95	249.70	21.40
253	-22.49	214.27	9.96

DATA SET	Yc	YF	TAU
254 FE	77.39	197.20	31.80
254	109.62	-1.68	-4.68
255 FE	81.91	205.79	38.13
255			
256 FE			
256			
257 FE	25.38	263.29	21.67
257	-300.97	467.64	3.41
258 FE	-4.10	360.33	35.60
258			
259 FE	100.54	110.90	26.49
259			
260 FE	28.73	219.89	18.47
260	-39.09	208.61	6.30
261 FE	-117.93	385.45	6.60
261	-2.45	375.71	16.27
262 FE	26.46	210.43	24.91
262	72.49	-11.51	-10.87
263 FE	63.20	182.51	11.92
263	-462.56	652.89	2.39
264 FE	70.57	187.37	24.44
264	79.66	344.06	60.58
265 FE	-16.73	262.06	16.39
265	-20.15	256.32	15.18
266 FE	4.01	315.91	39.56
266	61.47	-0.45	-3.93
267 FE			
267			
268 FE			
268			
269 FE	8.54	205.27	12.83
269			

DATA SET	Yc	Y.F	TAU
270 FE	-13.44	359.58	27.81
270	59.82	-3.77	-5.28
271 FE	96.02	134.90	53.33
271			
272 FE	60.24	189.43	14.55
272	109.82	-21.92	-10.37
273 FE			
273			
274 FE	-	-	-
274	71.81	-20.94	-14.63
275 FE	54.43	163.53	9.65
275			
276 FE	20.54	233.69	21.82
276	-52.82	217.97	6.96
277 FE			
277			
278 FE			
278			
279 FE	57.43	175.91	34.98
279	92.40	-0.021	-2.74
280 FE	52.42	164.05	25.19
280	67.18	-413.56	-119.25
281 FE	43.85	192.22	14.89
281	-3882.29	4039.69	1.28
282 FE	-72.79	337.96	15.66
282	-33.73	593.53	46.84
283 FE	71.74	140.29	15.47
283	70.14	138.63	14.63
284 FE	-8.82	265.48	28.11
284	51.23	-0.58	-3.97
285 FE	-	-	-
285			

DATA SET	Yc	YF	TAU
286 FE	-65.92	281.09	8.88
286			
287 FE	84.50	124.24	19.76
287			
288 FE	23.96	189.11	20.52
288	75.68	-0.11	-3.11
289 FE	97.39	163.15	23.28
289	84.58	138.83	13.75
290 FE	49.58	940.08	431.32
290			
291 FE	87.41	128.77	10.37
291	-1165.62	1346.40	1.59
292 FE	72.46	141.56	12.17
292	-14.24	192.31	4.49
293 FE	33.40	269.27	38.81
293	61.79	-71.15	2.79
294 FE	79.29	396.98	105.21
294	57.09	120.20	14.08
295 FE	82.34	201.25	20.45
295	-42.97	236.42	4.41
296 FE	106.38	156.41	33.93
296	131.39	-2.55	-6.22

APPENDIX I

TELEPHONIST TRAINING DATA OBTAINED BY DIRECT OBSERVATION

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T0324 HACKETT, TELEPHONIST EXPERIENCE DATA                              SUBJECT:-KN
 4
 37.0 184.0 44.0 162.0 53.0 215.0 61.0 150.0
T0325 HACKETT, TELEPHONIST TRAINING DATA FOR FIRST THREE WEEKS          SUBJECT:-JC
 12
 2.0 12.0 3.0 48.0 4.0 64.0 5.0 66.0
 8.0 94.0 9.0 104.0 11.0 96.0 12.0 112.0
 15.0 92.0 16.0 88.0 18.0 96.0 19.0 102.0
T0326 HACKETT, TELEPHONIST TRAINING DATA TO END OF TRAINING              SUBJECT:-JC
 20
 2.0 12.0 3.0 48.0 4.0 64.0 5.0 66.0
 8.0 94.0 9.0 104.0 11.0 96.0 12.0 112.0
 15.0 92.0 16.0 88.0 18.0 96.0 19.0 102.0
 22.0 142.0 23.0 124.0 24.0 126.0 25.0 113.0
 26.0 109.0 29.0 121.0 31.0 150.0 33.0 145.0
T0327 HACKETT, TELEPHONIST TRAINING DATA, ALL OBSERVATIONS               SUBJECT:-JC
 22
 2.0 12.0 3.0 48.0 4.0 64.0 5.0 66.0
 8.0 94.0 9.0 104.0 11.0 96.0 12.0 112.0
 15.0 92.0 16.0 88.0 18.0 96.0 19.0 102.0
 22.0 142.0 23.0 124.0 24.0 126.0 25.0 113.0
 26.0 109.0 29.0 121.0 31.0 150.0 33.0 145.0
 37.0 148.0 70.0 203.0
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T0321 HACKETT, TELEPHONIST TRAINING DATA FOR FIRST THREE WEEKS,      SUBJECT;=KN
 14
 2,0 70.0 3,0 98,0 4.0 96,0 5.0 92,0
 8,0 102,0 9,0 112.0 10.0 124.0 11,0 110,0
 12,0 96,0 15,0 90.0 16.0 124.0 17.0 158,0
 18,0 178,0 19,0 114.0
T0322 HACKETT, TELEPHONIST TRAINING DATA TO END OF TRAINING          SUBJECT;=KN
 21
 2,0 70.0 3,0 98,0 4.0 96,0 5,0 92,0
 8,0 102,0 9,0 112,0 10.0 124,0 11,0 110,0
 12,0 96,0 15,0 90,0 16,0 124.0 17,0 158,0
 18,0 178,0 19,0 114.0 22,0 152,0 23,0 176,0
 24.0 182,0 25,0 168.0 26.0 134,0 30,0 175,0
 35,0 143,0
T0323 HACKETT, TELEPHONIST TRAINING DATA, ALL OBSERVATIONS,          SUBJECT;=KN
 25
 2,0 70,0 3,0 98,0 4.0 96,0 5,0 92,0
 8,0 102,0 9,0 112.0 10.0 124,0 11.0 110,0
 12,0 96,0 15,0 90,0 16.0 124,0 17,0 158,0
 18,0 178,0 19,0 114.0 22,0 152,0 23,0 176,0
 24.0 182,0 25,0 168.0 26,0 134,0 30,0 175,0
 35.0 143,0 37,0 184.0 44,0 162,0 53.0 213,0
 61,0 150,0
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T0316 LAMB, TELEPHONIST EXPERIENCE DATA                                SUBJECT:=SJ
 7
 40.0 287.0 50.0 223.0 58.0 228.0 75.0 289.0
 108.0 231.0 129.0 236.0 158.0 264.0
T0317 HACKETT, TELEPHONIST TRAINING DATA FOR FIRST THREE WEEKS,        SUBJECT:=EB
 14
 2.0 72.0 3.0 66.0 4.0 86.0 5.0 110.0
 8.0 134.0 9.0 128.0 10.0 122.0 11.0 122.0
 12.0 144.0 15.0 72.0 16.0 136.0 17.0 98.0
 18.0 166.0 19.0 140.0
T0318 HACKETT, TELEPHONIST TRAINING DATA TO END OF TRAINING             SUBJECT:=EB
 21
 2.0 72.0 3.0 66.0 4.0 86.0 5.0 110.0
 8.0 134.0 9.0 128.0 10.0 122.0 11.0 122.0
 12.0 144.0 15.0 72.0 16.0 136.0 17.0 98.0
 18.0 166.0 19.0 140.0 22.0 145.0 23.0 160.0
 24.0 154.0 25.0 203.0 26.0 191.0 30.0 192.0
 32.0 167.0
T0319 HACKETT, TELEPHONIST TRAINING DATA, ALL OBSERVATIONS               SUBJECT:=EB
 27
 2.0 72.0 3.0 66.0 4.0 86.0 5.0 110.0
 8.0 134.0 9.0 128.0 10.0 122.0 11.0 122.0
 12.0 144.0 15.0 72.0 16.0 136.0 17.0 98.0
 18.0 166.0 19.0 140.0 22.0 145.0 23.0 160.0
 24.0 154.0 25.0 203.0 26.0 191.0 30.0 192.0
 32.0 167.0 37.0 179.0 44.0 180.0 53.0 203.0
 93.0 205.0 129.0 245.0 185.0 217.0
T0320 HACKETT, TELEPHONIST  EXPERIENCE DATA                             SUBJECT:=EB
 6
 37.0 179.0 44.0 180.0 53.0 203.0 93.0 205.0
 129.0 245.0 185.0 217.0
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T0311 LAMB, TELEPHONIST TRAINING, ALL OBSERVATIONS,                    SUBJECT:=LS
 27
 2,0 59.0 3,0 112,0 4.0 66,0 5,0 97,0
 8,0 128,0 9,0 104,0 10,0 112,0 11,0 88,0
 12.0 68,0 15,0 96.0 16.0 131,0 17.0 105,0
 18,0 128,0 19,0 102,0 22,0 139,0 23,0 129,0
 24.0 105,0 25,0 106,0 26,0 147,0 29,0 111,0
 33.0 143,0 37,0 133,0 47,0 159,0 54,0 148,0
 115,0 175,0 136,0 222,0 165,0 190,0
T0312 LAMB, TELEPHONIST EXPERIENCE DATA                           SUBJECT:-LS
  6
 37,0 133,0 47,0 159,0 54,0 148,0 115,0 175,0
 136,0 222,0 165,0 190,0
T0313 LAMB, TELEPHONIST TRAINING DATA TO END OF FIRST THREE WEEKS,  SUBJECT:=SJ
 13
 2,0 94,0 3,0 126,0 4.0 93,0 5,0 125,0
 8,0 140,0 9,0 117,0 10.0 193,0 11,0 175,0
 12,0 206,0 15,0 167,0 16,0 182,0 17,0 154,0
 18,0 168,0
T0314 LAMB, TELEPHONIST TRAINING DATA TO END OF TRAINING            SUBJECT:=SJ
 19
 2,0 94,0 3,0 126,0 4,0 93,0 5,0 125,0
 8,0 140,0 9,0 117,0 10,0 193,0 11,0 175,0
 12,0 206,0 15,0 167,0 16,0 182,0 17,0 154,0
 18,0 168,0 22,0 179,0 23,0 182,0 25,0 162,0
 26,0 155,0 30,0 191,0 33,0 223,0
T0315 LAMB, TELEPHONIST TRAINING DATA, ALL OBSERVATIONS,            SUBJECT:-SJ
 26
 2,0 94.0 3,0 126,0 4,0 93,0 5,0 125,0
 8,0 140.0 9,0 117,0 10,0 193,0 11,0 175,0
 12.0 206,0 15,0 167,0 16,0 182,0 17,0 154,0
 18,0 168,0 22,0 179,0 23,0 182,0 25,0 162,0
 26,0 155,0 30,0 191,0 33,0 223,0 40,0 287,0
 50.0 223,0 56,0 228,0 75,0 269,0 108,0 231,0
 129,0 236,0 158,0 264,0
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T0306 LAMB, TELEPHONIST TRAINING DATA TO END OF TRAINING,              SUBJECT:=KF
 21
 2.0 50.0 3.0 69.0 4.0 56.0 5.0 81.0
 8.0 95.0 9.0 116.0 10.0 86.0 11.0 106.0
 12.0 109.0 15.0 98.0 16.0 128.0 17.0 128.0
 18.0 143.0 19.0 148.0 22.0 144.0 23.0 133.0
 24.0 115.0 25.0 128.0 26.0 117.0 29.0 152.0
 33.0 104.0
T0307 LAMB, TELEPHONIST TRAINING DATA, ALL OBSERVATIONS,               SUBJECT:=KF
 27
 2.0 50.0 3.0 69.0 4.0 56.0 5.0 81.0
 8.0 95.0 9.0 116.0 10.0 86.0 11.0 106.0
 12.0 109.0 15.0 98.0 16.0 128.0 17.0 128.0
 18.0 143.0 19.0 148.0 22.0 144.0 23.0 133.0
 24.0 115.0 25.0 128.0 26.0 117.0 29.0 152.0
 35.0 104.0 37.0 142.0 47.0 191.0 54.0 163.0
 115.0 220.0 136.0 215.0 165.0 213.0
T0308 LAMB, TELEPHONIST EXPERIENCE DATA,                                SUBJECT:=KF
 6
 37.0 142.0 47.0 191.0 54.0 165.0 115.0 220.0
 136.0 215.0 165.0 213.0
T0309 LAMB, TELEPHONIST TRAINING DATA TO END OF FIRST THREE WEEKS,  SUBJECT:=LS
 14
 2.0 59.0 3.0 112.0 4.0 66.0 5.0 97.0
 8.0 128.0 9.0 104.0 10.0 112.0 11.0 88.0
 12.0 68.0 15.0 96.0 16.0 131.0 17.0 105.0
 18.0 128.0 19.0 102.0
T0310 LAMB, TELEPHONIST TRAINING DATA TO  END OF TRAINING,            SUBJECT:=LS
 21
 2.0 59.0 3.0 112.0 4.0 66.0 5.0 97.0
 8.0 128.0 9.0 104.0 10.0 112.0 11.0 88.0
 12.0 68.0 15.0 96.0 16.0 131.0 17.0 105.0
 18.0 128.0 19.0 102.0 22.0 139.0 23.0 129.0
 24.0 105.0 25.0 106.0 26.0 147.0 29.0 111.0
 33.0 143.0
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T0301 LAMB, TELEPHONIST TRAINING DATA TO END OF FIRST THREE WEEKS,  SUBJECT:=JJ
 14
 2,0 102,0 3,0 100,0 4,0 119,0 5,0 105,0
 8,0 117,0 9,0 136,0 10,0 131,0 11,0 147,0
 12,0 152,0 15,0 151,0 16,0 177,0 17,0 180,0
 18,0 154,0 19,0 100,0
T0302 LAMB, TELEPHONIST TRAINING DATA TO END OF TRAINING.            SUBJECT:=JJ
 20
 2,0 102,0 3,0 100,0 4,0 119,0 5,0 105,0
 8,0 117,0 9,0 136,0 10,0 131,0 11,0 147,0
 12,0 152,0 15,0 151,0 16,0 177,0 17,0 180,0
 18,0 154,0 19,0 100,0 22,0 155,0 23,0 166,0
 25,0 180,0 26,0 169,0 30,0 169,0 33,0 187,0
T0303 LAMB, TELEPHONIST TRAINING DATA, ALL OBSERVATIONS.             SUBJECT:=JJ
 27
 2,0 102,0 3,0 100,0 4,0 119,0 5,0 105,0
 8,0 117,0 9,0 136,0 10,0 131,0 11,0 147,0
 12,0 152,0 15,0 151,0 16,0 177,0 17,0 180,0
 18,0 154,0 19,0 100,0 22,0 155,0 23,0 166,0
 25,0 180,0 26,0 169,0 30,0 169,0 33,0 187,0
 40,0 265,0 50,0 166,0 58,0 214,0 75,0 289,0
 108,0 240,0 129,0 241,0 163,0 254,0
T0304 LAMB, TELEPHONIST EXPERIENCE DATA.                             SUBJECT:=JJ
  7
 40,0 265,0 50,0 166,0 58,0 214,0 75,0 289,0
 108,0 240,0 129,0 241,0 163,0 254,0
T0305 LAMB, TELEPHONIST TRAINING DATA TO END OF FIRST THREE WEEKS,  SUBJECT:=KF
 14
 2,0 50,0 3,0 69,0 4,0 56,0 5,0 81,0
 8,0 95,0 9,0 116,0 10,0 86,0 11,0 106,0
 12,0 109,0 15,0 98,0 16,0 128,0 17,0 128,0
 18,0 143,0 19,0 148,0
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