Influence of Initialization on the Performance of Metaheuristic Optimizers

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Abstract

Citation Detail:
Qian Li, San-Yang Liu, Xin-She Yang, Influence of initialization on the performance of metaheuristic optimizers, Applied Soft Computing, 2020, Volume 91, Article 106193.

https://doi.org/10.1016/j.asoc.2020.106193
Available online 24 February 2020, Article 106193

All metaheuristic optimization algorithms require some initialization, and the initialization for such optimizers is usually carried out randomly. However, initialization can have some significant influence on the performance of such algorithms. This paper presents a systematic comparison of 22 different initialization methods on the convergence and accuracy of five optimizers: differential evolution (DE), particle swarm optimization (PSO), cuckoo search (CS), artificial bee colony (ABC) algorithm and genetic algorithm (GA). We have used 19 different test functions with different properties and modalities to compare the possible effects of initialization, population sizes and the numbers of iterations. Rigorous statistical ranking tests indicate that 43.37% of the functions using the DE algorithm show significant differences for different initialization methods, while 73.68% of the functions using both PSO and CS algorithms are significantly affected by different initialization methods. The simulations show that DE is less sensitive to initialization, while both PSO and CS are more sensitive to initialization. In addition, under the condition of the same maximum number of function evaluations (FEs), the population size can also have a strong effect. Particle swarm optimization usually requires a larger population, while the cuckoo search needs only a small population size. Differential evolution depends more heavily on the number of iterations, a relatively small population with more iterations can lead to better results. Furthermore, ABC is more sensitive to initialization, while such initializa-

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tion has little effect on GA. Some probability distributions such as the beta distribution, exponential distribution and Rayleigh distribution can usually lead to better performance. The implications of this study and further research topics are also discussed in detail.

Keywords: Initialization, Differential Evolution, Particle Swarm Optimization, Cuckoo Search, Probability Distribution.

Acronyms

CEC	Congress of Evolutionary Computation
CS	Cuckoo Search
DE	Differential Evolution
DE-a	Adaptive Variant of DE
GA	Genetic Algorithm
LHS	Latin Hypercube Sampling
PSO	Particle Swarm Optimization
PSO-w	PSO with an Inertia Weight

1. Introduction

Many real-world optimization problems are very complex, subject to multiple nonlinear constraints. Such nonlinearity and multimodality can cause difficulties in solving these optimization problems. Both empirical observations and numerical simulations suggest that the final solution may depend on the initial starting points for multimodal optimization problems [1, 2]. This is especially true for gradient-based methods. In addition, for problems with non-smooth objective functions and constraints, gradient information may not be available. Hence, most traditional optimization methods struggle to cope with such challenging issues. A good alternative is to use metaheuristic optimization algorithms, such as particle swarm optimization (PSO) and cuckoo search (CS). These metaheuristic optimizers are gradient-free optimizers, which do not require any prior knowledge or rigorous mathematical properties, such as continuity and smoothness [1, 3].

In the past decade, various studies have shown that these metaheuristic algorithms are effective in solving different types of optimization problems, including noisy and dynamic problems [1, 4, 5, 6]. For example, engineering design problems can be solved by an improved variant of the PSO [7] and the connectivity of the internet of things (IoT) can be enhanced by a multi-swarm optimization algorithm [8]. In addition, the optimized energy consumption model for smart homes can be achieved by differential evolution (DE) [9], while the optimal dam and reservoir operation can be achieved by a hybrid of the bat algorithm (BA) and PSO [10]. A fuzzy-driven genetic algorithm [11] was used to solved a sequence segmentation problem, and a fuzzy genetic clustering algorithm was used to solve a dataset partition problem [12].

Almost all algorithms for optimization require some forms of initialization, where some educated guess or random initial solutions are generated. Ideally, the final optimal solutions found by algorithms should be independent on their initial choices. This is only true for a few special cases such as linear programs and convex optimization; however, a vast majority of problems are not linear or convex, thus such dependency can be a challenging issue. In fact, most algorithms will have different degrees of dependency on their initial setting, and the actual dependency can be problem-specific and algorithm-specific [13, 14]. For large-scale and multimodal problems, the effect of initialization is more obvious, and many algorithms may show differences in the probability of finding global optima on different initialization [15].

However, it still lacks a systematical study of initialization and how the initial distributions may affect the performance of algorithms under a given set of problems. The good news is that researchers start to realize the importance of initialization and have started to explore other possibilities with the aim to increase the diversity of the initial population [13]. For example, based on the guiding principle of covering the search space as uniformly as possible, some studies have preliminarily explored certain ideas of different initialization methods, including quasi-random initialization [16, 17, 18, 19], chaotic systems [20, 21], anti-symmetric learning methods [22], and Latin hypercube sampling [23, 24]. In some cases, these studies have improved the performance of algorithms such as PSO and genetic algorithms (GA), but there are still some serious issues. Specifically, quasi-random initialization is simple and easy to implement, but it suffers from the curse of dimensionality [19]; for chaos-based approaches, random sequences are generated by a few chaotic maps and fewer parameters (initial conditions), but they can inevitably have very sensitive dependence upon their initial conditions under certain conditions [25]. In addition, in the anti-symmetric learning method, twice the number of the population as the solution cohorts are used so as to select the solutions for the next generation, which doubles the computational cost. Though the Latin hypercube sampling is very effective at low dimensions, its performance can deteriorate significantly for higher-dimensional problems. We will discuss this issue in more detail later in this paper.

On the other hand, some researchers attempted to design some specific type of initialization in combination with a certain type of algorithm so as to solve a particular type of problems more efficiently. For example, Kondamadugula et al. [14] used a special sampling evolutionary algorithm and random sampling evolutionary algorithm to estimate parameters concerning digital integrated circuits; Li et al. [26] applied knowledge-based initialization to improve the performance of the genetic algorithm for solving the traveling salesman problem; Li et al. [27] used the degrees of nodes to initialization for network disintegration problem, and Puralachetty et al. [28] proposed a two-stage initialization approach for a PID controller tuning in a coupled tank-liquid system. However, these approaches do have some drawbacks. Firstly, such initialization requires sophisticated allocation of points, which may not be straightforward to implement and can thus increase the computational costs. Secondly, they may be suitable only for a particular type of problems or algorithms. Thirdly, such initialization is largely dependent on the experience of the user. Finally, there is no mathematical guidance about the ways of initialization in practice.

This motivates us to carry out a systematic study of different initialization methods and their effects on the algorithmic performance. The choice of 22 probability distributions are based on rigorous probability theory with the emphasis on different statistical properties. In addition, we have used five different metaheuristic optimization algorithms for this study, and they are differential evolution (DE), particle swarm optimization (PSO), cuckoo search (CS), artificial bee colony (ABC) algorithm and genetic algorithm (GA). There are over 100 different algorithms and variants in the literature [1, 2, 24], it is not possible to compare a good fraction of these algorithms. Therefore, the choice of algorithm has to focus on different search characteristics and representativeness of algorithms in the current literature. Differential evolution is a good representative of evolutionary algorithms, while particle swarm optimization is considered as the main optimizer of swarm intelligence based algorithms. In addition, the cuckoo search uses a long-tailed, Lévy flights-based search mechanism that has been shown to be more efficient in exploring the search space. Furthermore, artificial bee colony is used to represent the bee-based algorithms, while the genetic algorithm has been considered as a cornerstone for a vast majority of evolutionary algorithms.

Based on the simulations and analyses below, we can highlight the features and contributions of this paper as follows:

1. Numerical experiments show that, under the same condition of the maximum number of fitness evaluations(FEs), some algorithms require a large number of populations to reach the optimal solution, while others can find the optimal solution through multiple iterations under a small number of populations. In this paper, we make some recommendations concerning the number of the initial population and the maximum number of iterations of the five algorithms.
2. The initialization of 22 different probability distributions and their influence on the performance of the algorithm are studied systematically. It is found that some algorithms such as the differential evolution are not significantly affected by initialization, while others such as the particle swarm optimization are more sensitive to initialization. This may be related to the design mechanisms of these algorithms themselves, which is also an important indicator to measure the robustness of algorithms.
3. For the five algorithms under consideration, we have used a statistical ranking technique, together with a correlation test, to gain insight into the appropriate initialization methods for given benchmark functions.

Therefore, the rest of this paper is organized as follows. Section 2 briefly introduces the fundamentals of the three metaheuristic optimizers with some brief discussions of the other two optimizers, followed by the discussion of motivations and details of initialization methods in Section 3. Experimental results are presented in Section 4, together with the comparison of different initialization methods on some benchmark functions, including commonly used benchmarks and some recent CEC functions. Further experiments concerning key parameters of different algorithms are also carried out. Then, Section 5 discusses the correlation between the distributions of the initial population and their corresponding final solutions. Finally, Section 6 concludes with discussions about further research directions.

2. Metaheuristic Optimizers

Though traditional optimization algorithms can work well for local search, metaheuristic optimization algorithms have some main advantages for global optimization because they usually treat the problem as a black-box and thus can be flexible and easy to use [29]. Furthermore, such optimizers do not have strict mathematical requirements (e.g., differentiability, smoothness), so they are suitable for problems with different properties, including discontinuities and nonlinearity. Various studies have shown their effectiveness in different applications [29, 30, 31].

The initialization of a vast majority of metaheuristic optimization algorithms has been done by using uniform distributions. Although this approach is easy to implement, empirical observations suggest that uniform distributions may not be the best option in all applications. It is highly needed to study initialization systematically using different probability distributions. As there are many optimization algorithms, it is not possible to study all of them. Thus, this paper will focus on five algorithms: differential evolution (DE), particle swarm optimization (PSO), cuckoo search (CS), artificial bee colony (ABC) and genetic algorithm (GA). These algorithms are representative, due to the different search mechanisms and their richer characteristics.

2.1. Differential Evolution

Differential evolution (DE) is a representative evolutionary and heuristic algorithm [32], which has been used in many applications such as optimization, machine learning and pattern recognition [33]. Though differential evolution has a strong global search capability with a relatively high convergence rate for unimodal problems, the performance of DE can depend on its parameter setting. For highly nonlinear problems, its convergence rate can be low. To overcome such limitations, various mutation strategies and adaptive parameter control for F have been proposed to improve its performance[34]. In the DE algorithm, each individual is a candidate solution or a point in the D-dimensional search space, and the i-th individual can be represented as $x_i = (x_{i,1}, x_{i,2}, \cdots, x_{i,D})$. In essence, different mutation strategies typically generate a mutation vector $(v_{i,1}, v_{i,2}, \cdots\cdots, v_{i,D})$ by modifying the current solution vector in different ways.

Crossover is another strategy of modifying a solution. For example, the binomial crossover is a component-wise modification, controlled by a crossover parameter CR, which takes the following form:

$$u_{i,j} = \begin{cases} v_{i,j}, & \text{if} \quad \text{rand}(0,1) < CR \quad \text{or} \quad j = j_{rand}, \\ x_{i,j}, & \text{otherwise}, \end{cases} \tag{1}$$

where $x_{i,j}$ is the j-th dimension of the i-th individual solution. The updated vector can be expressed as $v_{i,j}$ after the mutation step, and $u_{i,j}$ corresponds to the j-th dimension of the i-th individual after crossover.

Among various variants of DE, Qin et al. [35] proposed a self-adaptive DE (SaDE) variant with four mutation strategies in its pool, which can be selected at different generations by a given criterion. More specifically, according to the success and failure of each mutation, a fixed learning period (LP) was used to update the probability of each mutation strategy being selected for the next generation. In addition, F was drawn from a normal distribution with a mean of 0.5 and standard deviation of 0.3; that is $N(0.5, 0.3^2)$. Similarly, CR was drawn from a normal distribution $N(CRm_k, 0.1)$, where CRm_k was calculated from previous LP generations. Though the performance of SaDE was good, its complexity had increased.

For the ease of implementation and comparison in this paper, we use a simplified adaptive DE (DE-a). Based on the idea of the SaDE algorithm, a simple adaptive DE (DE-a) algorithm is proposed in this paper. In the mutation pool, we use five mutation strategies as follows:

- DE/rand/1 [32]
$$v_{i,j} = x_{r_1,j} + F \cdot (x_{r_2,j} - x_{r_3,j}), \tag{2}$$
- DE/best/1
$$v_{i,j} = x_{best,j} + F \cdot (x_{r_1,j} - x_{r_2,j}). \tag{3}$$
- DE/current-to-best/1 [34]
$$v_{i,j} = x_{i,j} + F \cdot (x_{best,j} - x_{i,j}) + F \cdot (x_{r_2,j} - x_{r_3,j}). \tag{4}$$
- DE/best/2
$$v_{i,j} = x_{best,j} + F \cdot (x_{r_1,j} - x_{r_2,j}) + F \cdot (x_{r_3,j} - x_{r_4,j}). \tag{5}$$
- DE/rand/2
$$v_{i,j} = x_{r_1,j} + F \cdot (x_{r_2,j} - x_{r_3,j}) + F \cdot (x_{r_4,j} - x_{r_5,j}). \tag{6}$$

where $F \in [0, 2]$ is a parameter for mutation strength, and $x_{best,j}$ is the j-th dimension of the current best solution. Here, $x_{r_1,j}$, $x_{r_2,j}$, $x_{r_3,j}$, $x_{r_4,j}$ and $x_{r_5,j}$ represent 5 different individuals, which are selected randomly from the current population.

Both parameters CR and F are initialized to a set of discrete values. That is, $CR \in [0.4, 0.5, 0.6, 0.7, 0.8]$ and $F \in [0.5, 0.6, 0.7, 0.8, 0.9]$. The current mutation strategy and parameter settings are not updated if better solutions are found during the iterations. Otherwise, mutation strategies and parameters are randomly selected from the above sets or ranges. Our simplified variant becomes easier to implement and the performance is much better than the original DE, as observed from our simulations later. Therefore, we will use this variant for later simulations.

2.2. Particle Swarm Optimization

Particle swarm optimization (PSO) is a well-known swarm intelligence optimizer with good convergence [36], which is widely used in many applications [37]. However, it can have premature convergence for some problems, and thus various variants have been developed to remedy it with different degrees of improvement. Among different variants, an improved PSO with an inertia weight (PSO-w), proposed by Shi and Eberhart [38], is efficient and its main steps can be summarized as the following update equations:

$$v_i^{t+1} = w \cdot v_i^t + c_1 r_1^t (p_i^t - x_i^t) + c_2 r_2^t (p_g^t - x_i^t) \tag{7}$$

$$x_i^{t+1} = x_i^t + v_i^{t+1} \tag{8}$$

where v_i^t and x_i^t are the velocity vector and position vector, respectively, for particle i at iteration t. Here, p_i^t is the individual best solution of i-th individual in the previous t iterations, and p_g^t is the best solution of the current population. In Eq. (7), c_1 and c_2 are the two learning parameters, while r_1^t and r_2^t are two random numbers at the current iteration, drawn from a uniform distribution. In a special case when the inertia weight $w = 1$, this variant becomes the original PSO.

The value of w can affect the convergence rate significantly. If w is large, the algorithm can have a faster convergence rate, but it can easily fall into local optima, leading to premature convergence. Studies showed that a dynamically adjusted w with iteration t can be more effective. That is

$$w = w_{\max} - \frac{(w_{\max} - w_{\min}) \cdot t}{T_{\max}} \tag{9}$$

where $T_{\max}$ represents the maximum number of iterations, $w_{\min}$ and $w_{\max}$ are the minimum inertia weight and the maximum inertia weight, respectively. we will use PSO-w in the later experiments.

2.3. Cuckoo Search

Cuckoo search (CS) algorithm is a metaheuristic algorithm, developed by Xin-She Yang and Suash Deb [39], which was based on the behavior of some cuckoo species and their interactions with host species in terms of brooding parasitism. CS also uses Lévy flights instead of isotropic random walks, which can explore large search spaces more efficiently. As a result, CS has been applied in many applications such as engineering design [40], neural networks [41], semantic Web service composition [42], thermodynamic calculations [43] and so on.

Briefly speaking, the CS algorithm consists of two parts: local search and global search. The current individual x_i^t is modified to a new solution x_i^{t+1} by using the following global random walk:

$$x_i^{t+1} = x_i^t + \alpha \oplus L(s, \lambda), \tag{10}$$

where α is a factor controlling step sizes, and s is the step size. L is a random vector drawn from a Lévy distribution [29]. That is

$$L(s, \lambda) \sim \frac{\lambda \Gamma(\lambda) \sin(\pi\lambda/2)}{\pi} * \frac{1}{s^{1+\lambda}} \tag{11}$$

Here, '$\sim$' means that L is drawn as a random-number generator from the distribution on the right-hand of the equation. Γ is the Gamma function, while $1 < \lambda \leqslant 3$ is a parameter. One of the advantages of using Lévy flights is that it has a small probability of long jumps, which enables the algorithm to escape from any local optima and thus increases its exploration capability [1, 44]. The local search is mainly carried out by

$$x_i^{t+1} = x_i^t + \alpha s \otimes H(p_a - \varepsilon) \otimes (x_j^t - x_k^t) \tag{12}$$

where $H(u)$ is the Heaviside function. This equation modifies the solution x_i^t using two other solutions x_j^t and x_k^t. Here, the random number ε is drawn from a uniform distribution and s is the step size. A switching probability p_a is used to switch between these two search mechanisms, intending to balance global search and local search.

2.4. Other Optimizers

There are other optimizers that can be representative for the purpose of comparison. The genetic algorithm (GA) has been a cornerstone of almost all modern evolutionary algorithms, which consists of crossover, mutation and selection mechanisms. The GA has a wide range of applications such as pattern recognition [45], neural networks and control system optimization [46] as well as discrete optimization problems [47]. The literature on this algorithm is vast, thus we will not introduce it in detail here.

On the other hand, the artificial bee colony (ABC) algorithm was inspired by foraging behaviour of honey bees [48], and this algorithm has been applied in many applications [49, 50, 51]. A multi-objective version also exists [52]. Due to the page limit, we will not introduce this algorithm in detail. Readers can refer to the relevant literature [53].

We will use the above five algorithms in this paper for different initialization strategies.

3. Initialization Methods

The main objective of this paper is to investigate different probability distributions for initialization and their effects on the performance of the algorithms used.

3.1. Motivations of this work

Both existing studies and empirical observations suggest that initialization can play an important role in the convergence speed and accuracy of certain algorithms. A good set of initial solutions, especially, when the initial solutions that are near the true optimality by chance, can reduce the search efforts and thus increase the probability of finding the true optimality. As the location of the true optimality is unknown in advance, initialization is largely uniform in a similar manner as those for Monte Carlo simulations. However, for problems in higher dimensions, a small initial population may be biased and could lie sparsely in unpromising regions. In addition, the diversity of the initial population is also important, and different distributions may have different sampling emphasis, leading to different degrees of diversity. For example, some studies concerning genetic algorithms have shown some effects of initialization [54, 55].

Many initialization methods such as the Latin hypercube sampling (LHS) in the literature are mainly based on the idea of uniform spreading in the search space. They are easy to implement and can work well sometimes. For example, the two-dimensional landscape of the Bukin function is shown in Fig. 1. When the search space is in the area of $[-15, -5] \times [-6, 3]$, the PSO-w algorithm with an initial population obeying a uniform distribution can find the optimal solution in a few iterations. The distribution of the particles is shown in Fig. 2. For comparison, another run with an initial beta distribution has also been carried out as shown in Fig. 3. Specifically, the $\star$ indicates the real optimal solution at (-10,1), while the dots show the locations of the current population and ($\ast$) indicates the best solution in current population. Fig. 2(a) shows the initial population with a uniform distribution in the search domain, while these population converged near the optimal solution after 5 iterations by the PSO-w algorithm, as shown in Fig. 2(b) where the current best solution of the population is close to the real optimal solution. However, the initial population (as shown in Fig. 3(a)) drawn from a beta distribution could fall into a local optimum after 5 iterations as shown in Fig. 3(b). This clearly shows the effect and importance of initialization.

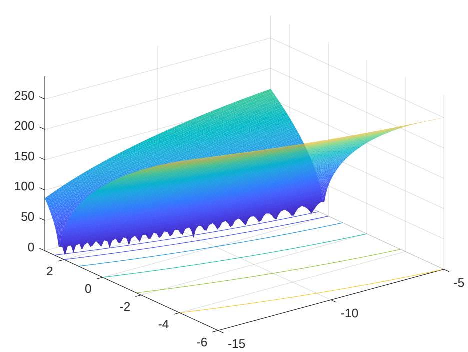
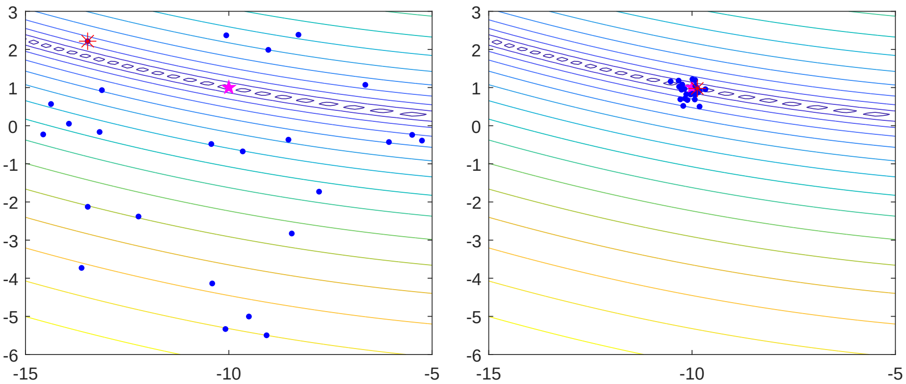
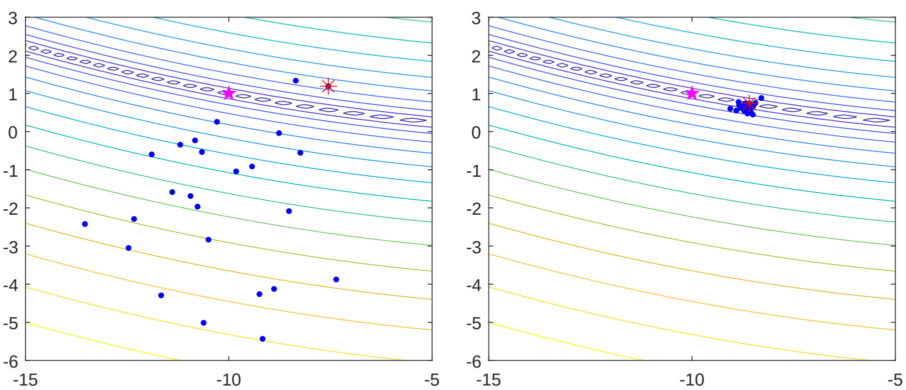


Figure 1: The landscape of Bukin Function N.6.



(a) (b)

Figure 2: (a) The initial population drawn from a uniform distribution where the blue dots are the locations of the initial population, and the red $\ast$ indicates the best solution found by the current population. The real optimal solution of this function is represented by $\bigstar$. (b) Distribution of the same population after 5 iterations by PSO-w, the population converges near the real optimal solution.



(a) (b)

Figure 3: (a) Initial population drawn from a beta distribution where the locations are marked with dots and the true optimality is marked with $\bigstar$. (b) The best solution $\ast$ found by PSO-w after 5 iterations is far from the true optimal solution, indicating premature convergence.

For the above function, initialization by a uniform distribution seems to give better results. However, for another function, uniform distributions may give worse results, even though uniform distributions are widely used. As an illustrative example, the best solution of the Michalewicz function is $f_{\min} = -1.801$ in two-dimensional space at [2.20319,1.57049] (see Fig. 4). If the initialization was done by a uniform distribution, it can lead to premature convergence as shown in Fig. 5, while the initialization by a beta distribution can lead to the global optimal solution after 5 iterations as shown in Fig. 6. Clearly, this shows that uniform distributions are not the best initialization method for all functions. For the same algorithm (such as PSO-w), different initialization methods can lead to different accuracies for different problems. This suggests that different initialization methods should be used for different problems. We will investigate this issue further in a more systematically way.

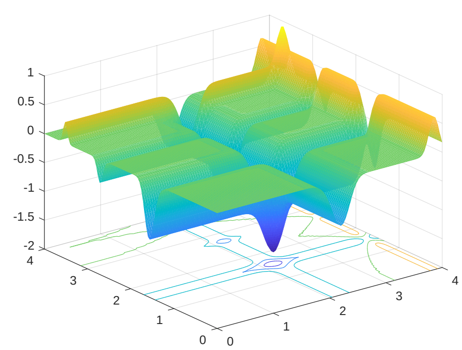


Figure 4: The landscape of the Michalewicz Function.

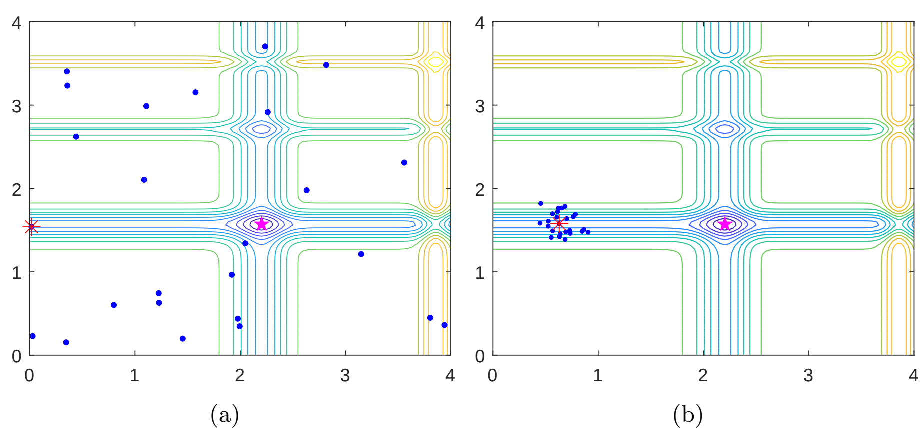


Figure 5: (a) Initial population drawn from a uniform distribution. (b) The location of the best solution $\ast$ found by PSO-w after 5 iterations is far from the true optimal solution $\bigstar$, leading to premature convergence.

In order to study the effect of initialization systematically, we will use a diverse range of

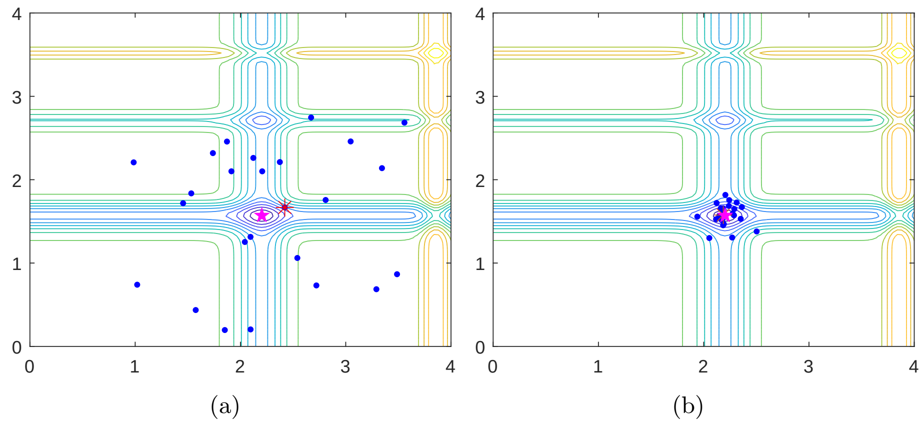


Figure 6: (a) Initial population drawn from a beta distribution. (b) The best solution ∗ found by PSO-w after 5 iterations is close to the true optimal solution ★.

different initialization methods such as Latin hypercube sampling and different probability distributions. We now briefly outline them in the rest of this section.

3.2. Details of initialization methods

Before we carry out detailed simulations, we now briefly outline the main initialization methods.

3.2.1. Latin hypercube sampling

Latin hypercube sampling (LHS) is a spatial filling mechanism. It creates a grid in the search space by dividing each dimension into equal interval segments, and then generates some random points within some interval. It utilizes ancillary variables to ensure that each of the variables to be represented is in a fully stratified feature space [56]. For example, if three sample points are needed in a two-dimensional (2D) parameter space, the three points may have four location scenarios (shown in Fig. 7). Obviously, these three points can also be scattered in the diagonal subspace of the 2D search space.

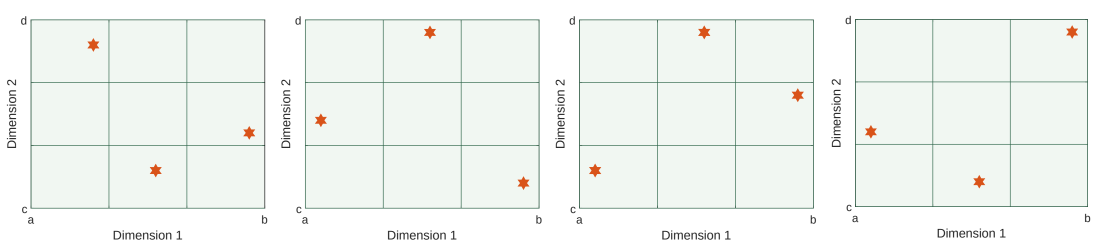


Figure 7: A 2D example of the LHS where three sampling points are distributed in four possible scenarios.

In the LHS, a set of samples are distributed so that they can sparsely distribute in the search space so as to effectively avoid the problem of over aggregation of sampling points. Studies show that such sampling can provide a better spread than uniform distributions, but it does not show a distinct advantage for higher-dimensional problems. So we will investigate this issue further.

3.2.2. Beta distribution

A beta distribution is a continuous probability distribution over the interval (0,1). Its probability density function (PDF) is given by

$$\mathrm{p}(x; a, b) = \frac{\Gamma(a+b)}{\Gamma(a) + \Gamma(b)} x^{a-1}(1-x)^{b-1}, \tag{13}$$

where $\Gamma(a)$ is the standard Gamma function. This distribution has two shape parameters $(a > 0, b > 0)$ that essentially control the shape of the distribution. Its notation is usually written as $X \sim Be(a, b)$. Its expected value is $\mu = \frac{a}{a+b}$ and its variance is $\frac{ab}{(a+b)(a+b+1)}$.

3.2.3. Uniform distribution

Uniform distributions are widely used in initialization, and a uniform distribution $U(a, b)$ on an interval $[a, b]$ is given by

$$p(x) = \begin{cases} \frac{1}{b-a}, & a < x < b, \\ 0, & \text{otherwise}, \end{cases} \tag{14}$$

where a and b are the limits of the interval. Its expectation or mean is $\frac{a+b}{2}$, and its variance is $\frac{(b-a)^2}{12}$.

3.2.4. Normal distribution

Gaussian normal distributions are among the most widely used distributions in various applications, though they are not usually used in initialization. The probability density function of this bell-shaped distribution can be written as

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp(-\frac{(x-\mu)^2}{2\sigma^2}), \tag{15}$$

with the mean of μ and the standard deviation σ. This distribution is often written as $\mathrm{N}(\mu, \sigma^2)$ where its mean determines the central location of the probability curve and its standard deviation σ determines the spread on both sides of the mean [29, 57]. Normal distributions can be approximated by other distributions and can be linked closely with other distributions such as the log-normal distribution, Student-t distribution and F-distribution.

3.2.5. Logarithmic normal distribution

Unlike the normal distribution, the Logarithmic normal distribution is an asymmetrical distribution. Its probability density function is

$$p(x) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\Big[-\frac{(\ln x - \mu)^2}{2\sigma^2}\Big]. \tag{16}$$

A random variable X obeying this distribution is often written as $\ln X \sim N(\mu, \sigma^2)$. Its expectation and variance are $\exp[\mu + \sigma^2/2]$ and $[\exp(\sigma^2) - 1]\exp[2\mu + \sigma^2]$, respectively.

3.2.6. Exponential distribution

An exponential distribution is asymmetric with a long tail, and its probability density function can be written as

$$p(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geqslant 0, \\ 0, & x < 0, \end{cases} \tag{17}$$

where $\lambda > 0$ is a parameter. Its mean and standard deviation are $1/\lambda$ and $1/\lambda^2$, respectively.

3.2.7. Rayleigh distribution

The probability density function of the Rayleigh distribution can be written as

$$p(x) = \frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}}, \quad x > 0, \tag{18}$$

whose mean and variance are $\sqrt{\frac{\pi}{2}}$ and $\frac{4-\pi}{2}\sigma^2 \approx 0.429\sigma^2$, respectively [58].

3.2.8. Weibull distribution

The Weibull distribution has a probability density function [57]

$$f(x; \lambda, k) = \begin{cases} \frac{k}{\lambda}\big(\frac{x}{\lambda}\big)^{k-1} e^{-(x/\lambda)^k}, & x \geqslant 0, \\ 0, & x < 0, \end{cases} \tag{19}$$

where λ is a scale parameter, and k is a shape parameter. This distribution can be considered as a generalization of a few other distributions. For example, $k = 1$ corresponds to an exponential distribution, while $k = 2$ leads to the Rayleigh distribution. Both its mean and variance are $\lambda\Gamma(1 + \frac{1}{k})$ and $\lambda^2[\Gamma(1 + \frac{2}{k}) - \Gamma(1 + \frac{1}{k})^2]$, respectively.

Based on the above different probability distributions, we will carry out various numerical experiments in the rest of this paper.

4. Numerical Experiments

4.1. Experimental settings

In order to investigate the possible influence of different initialization methods on the five algorithms (PSO-w, DE-a, CS, ABC, GA), a series of experiments have been carried out first using a set of nine benchmark functions as shown in Table 1. The experiments will focus first on the PSO-w, DE-a and CS, and then similar tests will be carried out for the ABC and GA. These benchmark functions are chosen based on their different properties such as their modal shapes and numbers of local optima. More specifically, f_1, f_3, f_6, and f_8 are continuous, unimodal functions, while f_2, f_4, f_5, f_7 and f_9 are multimodal functions. For example, the global minimum of f_1 lies in a narrow, parabolic valley, which can be difficult for many traditional algorithms. Functions f_2, f_4, f_5, and f_9 have many local minima that are widespread. The bowl-shaped function f_3 has D local minima with only one global optimum, while the Easom function has several local minima, and its global minimum lies in a small area in a relatively large search space. In addition, we will use 10 more recent benchmarks from CEC2014 and CEC2017 to be discussed in detail later.

Table 1: Basic Benchmark Functions.

Name	Function	Search Range	x^{opt}	Opt
Rosenbrock	$f_1(X) = \sum_{i=1}^{D-1} [100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2]$	$[-5, 5]^D$	$(1, 1, \cdots, 1)$	0
Ackley	$f_2(X) = -20\exp(-0.2\sqrt{\frac{1}{D}\sum_{i=1}^{D} x_i^2})$			
	$-\exp(\frac{1}{D}\sum_{i=1}^{D}\cos(2\pi x_i)) + 20 + e$	$[-10, 10]^D$	$(0, 0, \cdots, 0)$	0
Sphere	$f_3(X) = \sum_{i=1}^{D} x_i^2$	$[-5, 5]^D$	$(0, 0, \cdots, 0)$	0
Rastrigin	$f_4(X) = \sum_{i=1}^{D} [x_i^2 - 10\cos(2\pi x_i) + 10]$	$[-5.12, 5.12]^D$	$(0, 0, \cdots, 0)$	0
Griewank	$f_5(X) = \frac{1}{4000}\sum_{i=1}^{D} x_i^2 - \prod_{i=1}^{D}\cos(\frac{x_i}{\sqrt{i}}) + 1$	$[-600, 600]^D$	$(0, 0, \cdots, 0)$	0
Zakharov	$f_6(X) = \sum_{i=1}^{D} x_i^2 + (\frac{1}{2}\sum_{i=1}^{D} ix_i)^2 + (\frac{1}{2}\sum_{i=1}^{D} ix_i)^4$	$[-100, 100]^D$	$(0, 0, \cdots, 0)$	0
Alpine	$f_7(X) = \sum_{i=1}^{D} \|x_i \sin(x_i) + 0.1x_i\|$	$[-10, 10]^D$	$(0, 0, \cdots, 0)$	0
Easom	$f_8(X) = [-\prod_{i=1}^{D}\cos(x_i)]\exp(-\sum_{i=1}^{D}(x_i - \pi)^2)$	$[-100, 100]^D$	$(\pi, \pi, \cdots, \pi)$	-1
Schwefel	$f_9(X) = 418.98288727243369 * n - \sum_{i=1}^{D} x_i \sin(\sqrt{\|x_i\|})$	$[-500, 500]^D$	$420.96857 * (1, 1, \cdots, 1)$	0

For a fair comparison, we have set the same termination condition for all the algorithms with the maximum number of function evaluations (FEs) of 600000, each algorithm with certain initialization has 20 independent runs. For all the test functions, the dimensionality is set to $D = 30$. As there are so many sets of data generated, we have summarized the results as the 'Best', 'Mean', 'Var' (variance) and 'Dist'. Here, 'Dist' corresponds to the mean distance from the obtained solution x^{find} to the true global optimal solution x^{opt}. That is

$$\text{Dist} = \frac{\sum_{i=1}^{TN}\sum_{j=1}^{D}\left|x_{i,j}^{\text{find}} - x_j{}^{opt}\right|}{TN}, \tag{20}$$

where $TN = 20$ denotes the total number of runs in each set of experiments. This distance metric not only measures the distance of the results, but also measures the stability of the obtained solutions.

For the algorithm-dependent parameters, after some preliminary parametric studies, we have set CR and F to $[0.4, 0.5, 0.6, 0.7, 0.8]$ and $[0.5, 0.6, 0.7, 0.8, 0.9]$, respectively, for

DE-a. In the PSO-w, learning factors c_1 and c_2 are set to 1.5, and the inertia weight $w = 0.8$. For the CS, we have used $p_a = 0.25$ and $\lambda = 1.5$. In addition, the population size (NP) will be varied so as to see if it has any effect on the results.

4.2. Influence of population size and number of iterations

Before we can compare different initialization methods in detail, we have to figure out if there is any significant effect due to the number of the population (NP) used and the maximum number of iterations T. Many studies in the existing literature used different population sizes and numbers of iterations [59]. Though the total number of function evaluations for all functions and algorithms is set to 600 000, the maximum iteration T will vary with NP. Obviously, a larger NP will lead to a smaller T.

In order to make a fair comparison, all the algorithms are initialized by the same random initialization. Four functions with $D = 30$ are selected randomly to reduce the computational efforts. We have carried out numerical experiments and the results are summarized in Tables 2 to 4.

Table 2: Influence of the population size and maximum iteration number on the DE-a algorithm.

Fun	value	NP=100 T=6000	NP=200 T=3000	NP=300 T=2000	NP=600 T=1000	NP=1000 T=600	NP=2000 T=300	NP=3000 T=200
Rosenbrock	Best	0	5.09e-19	1.39e-09	2.53	12.225	19.929	22.198
	Mean	0.0987	0.1993	0.1993	7.2212	13.632	21.224	23.878
	Var	1.5057	0.7947	0.7947	215.2	1.2934	1.9251	1.2532
	Dist	0.2	0.0999	0.1003	5.6464	14.933	22.638	25.277
Sphere	Best	5.67e-197	9.71e-105	8.64e-70	7.19e-36	1.02e-22	9.05e-11	1.63e-07
	Mean	1.78e-187	1.57e-96	4.45e-65	7.74e-33	1.92e-19	1.72e-09	2.74e-06
	Var	0	3.63e-191	1.99e-128	2.88e-64	5.98e-37	1.18e-17	2.39e-11
	Dist	2.0039e-48	2.01e-48	1.48e-32	2.34e-16	9.17e-10	1.44e-4	5.88e-3
Rastrugin	Best	6.9647	18.271	91.987	113.07	112.94	130.32	140.17
	Mean	43.547	96.429	113.77	122.2	131.08	142.57	151.7
	Var	1108.7	558.39	95.447	48.535	77.995	41.862	64.614
	Dist	14.371	19.502	21.89	23.979	24.69	26.31	27.254
Griewank	Best	0	0	0	0	0	7.21e-12	7.07e-09
	Mean	1.11e-03	1.11e-03	2.22e-03	0	3.53e-03	1.11e-03	1.11e-03
	Var	7.34e-06	7.34e-06	1.21e-05	0	8.69e-05	7.34e-06	7.35e-06
	Dist	1.1368	1.1368	2.2735	8.60e-07	0.9227	1.1371	1.1572

Table 2 shows the experimental results of the DE-a algorithm with different NP and T. When $NP = 100$ and $T = 6000$, DE-a shows better performance in most cases. That means the accuracy of the DE-a algorithm depends more heavily on the number of iterations, and it manages to find the optimal solution with a small population size.

Table 3 summarizes the results for the PSO-w algorithm. We can see that the PSO-w algorithm performs well on the Rosenbrock, Rastrugin and Griewank functions when the size of population is 3000 and the number of iterations is 200. Only for the Sphere function, the PSO-w has the highest search accuracy when $NP = 600$ and $T = 1000$. The results show that the accuracy of the PSO-w may depend more on its population size.

Table 3: Influence of the population size and maximum iteration number on the PSO-w algorithm.

Fun	value	NP=100 T=6000	NP=200 T=3000	NP=300 T=2000	NP=600 T=1000	NP=1000 T=600	NP=2000 T=300	NP=3000 T=200
Rosenbrock	Best	27.141	17.382	7.7837	17.936	13.534	16.019	14.754
	Mean	36.055	28.803	24.005	21.842	18.815	19.304	18.085
	Var	264	161.56	18.62	5.035	7.1071	6.5569	2.1004
	Dist	27.465	26.791	25.076	23.145	20.193	20.678	19.607
Sphere	Best	2.46e-04	1.69e-08	9.77e-16	1.33e-36	4.56e-28	3.91e-18	4.12e-14
	Mean	2.32e-03	2.35e-07	1.14e-11	5.68e-34	2.84e-27	1.30e-17	1.44e-13
	Var	3.51e-06	1.30e-13	1.35e-21	1.14e-66	4.69e-54	6.21e-35	6.36e-27
	Dist	1.91e-01	1.78e-03	7.72e-06	6.91e-17	2.18e-13	1.54e-08	1.59e-06
Rastrugin	Best	28.59	17.913	22.884	19.899	12.935	12.935	8.9567
	Mean	44.819	35.542	33.732	32.187	25.073	22.287	18.26
	Var	91.411	98.105	41.561	68.803	85.093	48.604	30.551
	Dist	27.43	23.591	23.187	21.74	18.805	17.91	15.081
Griewank	Best	5.36e-05	5.12e-09	2.92e-14	0	0	2.22e-16	0
	Mean	1.14e-03	3.25e-03	5.77e-03	2.34e-03	3.69e-04	3.70e-04	3.69e-04
	Var	5.25e-06	1.28e-04	2.76e-04	1.09e-04	2.74e-06	2.73e-06	2.73e-06
	Dist	1.177	1.2848	1.4333	5.02e-01	3.79e-01	3.79e-01	3.79e-01

Table 4 shows that the CS algorithm has better performance under a small population and repeatedly iterations. Compared with DE, CS can find the optimal solution with a smaller size of population. This may be related to the design mechanism of the CS algorithm, which increases the diversity in the iteration process of the algorithm. This is one of the advantages of the CS algorithm.

Based on the above experiments, it is recommended that the population size and the number of maximum iterations be set as shown in Table 5. Thus, these parameter settings will be used in all the subsequent experiments.

4.3. Numerical results

In order to compare the possible effects of different initialization strategies for the first three algorithms (PSO-w, DE-a and CS), 22 different initialization methods have been tested, including 9 different distributions with different distribution parameters. As before, we have used different benchmarks with $D = 30$ and have run each algorithm independently for 20 times. Tables 6, 7 and 8 show the comparison results of the 'Best', 'Mean', 'Var' and 'Dist' obtained by the three algorithms.

As presented in these three tables, for nine benchmark functions and three different algorithms, four indicators with 22 different initialization methods have been tested and analyzed. It can be seen that initialization methods can have a great influence on the results of these algorithms. For some functions, some initialization methods are significantly better than others.

But for other functions, a quick look seems to give the impression that the results are not quite consistent. For example, it seems that some initialization methods can find the 'Best' fitness values with a higher accuracy, but their 'Mean' fitness values are less accurate.

Table 4: Influence of the population size and maximum iteration number on the CS algorithm.

Fun	value	NP=30 T=20000	NP=60 T=10000	NP=100 T=6000	NP=200 T=3000	NP=300 T=2000	NP=600 T=1000	NP=1000 T=600
Rosenbrock	Best	0	3.18e-13	2.76e-01	7.01	12.55	31.904	92.556
	Mean	3.22e-30	4.05e-09	2.6204	11.9608	16.78	33.51	105.21
	Var	1.04e-58	2.67e-16	1.5332	7.8448	7.1492	8.1804e-01	83.05
	Dist	5.55e-17	1.90e-05	4.3424	12.113	15.364	27.867	27.592
Sphere	Best	1.91e-139	4.48e-62	1.27e-32	5.91e-14	1.23e-08	1.42e-03	7.60e-02
	Mean	1.41e-136	2.54e-61	3.63e-32	9.10e-14	2.22e-08	2.03e-03	1.36e-01
	Var	1.47e-271	7.27e-122	1.86e-64	7.30e-28	5.04e-17	1.86e-07	5.14e-04
	Dist	3.16e-68	2.03e-30	8.20e-16	1.30e-06	6.50e-04	1.97e-01	1.62
Rastrugin	Best	0	12.791	24.333	47.727	55.124	77.96	89.599
	Mean	9.45e-01	16.8	34.625	57.695	68.146	89.385	102.36
	Var	8.83e-01	7.4984	16.161	36.615	33.234	38.407	54.137
	Dist	9.45e-01	14.626	22.947	30.64	33.247	36.986	40.746
Griewank	Best	0	0	0	2.71e-11	1.28e-06	2.48e-02	2.57e-02
	Mean	0	0	0	1.67e-10	2.49e-06	3.19e-02	3.66e-02
	Var	0	0	0	1.48e-20	2.72e-12	3.27e-07	2.05e-05
	Dist	5.28e-07	5.46e-07	5.19e-07	2.81e-04	3.60e-02	1.3211	4.5692

Table 5: Parameter settings for DE-a, PSO-w and CS.

algorithm	NP	T
DE-a	100	6000
PSO-w	3000	200
CS	30	10000

To make sense of the results, we have carried out some statistical analyses, including the Friedman ranking test [60], and the results are presented in the next subsection.

4.4. Comparison and Friedman rank test

Four indicators ('Best', 'Mean', 'Var', and 'Dist') are used to analyze the results of each algorithm from different perspectives. In order to compare the effects of the 22 initialization approaches, a useful non-parametric test, known as the Friedman rank test, is used for statistical analyses.

Let us start with function f_1 in Table 6. For the Friedman rank test to compare different initialization methods, the null hypothesis is that the effects of initialization methods for DE-a are all equal, while its alternative hypothesis is that at least one of the initialization methods may differ from at least one of the others. That is

$$\begin{aligned} &H_0 : I_1 = I_2 = I_3 = \cdots = I_{22} \\ &H_1 : \text{Not all the initialization effects are equal.} \end{aligned} \tag{21}$$

In essence, the Friedman rank test begins by ordering the ranks of the initialization methods for test indicators. For indicator 'Best', as can be seen from Fig. 6, different initializations produce the same results. So the first row of Table 9 is $11.5 = (1 + 2 + \cdots + 21 + 22)/22$. Calculate the rank values for each row with this method (Different values are sorted by traditional sorting methods). The rank values of different initialization methods for f_1 are listed in Table 9.

For such Friedman rank test, we usually focus on the mean rank of different indicators. In the 'mean' row in Table 9, we can see that the minimum value is 4.38 and there are three such values. For the ease of observation, the order or rank of initialization corresponding to these three values is 1. The second smallest is 7.38 that repeated five times, so their rank is 2. In this way, the orders of the initialization performance for all different functions and their corresponding two-side p-values are given in Table 10.

It can be seen clearly in Table 10 that different initialization methods may have different effects on different functions. Apart from functions f_2, f_8 and f_9, all the p-values of other six functions are far less than 0.05, so the null hypothesis should be rejected at the $\alpha = 0.05$ level. This means that the initialization will affect 2/3 of the functions, and only three functions (f_2, f_8 and f_9) are less influenced by initialization when the DE-a algorithm is used.

For the DE-a algorithm, it is easy to see in Table 10 which initialization method is more suitable for certain functions. Now the question is that which initialization method(s) may be better for the DE-a algorithm? We use the above results in Table 10 and treat the nine functions as observation samples. By comparing 22 different initialization methods using the Friedman rank test, the results are summarized in Table 11. The p-value is 0.617, which is much greater than 0.05, so we cannot reject the null hypothesis at the $\alpha = 0.05$ level. This means that the performance of the DE-a is not particularly sensitive to the initialization on most test functions, which implies that DE is a relatively stable and robust algorithm.

Similar to the analysis for the DE-a algorithm, we now use Friedman rank tests for the PSO-w. The sorted results of nine functions are shown in Table 12. The p-values for functions f_1, f_8 and f_9, are 0.344, 0.459, 0.984, respectively, which means that we cannot reject the null hypothesis. This indicates that different initialization methods have no obvious effect on these three functions. However, for functions f_3, f_4, f_6 and f_7, when the initial population distribution obeys $Be(3, 2)$, the accuracy of the solution is higher. In this case, its 'Best', 'Mean', 'Var' and 'Dist' values are all better than those by other distributions.

Considering all the benchmark functions, the overall ranks for the PSO-w algorithm are summarized in Table 13. The null hypothesis is that the effects of initialization methods for the PSO-w are equally well. The p-value is 0.001, which is less than 0.05, so we can reject the null hypothesis at the $\alpha = 0.05$ level. Thus, we can conclude that the performance of the PSO-w is sensitive to the initialization methods on test functions, and the three best initialization methods for the PSO-w algorithm are Random, $Be(2.5, 2.5)$ and LHS, respectively.

Similarly, we have carried out some analysis for the results by the CS. Table 14 shows the rank results of different initialization methods for the CS over functions $f_1 - f_9$. Apart from functions f_8 and f_9, the p-values of all other functions are far less than 0.05, which indicates that the CS algorithm is significantly affected by initialization. For most test functions, the most commonly used pseudo-random method is not the best initialization method. For example, the solution of f_6 under the beta distribution has the highest accuracy and stability.

In Table 15, the effect of different initialization methods on the CS is sorted. The null hypothesis is that the initialization methods for the CS are equally effective. The p-value is 0.00276, which is far less that 0.05, so we can confidently reject the null hypothesis at the $\alpha = 0.05$ level. In other words, the selection of initial population has a great influence on the CS algorithm. It seems that the best initialization method is the Beta distribution, followed by Rayleigh and Uniform distributions.

4.5. Test problems from CEC2014 and CEC2017 suites

In this section, some well-known single objective real-parameter numerical optimization problems from the CEC2014 and CEC2017 benchmark suites have also been tested. The CEC2014 test suite [61] contains 30 test problems where f_1 to f_3 are unimodal functions, f_4 to f_6 are multi-modal, f_7 to f_{22} are hybrid, and f_{23} to f_{30} are compositions. The CEC2017 suite contains mainly single objective real parameter bound-constrained numerical optimization benchmark problems [62]. The 30 benchmark functions are divided into four categories: unimodal functions (F_1-F_3), multimodal functions (F_4-F_{10}), hybrid functions (F_{11}-F_{20}) and composition functions (F_{21}-F_{30}). All these problems are the minimization of the objective function with in the regular domain of $[-100, 100]^D$ where D is the dimension of the problem. Obviously, it is time-consuming to test all these functions. In order to focus on the main objectives of this paper, we have selected some representative functions with different properties. These 10 selected benchmarks are seven from CEC2014 functions and three from the CEC2017 suite (see Table 16).

Each algorithm has been run independently 20 times for each initialization method, and the stopping criterion is the same for all runs with 600000 fitness evaluations. The population size NP and the maximum number of iterations T of DE-a are set to 300 and 2000, respectively. For the PSO-w, the NP is 2000 and the T is 300. The NP is set to 100 and the T is set to 6000 for CS. The parameters involved in three algorithms are the same as the previous settings. There is no explicit reference about the optimal solutions of CEC2014 and CEC217 suites, so the indicator 'Dist' is not presented in this part. The experimental results of these 30 dimensions are shown in Tables 17 to 19:

From these tables, we can see that the CEC2014 and CEC2017 benchmark problems are indeed more difficult and challenging than the problems considered in the previous subsection. In other words, different search areas have different characteristics and most of the CEC problems have many local optima. More detailed results regarding the quality of the solutions obtained are shown in Table 17, Table 18, and Table 19. From these results, it is clear that the performance of these algorithms for most functions were influenced by different initialization methods, but for a few functions, the performance is less affected by different initialization methods.

For all the results of these ten test functions, the Friedman test has used to rank all initialization methods of each algorithm. For the DE-a, the p values of ten functions are 0.191, 0.012, 0.822, 0.742, 0.912, 0.004, 0.0726, 0.858, 0.827, and 0.006, respectively. For the PSO-w, the p values of ten functions are 0.021, 0.024, 0.743, 0.001, 0.031,0.009, 0.004, 0.196, 0.001, and 0.001, respectively. For the CS, the p values of ten functions are 0.005, 0.036, 0.091, 0.006, 0.023, 0.006, 0.0001, 0.061, 0.275, and 0.013, respectively. The Friedman tests of all three algorithms are summarized in Table 20. The p value of the DE-a is 0.3045, which is greater than 0.05, so we cannot reject the null hypothesis at the $\alpha = 0.05$ level. It can be considered that various initialization methods have little influence on the DE-a algorithm. The p values of both the PSO-w and CS are all less than 0.05, this means that the effects of different initialization methods are significantly different. The robustness of three algorithms is completely consistent with the previous experimental results, which again shows that initialization is important and their detailed study is necessary.

Furthermore, the top three most suitable initialization methods are: $Be(2,3)$, Random and $Be(3,2)$ for the PSO-w algorithm. For the CS, $E(0.5)$, $Rayl(0.1)$, $E(0.1)$ perform better in solving the CEC2014 and CEC2017 problems. Although initialization has little effect on the DE-a algorithm, some methods are still preferred. The recommended initialization methods are $logn(0, 0.5)$, $logn(0, 1)$, and $Weib(1, 1.5)$. This seems to be slightly inconsistent with the earlier conclusion. The reason may be that these CEC2014 and CEC2017 problems are much more complex and multimodal. It can be expected that a combination of different initialization methods may be useful to enhance the diversity of the population.

Based on all the experimental results, for the 19 test functions in this paper, we can say that 43.37% of the functions using the DE-a algorithm show significant differences for different initialization methods, while 73.68% of the functions using the PSO-w and the CS are significantly affected by different initialization methods. In other words, initialization methods may have some effects on the performance of the algorithm, whether it is for less complex functions or more complex problems.

Table 6: Comparison of DE-a for functions f_1-f_9 with different initialization methods.

Fun	Value	$Be(3,2)$	$Be(2.5,2.5)$	$Be(2,3)$	$U(0,1)$	$N(0,1)$	$N(0.5,1)$	$N(0.5,0.5)$	$logn(0,1)$	$logn(.69,.25)$	$logn(0,0.5)$	$logn(0,2/3)$
f_1	Best	0	0	0	0	0	0	0	0	0	0	0
	Mean	0.9967	0.7973	0.7973	0.5980	0.7973	0.7973	0.5980	0.5980	0.3987	0.1993	0.3987
	Var	3.1368	2.6767	2.6767	2.133	2.6767	2.6767	2.133	2.133	1.5057	0.79466	1.5057
	Dist	0.4999	0.3999	0.3999	0.2999	0.3999	0.3999	0.2999	0.2999	0.2	0.09999	0.2
f_2	Best	2.66e-15	2.66e-15	2.66e-15	2.66e-15	2.66e-15	2.66e-15	2.66e-15	2.66e-15	2.66e-15	2.66E-15	2.66E-15
	Mean	6.04e-15	5.68e-15	5.86e-15	5.86e-15	5.68e-15	5.86e-15	5.51e-15	6.04e-15	5.33e-15	5.86e-15	5.51e-15
	Var	6.31e-31	1.69e-30	1.20e-30	1.20e-30	1.69e-30	1.20e-30	2.13e-30	6.31e-31	2.49e-30	1.20e-30	2.13e-30
	Dist	5.63e-14	5.28e-14	5.50e-14	5.47e-14	5.37e-14	5.71e-14	5.28e-14	5.61e-14	5.11e-14	5.44e-14	5.23e-14
f_3	Best	2.35e-194	3.08e-197	4.91e-195	8.12e-195	3.43e-195	1.69e-195	2.40e-195	7.07e-194	2.62e-191	1.62e-193	3.66e-193
	Mean	2.73e-189	2.67e-185	6.97e-186	3.27e-187	1.87e-187	1.03e-187	4.39e-188	1.42e-187	2.16e-185	8.86e-187	1.19e-186
	Var	0	0	0	0	0	0	0	0	0	0	0
	Dist	1.33e-94	5.54e-93	4.13e-93	1.22e-93	1.11e-93	4.91e-94	4.77e-94	7.61e-94	5.34e-93	1.93e-93	2.05e-93
f_4	Best	5.9698	3.9798	4.9748	6.9647	6.9647	4.9748	6.9647	2.9849	5.9698	7.9597	4.9748
	Mean	42.146	36.604	30.935	44.655	48.159	19.308	45.89	34.506	39.749	34.128	42.129
	Var	877.55	1124.6	998.76	1243.8	1229.1	349.48	896.15	943.9	819.12	661.84	1006.8
	Dist	13.865	11.948	13.167	13.421	14.901	10.458	16.187	12.368	14.037	14.138	13.541
f_5	Best	0	0	0	0	0	0	0	0	0	0	0
	Mean	4.31e-03	4.56e-03	6.04e-03	6.16e-03	4.19e-03	3.82e-03	5.67e-03	5.67e-03	3.82e-03	3.57e-03	4.68e-03
	Var	3.38e-05	1.94e-05	5.58e-05	8.77e-05	4.15e-05	4.05e-05	4.99e-05	3.78e-05	3.73e-05	6.40e-05	2.81e-05
	Dist	3.5522	4.3823	4.6998	4.5174	3.0828	2.9011	4.3142	4.5029	3.1007	2.2772	4.0719
f_6	best	7.14e-04	1.69e-04	7.76e-05	8.13e-03	2.52e-03	1.06e-03	1.81e-04	7.29e-04	2.88e-03	7.33e-03	9.60e-04
	Mean	0.7707	0.2982	0.1959	0.8781	0.6424	0.8083	0.6176	0.7647	0.6395	1.1053	0.3683
	Var	2.9675	1.2030	0.2235	1.8932	1.3720	2.2181	3.4087	1.8174	1.1824	7.0834	0.2632
	Dist	2.2541	1.0356	1.1638	2.9246	2.5203	2.5707	2.0005	2.7499	2.5098	3.1681	2.1418
f_7	Best	6.41e-179	2.30e-171	3.37e-202	9.60e-158	1.26e-175	3.15e-190	3.18e-157	6.17e-175	6.19e-191	1.62e-186	2.66e-193
	Mean	2.69e-16	4.97e-16	3.28e-16	1.81e-16	5.25e-16	1.25e-16	5.75e-16	2.75e-16	2.25e-16	3.22e-16	2.78e-16
	Var	1.92e-31	3.20e-31	1.60e-31	4.55e-32	5.29e-31	8.40e-32	1.76e-31	9.26e-32	6.79e-32	1.68e-31	1.21e-31
	Dist	3.3825	5.0903	4.4269	3.0651	5.8807	2.455	5.6116	3.9507	3.7535	3.8837	4.34
f_8	Best	0	0	0	0	0	0	0	0	0	0	0
	Mean	0	0	0	0	0	0	0	0	0	0	0
	Var	0	0	0	0	0	0	0	0	0	0	0
	Dist	1373.1	1317.2	1350.1	1333.4	1343.6	1354.2	1353.9	1256.2	1343.2	1315.9	1333.6
f_9	Best	-3.64e-12	-3.64e-12	118.44	-3.64e-12	-1208.1	-1330.4	-3.64e-12	-83319	-9507.6	-2585.2	-3167.7
	Mean	159.89	324.72	379	342.48	1716.7	2918.7	225.03	-22174	-3801	1872.5	732.47
	Var	18125	64990	42231	46360	2.92e+06	4.24e+06	38244	2.89e+08	7.38e+06	3.89e+06	3.45e+06
	Dist	976.72	1928.1	2315.2	2036.6	34256	24285	1374.6	60249	34954	8528.4	22632

Fun		$E(0.5)$	$E(0.1)$	$E(0.8)$	$Rayl(0.4)$	$Rayl(0.8)$	$Rayl(0.1)$	$Weib(1,1.5)$	$Weib(1.5,1)$	$Weib(1,1)$	$random$	LHS
f_1	Best	0	0	0	0	0	0	0	0	0	0	0
	Mean	0.5980	0.7973	1.3953	0.3987	0.1993	0.3987	0.7973	0.3987	0.1993	0.7973	0.7973
	Var	2.133	2.6767	3.806	1.5057	0.7947	1.5057	2.6767	1.5057	0.79466	2.6767	2.6767
	Dist	0.2999	0.3999	0.6999	0.2	0.09999	0.2	0.3999	0.2	0.09999	0.3999	0.3999
f_2	Best	2.66e-15	2.66e-15	2.66e-15	2.66e-15	2.66e-15	2.66e-15	2.66e-15	2.66e-15	6.22e-15	2.66e-15	6.22e-15
	Mean	5.86e-15	5.68e-15	6.04e-15	5.86e-15	5.51e-15	5.51e-15	5.68e-15	5.68e-15	6.22e-15	6.04e-15	6.22e-15
	Var	1.20e-30	1.69e-30	6.31e-31	1.20e-30	2.13e-30	2.13e-30	1.69e-30	1.69e-30	0	6.31e-31	0
	Dist	5.55e-14	5.34e-14	5.71e-14	5.39e-14	5.35e-14	5.36e-14	5.27e-14	5.43e-14	5.79e-14	5.66e-14	5.70e-14
f_3	Best	1.72e-192	3.16e-193	2.30e-194	2.25e-194	1.10e-194	1.73e-194	1.95e-193	1.29e-194	1.18e-195	2.14e-195	6.84e-195
	Mean	4.73e-187	1.06e-185	8.21e-187	4.64e-188	1.06e-184	6.70e-188	1.53e-186	4.18e-188	2.44e-187	9.04e-188	3.05e-185
	Var	0	0	0	0	0	0	0	0	0	0	0
	Dist	1.36e-93	5.21e-93	1.25e-93	3.99e-94	1.09e-92	4.95e-94	1.54e-93	3.80e-94	8.66e-94	6.81e-94	6.29e-93
f_4	Best	4.9748	2.9849	4.9748	2.9849	5.9698	5.9698	4.9748	5.9698	4.9748	3.9798	4.9748
	Mean	42.129	33.681	25.801	36.415	35.321	41.014	31.766	33.768	40.713	35.916	44.553
	Var	1148	790.32	782.65	1050.8	1068.3	983.76	1127.1	1090.5	1115.5	1184	1290.4
	Dist	14.202	12.842	12.735	13.968	12.871	14.386	12.713	13.309	13.435	13.335	14.058
f_5	Best	0	0	0	0	0	0	0	0	0	0	0
	Mean	6.40e-03	4.44e-03	3.70e-03	2.46e-03	6.16e-03	2.96e-03	2.34e-03	3.82e-03	5.18e-03	6.16e-03	3.08e-03
	var	5.19e-05	2.91e-05	2.46e-05	2.30e-05	5.78e-05	2.28e-05	1.82e-05	4.12e-05	2.93e-05	4.31e-05	3.25e-05
	Dist	4.6522	3.7506	3.3293	2.106	4.5919	2.5793	1.9941	2.5779	4.1793	4.9742	2.4003
f_6	Best	3.23e-03	1.09e-03	3.89e-03	5.40e-04	1.19e-03	3.29e-03	5.35e-04	2.03e-03	3.35e-03	5.32e-04	5.10e-04
	Mean	1.0868	0.3263	0.3506	0.0958	0.6177	0.2773	0.7498	0.9980	0.6868	3.3764	0.6231
	Var	3.8065	0.4366	0.3968	0.0149	2.1068	0.2543	0.9155	4.2390	3.0450	128.9300	1.0124
	Dist	3.1982	1.5674	1.9328	1.0814	2.3737	1.7252	2.9348	3.0664	2.3291	3.7989	2.5342
f_7	Best	1.23e-151	1.19e-166	6.09e-174	1.75e-182	1.64e-150	5.93e-190	3.33e-168	5.81e-173	3.47e-168	4.69e-175	7.21e-187
	Mean	4.11e-16	4.27e-16	2.69e-16	2.03e-16	3.86e-16	1.50e-15	2.86e-16	3.05e-16	3.58e-16	1.61e-16	3.14e-16
	Var	2.80e-31	3.79e-31	1.63e-31	9.48e-32	1.72e-31	7.74e-30	1.89e-31	9.61e-32	1.93e-31	6.00e-32	1.05e-31
	Dist	4.5089	4.716	4.3668	3.5245	4.3919	8.8051	4.1528	4.1528	4.554	3.1052	4.0508
f_8	Best	0	0	0	0	0	0	0	0	0	0	0
	Mean	0	0	0	0	0	0	0	0	0	0	0
	Var	0	0	0	0	0	0	0	0	0	0	0
	Dist	1348.7	1319.5	1317	1340.6	1317.1	1295.9	1386.8	1322	1354.3	1310.2	1333.8
f_9	Best	-3.64e-12	118.44	-3.64e-12	-3.64e-12	-3.64e-12	236.88	-3.64e-12	-13182	-4568.3	-3.64e-12	-3.64e-12
	Mean	265.5	484.61	2642	370.12	1599.4	983.04	1034.8	-6474.3	1346.3	222.07	318.8
	Var	39026	88492	3.88e+06	66401	3.02e+06	6.10e+05	2.86e+06	1.40e+07	6.37e+06	53850	48821
	Dist	1566.4	2904.8	12783	2197.8	4777.1	6005	3977.6	40105	25694	1293.4	1891.9

Table 7: Comparison of PSO-w for functions f_1-f_9 with different initialization methods.

Fun	Value	$Be(3,2)$	$Be(2.5,2.5)$	$Be(2,3)$	$U(0,1)$	$N(0,1)$	$N(0.5,1)$	$N(0.5,0.5)$	$logn(0,1)$	$logn(.69,.25)$	$logn(0,0.5)$	$logn(0,2/3)$
f_1	Best	1.90e-08	16.383	17.351	14.452	17.431	16.145	17.047	14.375	15.049	1.9259	8.11e-05
	Mean	12.924	18.405	18.932	18.724	19.349	18.744	21.668	20.457	18.064	15.289	14.776
	Var	614.41	1.5182	1.3712	2.8465	2.5575	3.8338	189.06	134.23	2.7577	32.079	37.001
	Dist	4.0164	20.043	20.546	20.248	20.682	20.308	20.005	18.914	19.624	16.927	16.334
f_2	Best	4.75e-07	5.49e-07	3.59e-07	6.52e-07	7.02e-07	7.75e-07	4.56e-07	7.76e-07	1.09e-06	4.39e-07	4.40e-07
	Mean	1.53e-01	1.07e-01	4.70e-02	8.50e-04	3.66e-03	3.20e-02	1.16e-01	7.52e-02	6.06e-02	5.96e-02	1.89e-01
	Var	1.37e-01	1.28e-01	4.37e-02	8.82e-06	6.29e-05	2.04e-02	1.26e-01	1.13e-01	6.65e-02	6.65e-02	2.21e-01
	Dist	3.09e-01	2.79e-01	9.16e-02	4.78e-03	1.96e-02	4.92e-02	2.68e-01	2.28e-01	1.47e-01	1.42e-01	5.06e-01
f_3	Best	4.77e-14	4.50e-14	8.96e-14	6.51e-14	8.40e-14	6.46e-14	1.02e-13	4.78e-14	6.51e-14	5.85e-14	7.13e-14
	Mean	1.49e-13	1.26e-13	1.98e-13	1.39e-13	1.42e-13	1.72e-13	2.09e-13	1.81e-13	1.50e-13	1.57e-13	1.74e-13
	Var	6.53e-27	2.35e-27	7.91e-27	3.49e-27	1.37e-27	3.42e-27	1.63e-26	8.98e-27	3.88e-27	3.65e-27	5.97e-27
	Dist	1.68e-06	1.53e-06	1.92e-06	1.60e-06	1.63e-06	1.79e-06	1.97e-06	1.81e-06	1.67e-06	1.70e-06	1.82e-06
f_4	Best	16.915	6.9659	17.91	9.9548	10.947	12.938	15.923	13.931	12.936	15.92	13.931
	Mean	27.668	16.933	28.767	22.39	24.428	21.397	22.79	23.14	21.165	27.765	25.275
	Var	43.693	35.101	62.087	39.554	37.565	41.602	28.926	42.563	25.969	35.863	47.994
	Dist	19.907	14.339	20.704	17.617	18.361	16.823	17.618	17.468	16.475	19.509	18.91
f_5	Best	1.43e-10	1.48e-10	2.01e-10	2.99e-10	1.62e-10	1.36e-10	4.01e-10	2.38e-10	2.22e-10	9.77E-11	2.41E-10
	Mean	8.99e-03	8.62e-03	8.00e-03	1.12e-02	8.62e-03	9.85e-03	9.36e-03	1.02e-02	1.03e-02	9.11e-03	9.59e-03
	Var	6.34e-05	3.09e-05	9.18e-05	1.28e-04	7.49e-05	1.12e-04	8.82e-05	9.96e-05	1.07e-04	1.14e-04	1.31e-04
	Dist	7.3018	7.0998	5.8384	7.0651	6.3561	6.5636	7.0443	7.454	6.9119	6.0607	6.2669
f_6	Best	9.18e-05	4.88e-06	6.42e-05	4.53e-05	7.86e-05	3.58e-05	5.92e-05	7.84e-05	4.38e-05	1.87e-04	1.82e-04
	Mean	9.54e-04	1.57e-04	1.35e-03	4.32e-04	3.94e-04	8.71e-04	9.79e-04	9.28e-04	6.74e-04	1.30e-03	1.78e-03
	Var	1.05e-06	2.82E-08	1.29e-06	6.05e-08	8.45e-08	1.41e-06	1.08e-06	1.01e-06	3.98e-07	1.46e-06	1.28e-05
	Dist	1.19e-01	4.75e-02	1.46e-01	8.63e-02	8.08e-02	1.15e-01	1.23e-01	1.19e-01	1.05e-01	1.47e-01	1.40e-01
f_7	Best	1.29e-02	1.76e-03	1.78e-02	3.82e-03	5.48e-03	5.26e-03	6.30e-03	9.67e-03	3.52e-03	1.23e-02	1.30e-02
	Mean	6.99e-02	1.85e-02	4.51e-02	2.58e-02	3.26e-02	2.32e-02	5.06e-02	3.72e-02	3.13e-02	5.30e-02	5.97e-02
	Var	3.52e-03	1.58e-04	6.32e-04	4.34e-04	3.76e-04	3.02e-04	3.33e-03	3.05e-04	6.72e-04	4.64e-03	1.54e-03
	Dist	39.429	14.155	35.415	19.585	21.599	19.558	26.302	23.057	19.963	29.271	32.17
f_8	Best	0	0	0	0	0	0	0	0	0	0	0
	Mean	0	0	0	0	0	0	0	0	0	0	0
	Var	0	0	0	0	0	0	0	0	0	0	0
	Dist	1091.3	1012.2	1158.5	1514.6	5500.2	4678.4	2247.6	6917.9	9429.8	3703	4829.9
f_9	Best	2053.3	2112.8	2546.5	2151.6	-6422.2	-3147.9	4490.9	-1.04e+05	-13842	-2612.6	-18992
	Mean	2811.1	3320.5	3670.8	2859.7	-2435.2	-1415.8	5569	-51602	-10275	57.926	-7186.7
	Var	1.70e+05	4.02e+05	1.91e+05	1.54e+05	2.11e+06	1.65e+06	2.69e+05	4.07e+08	3.66e+06	1.91e+06	1.25e+07
	Dist	6586.7	9418.9	14068	9433.5	37369	30269	17740	98148	37286	18758	31444

Fun		$E(0.5)$	$E(0.1)$	$E(0.8)$	$Rayl(0.4)$	$Rayl(0.8)$	$Rayl(0.1)$	$Weib(1,1.5)$	$Weib(1.5,1)$	$Weib(1,1)$	$random$	LHS
f_1	Best	16.449	13.908	13.012	17.453	7.95e-04	13.064	2.46e-01	6.1102	1.3975	14.909	16.213
	Mean	19.28	18.472	17.983	18.637	16.005	19.003	20.822	18.077	20.893	18.03	18.369
	Var	1.6701	3.7361	6.84837	1.3217	25.016	4.4588	358.48	13.88	183.51	3.565	2.2941
	Dist	20.954	19.898	19.395	20.132	17.446	20.556	15.883	19.229	19.228	19.393	19.624
f_2	Best	4.60e-07	5.37e-07	5.14e-07	3.88e-07	3.89e-07	5.01e-07	3.82e-07	3.92e-07	4.43e-07	4.80E-07	6.11E-07
	Mean	1.36e-01	6.11e-02	4.66e-02	4.67e-02	8.24e-02	1.88e-01	1.94e-03	1.52e-01	1.97e-01	6.15e-02	2.26e-05
	Var	1.70e-01	6.64e-02	4.34e-02	4.34e-02	1.36e-01	2.46e-01	5.95e-05	1.37e-01	1.87e-01	6.70e-02	1.42e-09
	Dist	3.71e-01	1.48e-01	8.26e-02	8.35e-02	2.77e-01	5.62e-01	1.06e-02	3.07e-01	5.34e-01	1.59e-01	1.35e-04
f_3	Best	9.53e-14	4.69e-14	8.81e-14	4.12e-14	6.00e-14	3.43e-14	4.76e-14	7.89e-14	8.11e-14	4.34e-14	7.52e-14
	Mean	1.61e-13	1.53e-13	1.66e-13	1.38e-13	1.63e-13	1.71e-13	1.48e-13	1.65e-13	1.68e-13	1.66e-13	1.21e-13
	Var	3.76e-27	5.19e-27	2.13e-27	4.08e-27	4.76e-27	5.07e-27	4.41e-27	4.80e-27	6.63e-27	5.33e-27	1.70e-27
	Dist	1.76e-06	1.67e-06	1.76e-06	1.59e-06	1.73e-06	1.74e-06	1.64e-06	1.74e-06	1.78e-06	1.77e-06	1.53e-06
f_4	Best	12.934	20.931	14.929	10.959	14.006	26.864	17.91	14.935	11.94	11.944	15.921
	Mean	25.526	38.759	21.196	19.801	25.846	38.663	29.556	23.736	22.745	20.113	22.641
	Var	60.968	113.88	21.165	28.328	53.31	63.94	63.415	25.204	61.3	24.206	34.464
	Dist	18.662	23.836	16.721	16.122	19.268	23.936	20.801	17.967	17.626	16.326	17.47
f_5	Best	1.13e-10	3.15e-10	1.47e-10	2.58e-10	2.43e-10	3.14e-10	2.19e-10	2.86e-10	1.38e-10	1.19e-10	2.52e-10
	Mean	6.64e-03	7.14e-03	9.11e-03	1.29e-02	1.22e-02	1.08e-02	1.41e-02	8.37e-03	9.96e-03	8.74e-03	1.24e-02
	Var	1.32e-04	7.52e-05	5.68e-05	1.95e-04	1.62e-04	1.44e-04	1.86e-04	1.77e-04	2.39e-04	9.21e-05	1.90e-04
	Dist	4.153	5.4522	6.5607	8.0532	7.823	6.8695	8.2484	5.5116	6.1723	5.7461	7.3272
f_6	Best	1.04e-04	7.86e-05	4.82e-05	2.44e-05	1.76e-04	2.22e-04	5.56e-05	4.07e-05	4.57e-05	2.27e-05	7.34e-05
	Mean	7.91e-04	4.89e-03	7.57e-04	1.86e-04	1.47e-03	3.98e-03	1.04e-03	4.37e-04	7.40e-04	4.28e-04	3.26e-04
	Var	7.00e-07	4.87e-05	3.41e-07	2.93e-08	1.48e-06	2.01e-05	8.13e-07	7.49e-08	7.59e-07	1.45e-07	5.26e-08
	Dist	1.07e-01	2.59e-01	1.09e-01	5.41e-02	1.56e-01	2.41e-01	1.28e-01	8.73e-02	1.06e-01	8.21e-02	7.31e-02
f_7	Best	4.07e-03	2.29e-02	2.09e-03	2.17e-03	9.49e-03	1.61e-02	7.42e-03	1.11e-02	1.20e-02	7.28e-03	6.92e-03
	Mean	3.42e-02	6.99e-02	2.67e-02	2.77e-02	3.65e-02	7.81e-02	5.73e-02	2.87e-02	4.88e-02	3.32e-02	2.88e-02
	Var	8.60e-04	1.34e-03	3.08e-04	8.30e-04	6.89e-04	2.96e-03	2.39e-03	6.56e-04	1.36e-03	8.67e-04	2.02e-04
	Dist	21.761	44.023	18.272	17.403	31.41	51.998	32.112	22.349	25.514	20.956	20.691
f_8	Best	0	0	0	0	0	0	0	0	0	0	0
	Mean	0	0	0	0	0	0	0	0	0	0	0
	Var	0	0	0	0	0	0	0	0	0	0	0
	Dist	2144.8	2460.6	3292.1	1221.1	3123.6	2346.7	3268.4	6778.9	4051.7	1487	1562.7
f_9	Best	3851.7	2151.8	-3501.2	2211.4	478.1	2526.7	495.23	-19232	-8430.5	1934.6	1638.4
	Mean	5721.5	2866.3	-686.19	3238.6	2372.5	2988.6	2509.5	-13907	-4593.5	2693.6	2569.2
	Var	1.08e+06	2.47e+05	2.19e+06	3.00e+05	1.41e+06	93053	1.04e+06	6.64e+06	3.64e+06	1.48e+05	1.84e+05
	Dist	18835	11125	25640	9832.4	14121	15581	17327	48313	30333	9012.9	7984

Table 8: Comparison of CS for functions f_1-f_9 with different initialization methods.

Fun	Value	$Be(3,2)$	$Be(2.5,2.5)$	$Be(2,3)$	$U(0,1)$	$N(0,1)$	$N(0.5,1)$	$N(0.5,0.5)$	$logn(0,1)$	$logn(.69,.25)$	$logn(0,0.5)$	$logn(0,2/3)$
f_1	Best	7.49e-25	2.51e-12	4.78e-12	5.09e-12	4.43e-13	3.88e-12	5.26e-12	1.25e-23	9.93e+05	5.85e-23	9.71e-29
	Mean	6.52e-03	9.11e-10	4.08e-08	1.99e-01	1.99e-01	1.52e-08	4.30e-09	3.00e-07	1.10e+06	2.02e-01	1.39e-04
	Var	6.24e-04	5.33e-18	2.01e-14	7.95e-01	7.95e-01	2.58e-15	1.35e-16	1.08e-12	2.58e+09	7.94E-01	1.89E-07
	Dist	4.93e-02	1.29e-05	7.41e-05	1.00e-01	1.00e-01	5.69e-05	3.88e-05	1.34e-04	1.16e+02	1.28e-01	2.42e-03
f_2	Best	2.66e-15	2.66e-15	2.66e-15	2.66e-15	2.66e-15	2.66e-15	2.66e-15	2.66e-15	1.70e+01	2.66e-15	2.66e-15
	Mean	9.31e-02	2.66e-15	2.66e-15	2.66e-15	2.66e-15	2.66e-15	9.31e-02	2.66e-15	1.71e+01	2.66e-15	4.66e-02
	Var	8.22e-02	0	0	0	0	0	8.22e-02	0	7.48e-03	0	4.34e-02
	Dist	1.64e-01	3.15e-14	3.22e-14	3.20e-14	3.13e-14	3.17e-14	1.64e-01	3.22e-14	2.84e+02	3.12e-14	8.21e-02
f_3	Best	5.16e-69	5.03e-70	3.44e-69	3.26e-69	1.78e-69	9.30e-70	3.90e-69	3.10e-69	625	1.27e-68	8.41e-69
	Mean	4.59e-67	9.48e-68	1.43e-67	1.83e-67	5.45e-67	2.18e-67	1.91e-67	4.46e-67	680	5.06e-67	2.31e-67
	Var	1.88e-132	1.98e-134	9.08e-134	5.36e-134	1.80e-132	1.59e-133	1.10e-133	3.59e-133	894.74	1.04e-132	5.64e-134
	Dist	1.81e-33	1.01e-33	1.27e-33	1.56e-33	2.09e-33	1.66e-33	1.52e-33	2.29e-33	136	2.38e-33	1.84e-33
f_4	Best	3.3086	2.9396	3.7509	2.0714	5.7085	2.0919	0.27923	4.5855	743.94	7.8894	7.0934
	Mean	8.5715	6.2083	8.9598	8.4141	10.208	8.6322	7.9627	10.67	819.93	12.461	11.592
	Var	8.845	6.8603	9.6093	7.8935	12.2	10.466	11.069	5.7495	982.17	5.9648	7.5269
	Dist	7.3607	5.1551	7.7512	7.3389	8.9385	7.5962	7.0386	9.3112	145.97	10.095	10.356
f_5	Best	0	0	0	0	0	0	0	0	2071	0	0
	Mean	7.39e-04	4.93e-04	0	0	3.69e-04	0	3.69e-04	0	25.5	0	7.39e-04
	Var	5.18e-06	4.86e-06	0	0	2.74e-06	0	2.74e-06	0	28030	0	5.18e-06
	Dist	7.57e-01	4.29e-01	5.37e-07	5.54e-07	3.79e-01	5.51e-07	3.79e-01	5.79e-07	16350	5.60e-07	7.58e-01
f_6	Best	7.74e-04	1.70e-04	5.83e-04	1.93e-03	4.82e-03	4.74e-03	3.50e-03	3.64e-03	4.73e+16	2.61e-03	3.75e-03
	Mean	1.46e-02	1.40e-03	1.54e-02	1.33e-02	7.30e-02	8.48e-02	6.46e-02	5.06e-02	1.63e+17	4.02e-02	1.20e-01
	Var	1.56e-04	8.78e-07	3.87e-04	9.05e-05	1.42e-02	7.70e-03	1.40e-02	2.82e-03	3.98e+33	1.41e-03	4.09e-02
	Dist	4.78e-01	1.52e-01	4.30e-01	4.68e-01	9.64e-01	1.14	9.09e-01	8.81e-01	3.00e+03	7.52e-01	1.18
f_7	Best	3.26e-03	1.45e-04	9.22e-02	3.62e-01	3.53e-01	5.04e-01	7.64e-01	1.50	1.11e+02	6.13e-01	8.81e-01
	Mean	5.12e-01	3.98e-01	5.68e-01	8.89e-01	1.82	1.89	1.66	2.39	1.23E+02	1.79	1.91
	Var	1.17e-01	1.35e-01	1.17e-01	2.61e-01	9.15e-01	4.46e-01	3.13e-01	2.63e-01	5.00e+01	3.30e-01	2.97e-01
	Dist	5.13e+01	3.28e+01	5.30e+01	5.77e+01	9.44e+01	9.15e+01	8.32e+01	1.16e+02	2.99e+02	1.27e+02	1.22e+02
f_8	Best	-1.65e-01	-2.52e-01	-1.70e-01	-2.89e-01	-2.12e-01	-1.45e-01	-1.49e-01	-9.56e-02	-1.76e-01	-1.44e-01	-1.26e-01
	Mean	-8.40e-02	-9.80e-02	-8.44e-02	-8.85e-02	-9.34e-02	-7.81e-02	-7.59e-02	-6.32e-02	-7.99e-02	-7.47e-02	-6.86e-02
	Var	8.11e-04	1.62e-03	1.41e-03	2.67e-03	1.72e-03	1.09e-03	7.88e-04	2.26e-04	1.33e-03	7.58e-04	5.41e-04
	Dist	1.70e+03	1.51e+03	1.82e+03	1.78e+03	1.85e+03	1.86e+03	1.83e+03	1.79e+03	1.85e+03	1.93e+03	1.84e+03
f_9	Best	3.95e-04	385.34	724.82	9.0398	-38.177	243.62	143.57	-25579	-4394.8	3.82e-04	3.82e-04
	Mean	280.18	722.53	1174.2	613.81	1459.4	820.01	749.11	-4934.2	-266.16	51.42	307.49
	Var	53694	55598	86599	93668	3.50e+05	1.22e+05	89298	3.88e+07	3.81e+06	24883	3.10e+05
	Dist	1222.4	3459.5	5595.9	3023.7	13250	4955.3	3812.9	38282	30198	229.96	1692.6

Fun		$E(0.5)$	$E(0.1)$	$E(0.8)$	$Rayl(0.4)$	$Rayl(0.8)$	$Rayl(0.1)$	$Weib(1,1.5)$	$Weib(1.5,1)$	$Weib(1,1)$	$random$	LHS
f_1	Best	4.24e-12	1.00e-11	8.65e-12	1.25e-12	1.44e-25	7.96e-12	2.28e-25	1.18e-21	1.01e-13	1.38e-12	3.11e-12
	Mean	5.47e-10	1.99e-01	1.99e-01	6.55e-10	1.37e-01	1.99e-01	1.07e-02	5.04e-04	9.87e-02	1.99e-01	1.99e-01
	Var	1.14e-18	7.95e-01	7.95e-01	1.63e-18	3.72e-01	7.95e-01	2.25e-03	5.07e-06	1.89e-01	7.95e-01	7.95e-01
	Dist	1.72e-05	1.00e-01	1.00e-01	1.25e-05	2.34e-01	1.00e-01	7.05e-02	1.30e-02	2.15e-01	1.00e-01	1.00e-01
f_2	Best	2.66e-15	2.66e-15	2.66e-15	2.66e-15	2.66e-15	2.66e-15	2.66e-15	2.66e-15	2.66e-15	2.66e-15	2.66e-15
	Mean	4.66e-02	4.66e-02	4.66e-02	5.78e-02	2.66e-15	5.78e-02	5.78e-02	4.66e-02	2.66e-15	9.31e-02	2.66e-15
	Var	4.34e-02	4.34e-02	4.34e-02	6.67e-02	0	6.67e-02	6.67e-02	4.34e-02	0	8.22e-02	0
	Dist	8.21e-02	8.21e-02	8.21e-02	1.31e-01	3.16e-14	1.31e-01	1.31e-01	8.21e-02	3.21e-14	1.64e-01	3.16e-14
f_3	Best	3.27e-69	7.05e-69	3.35e-70	2.84e-70	2.82e-69	1.38e-69	3.33e-69	3.47e-69	2.32e-69	6.93e-69	5.24e-70
	Mean	3.29e-67	7.26e-67	1.51e-67	1.06e-67	1.03e-66	7.00e-67	1.78e-67	4.17e-67	1.29e-66	1.85e-67	1.18e-67
	Var	1.86e-133	2.26e-132	3.04e-134	5.92e-134	1.13e-131	1.90e-132	1.46e-133	2.45e-133	1.76e-131	1.27e-133	1.24e-134
	Dist	2.01e-33	2.71e-33	1.47e-33	1.01e-33	2.59e-33	2.65e-33	1.46e-33	2.37e-33	2.44e-33	1.51e-33	1.32e-33
f_4	Best	6.9979	6.6867	3.886	5.1891	5.5713	6.5398	2.8385	4.268	2.9355	2.1314	1.4139
	Mean	10.761	11.877	9.3383	8.1427	10.574	11.839	7.9303	10.917	9.9632	7.3846	8.8921
	Var	4.4353	6.3303	8.5221	3.2042	10.417	6.4774	6.2093	11.118	9.9866	6.608	10.793
	Dist	9.1505	10.289	8.0989	7.0184	8.9637	10.381	6.834	9.4743	8.6558	6.575	7.8213
f_5	Best	0	0	0	0	0	0	0	0	0	0	0
	Mean	0	0	0	0	0	0	0	0	3.69e-04	3.69e-04	0
	Var	0	0	0	0	0	0	0	0	2.74e-06	2.74e-06	0
	Dist	5.39e-07	5.29e-07	5.37e-07	5.48e-07	5.84e-07	5.47e-07	5.50e-07	5.22e-07	3.79e-01	3.79e-01	5.65e-07
f_6	Best	8.88e-04	5.10e-03	2.52e-03	6.73e-04	1.79e-03	2.01e-03	1.58e-03	3.00e-03	3.98e-03	9.87e-04	2.70e-03
	Mean	4.04e-02	6.87e-02	3.49e-02	4.46e-03	3.41e-02	1.07e-01	5.22e-02	3.59e-02	4.57e-02	1.61e-02	1.46e-02
	Var	1.16e-03	7.29e-03	1.25e-03	1.61e-05	7.68e-04	2.57e-02	5.11e-03	1.61e-03	1.58e-03	2.89e-04	1.81e-04
	Dist	7.96e-01	9.85e-01	6.99e-01	2.58e-01	7.53e-01	1.13	8.43e-01	7.03e-01	8.26e-01	4.88e-01	4.69e-01
f_7	Best	2.75e-01	1.63e-02	5.31e-01	2.51e-03	6.82e-01	1.28e-03	6.44e-01	8.43e-01	4.79e-01	9.15E-02	1.06E-02
	Mean	1.22	1.31	2.06	6.86e-01	1.88	1.22	1.97	2.37	2.17	9.86	9.49e-01
	Var	3.07e-01	3.52e-01	7.59e-01	2.90e-01	4.29e-01	2.14e-01	8.69e-01	5.54e-01	6.51e-01	2.29e-01	3.14e-01
	Dist	7.26e+01	1.58e+02	9.52e+01	4.94e+01	1.26e+02	1.44e+02	1.06e+02	1.21e+02	9.47e+01	6.20e+01	5.93e+01
f_8	Best	-1.24e-01	-2.16e-01	-1.34e-01	-2.31e-01	-1.44e-01	-1.92e-01	-1.64e-01	-1.49e-01	-1.48e-01	-1.42e-01	-2.43e-01
	Mean	-7.16e-02	-8.33e-02	-6.78e-02	-7.88e-02	-7.21e-02	-6.99e-02	-8.58e-02	-7.88e-02	-7.05e-02	-7.94e-02	-1.01e-01
	Var	6.77e-04	2.24e-03	7.63e-04	1.70e-03	7.19e-04	1.30e-03	1.11e-03	1.07e-03	9.00e-04	1.06e-03	3.08e-03
	Dist	1.82e+03	1.73e+03	1.89e+03	1.74e+03	1.80e+03	1.83e+03	1.81e+03	1.80e+03	1.84e+03	1.74e+03	1.60e+03
f_9	Best	520.68	1793.7	175.86	437.43	3.82e-04	1594	3.82e-04	-12349	-1523.4	459.12	182.46
	Mean	968.22	2160.1	578.87	810.59	50.817	1998.4	295.42	-2165.4	615.8	771.45	791.25
	Var	71115	46239	98524	66299	13395	53307	1.12e+05	1.71e+07	5.24e+05	53597	94734
	Dist	4857.6	11135	2806.9	3851.3	203.1	11048	1385.4	33789	8227.6	3437.3	3903.5

Table 9: Friedman rank values of different initialization methods for f_1.

Rank	$Be(3,2)$	$Be(2.5,2.5)$	$Be(2,3)$	$U(0,1)$	$N(0,1)$	$N(0.5,1)$	$N(0.5,0.5)$	$logn(0,1)$	$logn(.69,.25)$	$logn(0,0.5)$	$logn(0,2/3)$
Best	11.5	11.5	11.5	11.5	11.5	11.5	11.5	11.5	11.5	11.5	11.5
Mean	21	16.5	16.5	10.5	16.5	16.5	10.5	10.5	6	2	6
Var	21	16.5	16.5	10.5	16.5	16.5	10.5	10.5	6	2	6
Dist	21	16.5	16.5	10.5	16.5	16.5	10.5	10.5	6	2	6
sum	74.5	61	61	43	61	61	43	43	29.5	17.5	29.5
mean	18.63	15.25	15.25	10.75	15.25	15.25	10.75	10.75	7.38	4.38	7.38
Rank	$E(0.5)$	$E(0.1)$	$E(0.8)$	$Rayl(0.4)$	$Rayl(0.8)$	$Rayl(0.1)$	$Weib(1,1.5)$	$Weib(1.5,1)$	$Weib(1,1)$	$random$	LHS
Best	11.5	11.5	11.5	11.5	11.5	11.5	11.5	11.5	11.5	11.5	11.5
Mean	10.5	16.5	22	6	2	6	16.5	6	2	16.5	16.5
Var	10.5	16.5	22	6	2	6	16.5	6	2	16.5	16.5
Dist	10.5	16.5	22	6	2	6	16.5	6	2	16.5	16.5
sum	43	61	77.5	29.5	17.5	29.5	61	29.5	17.5	43	43
mean	10.75	15.25	19.38	7.38	4.38	7.38	15.25	7.38	4.38	15.25	15.25

Table 10: Ranks of different initialization methods for DE-a over functions $f_1 - f_9$.

Fun	$p-value$	$Be(3,2)$	$Be(2.5,2.5)$	$Be(2,3)$	$U(0,1)$	$N(0,1)$	$N(0.5,1)$	$N(0.5,0.5)$	$logn(0,1)$	$logn(.69,.25)$	$logn(0,0.5)$	$logn(0,2/3)$
f_1	0.000	5	4	4	3	4	4	3	3	2	1	2
f_2	0.957	17	5	14	13	9	20	4	16	1	12	2
f_3	0.008	3	11	11	8	7	2	1	9	18	12	15
f_4	0.002	13	6	5	18	19	1	17	4	11	10	11
f_5	0.000	10	9	15	18	11	6	14	13	6	8	11
f_6	0.000	11	3	1	16	12	14	6	13	10	20	5
f_7	0.000	9	18	11	5	18	1	19	7	2	10	6
f_8	0.459	21	7	17	10	15	19	18	1	14	4	11
f_9	0.337	1	6	8	5	15	17	2	9	9	13	10
Fun		$E(0.5)$	$E(0.1)$	$E(0.8)$	$Rayl(0.4)$	$Rayl(0.8)$	$Rayl(0.1)$	$Weib(1,1.5)$	$Weib(1.5,1)$	$Weib(1,1)$	$random$	LHS
f_1		3	4	6	2	1	2	4	2	1	4	4
f_2		15	6	19	10	7	8	3	11	22	18	21
f_3		12	17	10	4	16	6	13	2	5	3	14
f_4		15	2	3	7	9	14	6	9	12	8	16
f_5		19	9	5	2	16	3	1	7	12	17	4
f_6		19	4	7	2	9	5	8	18	15	17	6
f_7		18	20	13	4	15	17	14	12	16	3	8
f_8		16	8	5	13	6	2	22	9	20	3	12
f_9		3	12	18	7	14	16	11	9	14	3	4

Table 11: Friedman ranks of different initialization methods for DE-a.

$Be(3,2)$	$Be(2.5,2.5)$	$Be(2,3)$	$U(0,1)$	$N(0,1)$	$N(0.5,1)$	$N(0.5,0.5)$	$logn(0,1)$	$logn(.69,.25)$	$logn(0,0.5)$	$logn(0,2/3)$
18	3	14	19	21	11	9	7	4	10	5
$E(0.5)$	$E(0.1)$	$E(0.8)$	$Rayl(0.4)$	$Rayl(0.8)$	$Rayl(0.1)$	$Weib(1,1.5)$	$Weib(1.5,1)$	$Weib(1,1)$	$random$	LHS
22	12	16	1	15	2	13	6	20	8	17

Table 12: Ranks of different initialization methods for PSO-w over functions $f_1 - f_9$.

Fun	$p-value$	$Be(3,2)$	$Be(2.5,2.5)$	$Be(2,3)$	$U(0,1)$	$N(0,1)$	$N(0.5,1)$	$N(0.5,0.5)$	$logn(0,1)$	$logn(.69,.25)$	$logn(0,0.5)$	$logn(0,2/3)$
f_1	0.344	2	11	17	13	21	19	22	16	8	4	1
f_2	0.02	18	17	5	4	6	8	14	16	13	9	20
f_3	0.000	10	1	21	4	5	15	22	18	9	7	20
f_4	0.000	18	1	19	6	8	7	12	11	4	16	15
f_5	0.002	9	8	3	17	4	10	15	16	14	5	12
f_6	0.000	16	1	17	5	7	11	15	13	8	18	19
f_7	0.000	20	1	16	4	8	3	14	11	7	18	19
f_8	0.459	2	1	3	6	19	17	9	21	22	15	18
f_9	0.984	3	17	20	4	18	7	21	11	8	12	13

Fun		$E(0.5)$	$E(0.1)$	$E(0.8)$	$Rayl(0.4)$	$Rayl(0.8)$	$Rayl(0.1)$	$Weib(1,1.5)$	$Weib(1.5,1)$	$Weib(1,1)$	$random$	LHS
f_1		20	10	5	14	3	18	12	6	15	7	9
f_2		19	10	7	3	12	22	1	15	21	11	2
f_3		16	8	14	2	12	11	6	17	19	13	3
f_4		14	21	5	2	17	22	20	10	13	3	9
f_5		1	7	6	22	19	18	20	11	13	2	21
f_6		12	21	10	2	20	22	14	6	9	3	4
f_7		9	21	2	5	13	22	17	10	15	12	6
f_8		8	11	14	4	12	10	13	20	16	5	7
f_9		22	14	15	19	5	16	6	9	10	1	2

Table 13: Friedman ranks of different initialization methods for PSO-w.

$Be(3,2)$	$Be(2.5,2.5)$	$Be(2,3)$	$U(0,1)$	$N(0,1)$	$N(0.5,1)$	$N(0.5,0.5)$	$logn(0,1)$	$logn(.69,.25)$	$logn(0,0.5)$	$logn(0,2/3)$
9	2	15	4	8	10	21	19	7	12	20

$E(0.5)$	$E(0.1)$	$E(0.8)$	$Rayl(0.4)$	$Rayl(0.8)$	$Rayl(0.1)$	$Weib(1,1.5)$	$Weib(1.5,1)$	$Weib(1,1)$	$random$	LHS
16	17	6	5	14	22	13	11	18	1	3

Table 14: Ranks of different initialization methods for CS over functions $f_1 - f_9$.

Fun	$p-value$	$Be(3,2)$	$Be(2.5,2.5)$	$Be(2,3)$	$U(0,1)$	$N(0,1)$	$N(0.5,1)$	$N(0.5,0.5)$	$logn(0,1)$	$logn(.69,.25)$	$logn(0,0.5)$	$logn(0,2/3)$
f_1	0.000	8	2	9	18	14	6	7	5	22	16	4
f_2	0.000	19	3	10	7	2	6	20	9	22	1	11
f_3	0.000	16	1	5	6	14	8	9	13	22	20	11
f_4	0.001	7	3	10	5	16	8	6	13	22	18	21
f_5	0.000	20	19	3	10	15	9	16	13	22	11	21
f_6	0.000	4	1	3	5	19	20	16	15	22	11	21
f_7	0.000	2	1	3	6	13	12	10	17	22	14	16
f_8	0.709	2	1	8	4	11	21	13	15	14	18	19
f_9	0.196	3	8	21	6	18	19	10	11	12	2	5

Fun		$E(0.5)$	$E(0.1)$	$E(0.8)$	$Rayl(0.4)$	$Rayl(0.8)$	$Rayl(0.1)$	$Weib(1,1.5)$	$Weib(1.5,1)$	$Weib(1,1)$	$random$	LHS
f_1		3	19	20	1	12	21	11	10	13	15	17
f_2		12	13	14	16	4	17	18	15	8	21	5
f_3		12	21	4	2	18	17	7	15	19	10	3
f_4		14	17	11	4	15	19	1	20	12	2	9
f_5		5	2	4	7	14	6	8	1	17	18	12
f_6		9	18	10	2	8	17	13	12	14	6	7
f_7		8	11	18	4	19	9	20	21	15	5	7
f_8		16	5	22	6	12	17	7	9	20	10	3
f_9		17	22	7	15	1	20	4	13	14	9	16

Table 15: Friedman ranks of different initialization methods on the CS.

$Be(3,2)$	$Be(2.5,2.5)$	$Be(2,3)$	$U(0,1)$	$N(0,1)$	$N(0.5,1)$	$N(0.5,0.5)$	$logn(0,1)$	$logn(.69,.25)$	$logn(0,0.5)$	$logn(0,2/3)$
6	1	4	3	17	11	13	15	22	16	20

$E(0.5)$	$E(0.1)$	$E(0.8)$	$Rayl(0.4)$	$Rayl(0.8)$	$Rayl(0.1)$	$Weib(1,1.5)$	$Weib(1.5,1)$	$Weib(1,1)$	$random$	LHS
9	18	12	2	10	21	7	14	19	8	5

Table 16: CEC2014 and CEC2017 single objective optimization problems.

Type	No.	Function	Opt
Unimodal	f_1	Rotated High Conditioned Elliptic Function	100
Multimodal	f_7	Shifted and Rotated Griewanks Function	700
	f_{13}	Shifted and Rotated HappyCat Function	1300
Hybrid	f_{18}	Hybrid Function 2 (N=3)	1800
	f_{20}	Hybrid Function 4 (N=4)	2000
Composition	f_{23}	Composition Function 1 (N=5)	2300
	f_{25}	Composition Function 3 (N=3)	2500
Multimodal	F_8	Shifted and Rotated Non-Continuous Rastrigins function	800
Hybrid	F_{14}	Hybrid Function 4 (N=4)	1400
Composition	F_{22}	Composition Function 2 (N=3)	2300

Table 17: Comparison of DE-a for CEC functions with different initialization methods.

Fun	Value	$Be(3,2)$	$Be(2.5,2.5)$	$Be(2,3)$	$U(0,1)$	$N(0,1)$	$N(0.5,1)$	$N(0.5,0.5)$	$logn(0,1)$	$logn(.69,.25)$	$logn(0,0.5)$	$logn(0,2/3)$
f_1	Best	1.84e+06	1.25e+06	1.59e+06	2.11e+06	1.50e+06	2.32e-06	2.10e+06	2.15e+06	2.00e+06	1.86e+06	1.95e+06
	Mean	3.19e+06	2.74e+06	3.56e+06	3.52e+06	3.43e+06	3.55e+06	3.80e+06	3.74e+06	3.79e+06	3.31e+06	3.58e+06
	Var	1.47e+12	1.145e+12	1.46e+12	1.65e+12	1.38e+12	1.11e+12	1.42e+12	1.21e+12	2.15e+12	1.28eE+12	1.86e+12
f_7	Best	700	700	700	700	700	700	700	700	700	700	700
	Mean	700.01	700.00	700.00	700.01	700.00	700.01	700.00	700.00	700.00	700.00	700.01
	Var	5.60e-05	2.52e-05	5.10e-05	6.36e-05	3.61e-05	3.38e-05	3.16e-05	5.12e-05	3.27e-05	3.59e-05	7.44e-05
f_{13}	Best	1300.19	1300.20	1300.21	1300.20	1300.22	1300.19	1300.20	1300.21	1300.15	1300.15	1300.19
	Mean	1300.26	1300.27	1300.27	1300.27	1300.27	1300.256	1300.27	1300.26	1300.26	1300.26	1300.28
	Var	1.81e-03	1.35e-03	1.66e-03	1.53e-03	1.53e-03	1.18e-03	1.51e-03	9.34e-04	2.14e-03	2.04e-03	1.52e-03
f_{18}	Best	1866.1	1888.3	1867.5	1870.2	1864	1883.6	1885.5	1881.5	1836.6	1894.1	1877
	Mean	2105.6	1984	2023.5	1966.4	2071.5	2199.6	2012.3	14458	1958.6	2202.2	1974.8
	Var	1.60e+05	13063	51432	10016	96318	4.55e+05	15738	3.10e+09	3500.4	6.84e+05	7250.3
f_{20}	Best	2037.16	2038.13	2033.42	2034.92	2042.43	2036.96	2043.04	2036.83	2037.90	2037.88	2039.17
	Mean	2048.25	2048.19	2049.18	2049.54	2050.26	2050.48	2051.85	2048.66	2050.31	2048.58	2049.95
	Var	36.18	52.19	51.87	41.27	36.47	34.04	17.77	31.20	33.69	28.20	25.30
f_{23}	Best	2615.24	2615.24	2615.24	2615.24	2615.24	2615.24	2615.24	2615.24	2615.24	2615.24	2615.24
	Mean	2615.24	2615.24	2615.24	2615.24	2615.24	2615.24	2615.24	2615.24	2615.24	2615.24	2615.24
	Var	2.18e-25	2.18e-25	2.18e-25	2.18e-25	2.18-25	2.18e-25	2.18e-25	2.18e-25	2.18-25	2.18-25	2.18e-25
f_{25}	Best	2703.26	2702.77	2702.78	2702.80	2702.82	2702.97	2703.03	2702.75	2702.92	2703.20	2702.91
	Mean	2704.02	2703.80	2703.99	2704.12	2703.62	2703.95	2704.11	2703.92	2704.12	2704.18	2703.76
	Var	3.39e-01	2.11e-01	7.17e-01	3.74e-01	1.93e-01	4.05e-01	4.08e-01	4.89e-01	6.52e-01	3.33e-01	2.94e-01
F_8	Best	942.05	945.78	947.19	957.68	944.1	948.11	939.79	939.35	918.84	919.51	939.27
	Mean	961.36	961.92	964.58	967.55	964.98	964.55	964.48	963.77	963.84	962.48	962.33
	Var	100.89	59.664	79.826	73.786	167.79	54.317	154.33	165.39	223.97	214.74	113.96
F_{14}	Best	1457	1463.3	1466.5	1466.1	1458.4	1467.1	1466.6	1460.5	1466.6	1457.8	1463
	Mean	1473.8	1474.8	1474.3	1476.8	1475.4	1478.2	1477.7	1474.9	1476.2	1474.8	1478.8
	Var	52.13	37.66	19.04	37.37	36.14	33.11	33.88	51.01	30.92	66.27	64.09
F_{22}	Best	2300	2300	2300	2300	2300	2300	2300	2300	2300	2300	2300
	Mean	2300.2	2300.1	2300.1	2300.1	2300.1	2300.1	2300	2300	2300.1	2300	2300.1
	Var	5.79e-01	3.01e-01	3.01e-01	3.13e-01	3.01e-01	3.01e-01	1.41e-25	1.52e-25	3.03e-01	1.74e-25	3.01e-01

Fun		$E(0.5)$	$E(0.1)$	$E(0.8)$	$Rayl(0.4)$	$Rayl(0.8)$	$Rayl(0.1)$	$Weib(1,1.5)$	$Weib(1.5,1)$	$Weib(1,1)$	$random$	LHS
f_1	Best	1.78e+06	2.01e+06	1.87e+06	1.62e+06	2.11e+06	1.19e+06	2.08e+06	1.78e+06	1.98e+06	1.44e+06	1.25e+06
	Mean	3.94e+06	3.62e+06	3.22e+06	3.41e+06	3.46e+06	3.44e+06	3.42e+06	3.26e+06	3.59e+06	3.58e+06	3.20e+06
	Var	2.64e+12	8.27e+11	1.23e+12	1.56e+12	8.22e+11	2.11e+12	1.17e+12	1.23e+12	1.62e+12	2.35e+12	1.39e+12
f_7	Best	700	700	700	700	700	700	700	700	700	700	700
	Mean	700.00	700.01	700.01	700.01	700.00	700.01	700.01	700.01	700.00	700.01	700.00
	Var	3.85e-05	1.39e-04	8.51e-05	1.17e-04	4.02e-05	7.73e-05	5.43e-05	7.84e-05	4.18e-05	4.46e-05	5.00e-05
f_{13}	Best	1300.17	1300.18	1300.20	1300.19	1300.16	1300.16	1300.18	1300.17	1300.20	1300.20	1300.20
	Mean	1300.27	1300.25	1300.27	1300.26	1300.26	1300.26	1300.25	1300.25	1300.26	1300.26	1300.28
	Var	1.92e-03	1.85e-03	1.34e-03	1.28e-03	1.97e-03	2.15e-03	2.66e-03	2.07e-03	1.34e-03	1.58e-03	2.05e-03
f_{18}	Best	1855.9	1854.5	1867.8	1857.5	1872.6	1850	1887.2	1881.2	1835	1851.4	1856.6
	Mean	2054.1	1936.1	1993.3	1958.8	2031.2	1962.6	1970.6	2043.3	1948.6	1958	3449
	Var	68834	6992.8	28191	3191.8	69288	21981	5744.2	97484	4970.3	5558.5	4.29E+07
f_{20}	Best	2035.72	2044.01	2029.32	2034.46	2036.93	2033.82	2034.80	2041.47	2040.84	2035.73	2041.37
	Mean	2049.25	2052.33	2048.86	2048.19	2051.34	2049.18	2048.36	2048.28	2051.39	2049.50	2049.10
	Var	31.46	11.49	49.09	62.84	37.90	50.68	37.71	16.13	38.44	45.16	44.02
f_{23}	Best	2615.24	2615.24	2615.24	2615.24	2615.24	2615.24	2615.24	2615.24	2615.24	2615.24	2615.24
	Mean	2615.24	2615.24	2615.24	2615.24	2615.24	2615.24	2615.24	2615.24	2615.24	2615.24	2615.24
	Var	2.18e-25	2.18e-25	2.18e-25	2.18e-25	2.18-25	2.18e-25	2.18e-25	2.18e-25	2.18-25	2.18-25	2.18e-25
f_{25}	Best	2702.83	2702.83	2703.16	2702.92	2703.19	2702.61	2703.25	2702.83	2703.03	2703.13	2703.13
	Mean	2704.11	2703.92	2704.15	2703.79	2703.98	2703.34	2704.06	2703.77	2703.93	2703.89	2703.92
	Var	5.91e-01	5.75e-01	4.34e-01	3.44e-01	3.78e-01	3.65e-01	2.39e-01	3.29e-01	2.56e-01	2.14e-01	3.26e-01
F_8	Best	944.34	941.11	949.11	932.02	948.63	943.51	936.27	940.6	945.19	941.23	948.74
	Mean	963.73	961.87	963.83	960.52	965.03	962.46	960.09	965.05	966.09	963.98	964.64
	Var	86.325	120.04	63.67	135.94	77.99	129.63	154.77	140.02	80.76	74.60	103.69
F_{14}	Best	1466.6	1466	1462.9	1462.7	1464.6	1453.3	1461.6	1462.9	1463.5	1460.6	1456.4
	Mean	1474.2	1475.4	1474.4	1476.6	1475.9	1475.7	1473.9	1474.9	1475	1474.1	1475.6
	Var	21.49	28.57	26.34	37.29	32.21	70.68	51.38	48.05	36.61	42.83	57.64
F_{22}	Best	2300	2300	2300	2300	2300	2300	2300	2300	2300	2300	2300
	Mean	2300.2	2300	2300.2	2300	2300.2	2639.6	2300	2300	2300	2300.1	2300.1
	Var	5.70e-01	1.52e-25	5.70e-01	1.96e-25	5.70e-01	2.30e+06	1.85e-25	1.31e-25	1.85e-25	3.01e-01	3.03e-01

Table 18: Comparison of PSO-w for CEC functions with different initialization methods.

Fun	Value	$Be(3,2)$	$Be(2.5,2.5)$	$Be(2,3)$	$U(0,1)$	$N(0,1)$	$N(0.5,1)$	$N(0.5,0.5)$	$logn(0,1)$	$logn(.69,.25)$	$logn(0,0.5)$	$logn(0,2/3)$
	Best	3.09e+05	2.08e+05	45809	3.20e+05	2.05e+05	2.81e+05	1.95e+05	1.70e+05	1.93e+05	2.46e+05	2.18e+05
f_1	Mean	5.11e+06	3.17e+06	1.73e+06	2.53e+06	2.41e+06	2.50e+06	4.62e+06	1.73e+06	2.81e+06	1.63e+06	2.62e+06
	Var	3.43e+13	1.11e+13	2.81e+12	3.49e+12	4.12e+12	8.05e+12	2.83e+13	2.35e+12	2.69e+13	1.89e+12	6.83e+12
	Best	700	700	700	700	700	700	700	700	700	700	700
f_7	Mean	700.02	700.02	700.01	700.01	700.02	700.02	700.01	700.02	700.02	700.01	700.02
	Var	3.69e-04	2.34e-04	2.64e-04	1.34e-04	1.64e-04	3.55e-04	1.54e-04	2.15e-04	2.44e-04	1.45e-04	2.69e-04
	Best	1300.2	1300.2	1300.2	1300.3	1300.3	1300.3	1300.2	1300.3	1300.2	1300.2	1300.2
f_{13}	Mean	1300.4	1300.4	1300.4	1300.4	1300.4	1300.4	1300.4	1300.4	1300.4	1300.4	1300.4
	Var	7.83e-03	1.14e-02	8.75e-03	6.54e-03	1.12e-02	1.12e-02	1.59e-02	7.42e-03	8.69e-03	7.69e-03	9.06e-03
	Best	1866.1	1888.3	1867.5	1870.2	1864	1883.6	1885.5	1881.5	1836.6	1894.1	1877
f_{18}	Mean	2105.6	1984	2023.5	1966.4	2071.5	2199.6	2012.3	14458	1958.6	2202.2	1974.8
	Var	1.60e+05	13063	51432	10016	96318	4.55e+05	15738	3.10e+09	3500.4	6.84e+05	7250.3
	Best	2050.3	2076	2066.1	2093	2090	2119.3	2084.9	2108.9	2103.6	2107.4	2068.1
f_{20}	Mean	2155.6	4383.9	2170.6	2199.4	2212.9	2180.2	2176.3	2174.8	2188.7	2184.7	2175.6
	Var	2155.7	9.94e+07	3548	5516.9	6635.3	2236.4	3755.2	2517.2	2758.6	5166.8	1918.3
	Best	2500	2500	2500	2500	2615.4	2500	2500	2615.4	2500	2500	2615.4
f_{23}	Mean	2500	2500	2500	2500	2615.7	2610	2528.9	2615.7	2586.9	2592.7	2615.8
	Var	3.09E-09	2.91E-09	3.21E-09	3.70E-09	2.19e-02	670.33	2639.8	5.182e-02	2646.8	2261.1	1.31e-01
	Best	2700	2700	2700	2700	2700	2700	2700	2700	2700	2700	2700
f_{25}	Mean	2700	2700	2700	2700	2707.4	2704.4	2710.1	2702.6	2702.2	2702.7	
	Var	4.21e-12	1.96e-12	3.68e-13	1.17e-11	18.138	20.167	3.82e-08	15.2	14.12	9.8308	15.881
	Best	888.55	900.49	872.63	866.66	888.55	887.56	885.57	898.50	883.58	906.47	900.49
F_8	Mean	933.62	933.48	928.55	932.13	934.52	935.56	929.19	931.33	935.07	951.04	941.56
	Var	513.41	570.29	745.33	773.66	552.95	576.55	595.09	483.40	578.83	589.91	450.36
	Best	1.47e+03	1.47e+03	1.48e+03	1.46e+03	1.48e+03	1.48e+03	1.48e+03	1.49e+03	1.48e+03	1.46e+03	1.50e+03
F_{14}	Mean	1.96e+03	2.01e+03	1.55e+03	1.53e+03	2.44e+03	2.07e+03	1.55e+03	3.14e+03	1.54e+03	1.54e+03	1.58e+03
	Var	3.68e+06	4.59e+06	3.89e+03	1.30e+03	1.59e+07	5.46e+06	2.75e+03	2.05e+07	2.08e+03	1.99e+03	7.95e+03
	Best	2.30e+03	2.30e+03	2.30e+03	2.30e+03	2.30e+03	2.30e+03	2.30e+03	2.30e+03	2.30e+03	2.30e+03	2.30e+03
F_{22}	Mean	2.30e+03	2.30e+03	2.30e+03	2.47e+03	2.30e+03	2.30e+03	2.30e+03	2.30e+03	2.61e+03	2.30e+03	2.30e+03
	Var	2.01	2.40	1.97	5.85e+05	2.57	0.82	0.80	2.15	9.85e+05	2.96	2.13

Fun		$E(0.5)$	$E(0.1)$	$E(0.8)$	$Rayl(0.4)$	$Rayl(0.8)$	$Rayl(0.1)$	$Weib(1,1.5)$	$Weib(1.5,1)$	$Weib(1,1)$	$random$	LHS
	Best	2.88e+05	2.33e+05	2.04e+05	56405	3.23e+05	1.93e+05	1.86e+05	95827	1.64e+05	74981	2.49e+05
f_1	Mean	2.96e+06	1.53e+06	2.52e+06	3.51e+06	2.22e+06	2.29e+06	2.65e+06	1.86e+06	1.98e+06	3.25e+06	3.41e+06
	Var	1.11e+13	2.39e+12	3.64e+12	1.11e+13	2.87e+12	3.99e+12	9.50e+12	3.63e+12	3.84e+12	1.71e+13	2.23e+13
	Best	700	700	700	700	700	700	700	700	700	700	700
f_7	Mean	700.02	700.01	700.01	700.01	700.02	700.01	700.01	700.02	700.01	700.01	700.01
	Var	3.36e-04	2.06e-04	1.75e-04	1.93e-04	4.02e-04	7.84e-05	8.82e-05	3.44e-04	1.71e-04	2.37e-04	1.03e-04
	Best	1300.2	1300.2	1300.2	1300.3	1300.2	1300.2	1300.2	1300.2	1300.3	1300.3	1300.2
f_{13}	Mean	1300.4	1300.3	1300.4	1300.4	1300.4	1300.4	1300.4	1300.4	1300.4	1300.4	1300.4
	Var	8.16e-03	8.75e-03	1.01e-02	7.84e-03	1.17e-02	1.15e-02	8.54e-03	8.97e-03	9.26e-03	5.34e-03	1.12e-02
	Best	1855.9	1854.5	1867.8	1857.5	1872.6	1850	1887.2	1881.2	1835	1851.4	1856.6
f_{18}	Mean	2054.1	1936.1	1993.3	1958.8	2031.2	1962.6	1970.6	2043.3	1948.6	1958	3449
	Var	68834	6992.8	28191	3191.8	69288	21981	5744.2	97484	4970.3	5558.5	4.29E+07
	Best	2094.5	2081.9	2078	2068.4	2080	2091	2089.2	2110.1	2062.6	2070.5	2065.2
f_{20}	Mean	2182.1	2200.1	2194.8	4627.2	2174.5	2193.8	2169.4	2174	2153.8	2164.9	2192.8
	Var	3888.9	4664	2587.6	1.22e+08	2677.6	4364.7	1978.1	4444.5	2943.2	3629.6	3892.2
	Best	2500	2500	2500	2500	2500	2500	2615.4	2615.4	2500	2500	2500
f_{23}	Mean	2500	2500	2586.8	2500	2598.4	2500	2615.7	2615.7	2609.8	2500	2500
	Var	8.84e-09	8.83e-10	2643.6	3.47e-09	1798.8	1.06e-09	3.9262e-02	5.20e-02	668.29	2.72e-09	7.06e-09
	Best	2700	2700	2700	2700	2700	2700	2700	2700	2700	2700	2700
f_{25}	Mean	2700	2700	2702.2	2700	2700.7	2700	2702	2710.6	2705.1	2700	2700
	Var	6.44e-11	3.05e-13	11.943	1.39e-12	3.2384	8.57e-13	10.414	14.051	7.8027	4.27e-12	6.92e-12
	Best	881.59	908.45	905.47	892.53	894.52	920.53	902.49	898.06	881.59	883.58	900.49
F_8	Mean	928.45	951.24	938.39	946.45	939.50	953.56	939.54	932.66	931.84	931.64	935.43
	Var	482.66	818.25	473.99	578.76	846.19	503.99	303.78	593.32	761.96	470.65	453.12
	Best	1.49e+03	1.50e+03	1.49e+03	1.46e+03	1.49e+03	1.48e+03	1.49e+03	1.48e+03	1.49e+03	1.46e+03	1.48e+03
F_{14}	Mean	1.54e+03	1.55e+03	2.25e+03	2.30e+03	2.26e+03	2.84e+03	4.14e+03	2.12e+03	3.25e+03	1.54e+03	1.55e+03
	Var	1.22e+03	1.49e+03	9.58e+06	1.15e+07	1.05e+07	3.12e+07	8.67e+07	6.98e+06	5.79e+07	1.47e+03	1.39e+03
	Best	2.30e+03	2.30e+03	2.30e+03	2.30e+03	2.30e+03	2.30e+03	2.30e+03	2.30e+03	2.30e+03	2.30e+03	2.30e+03
F_{22}	Mean	2.30e+03	4.25e+03	2.30e+03	2.30e+03	2.30e+03	4.02e+03	2.30e+03	2.30e+03	2.30e+03	2.30e+03	2.30e+03
	Var	3.22	5.18e+06	3.30	13.27	1.42	5.92e+06	4.31	2.12	2.52	1.48	2.70

Table 19: Comparison of CS for CEC functions with different initialization methods.

Fun	Value	$Be(3,2)$	$Be(2.5,2.5)$	$Be(2,3)$	$U(0,1)$	$N(0,1)$	$N(0.5,1)$	$N(0.5,0.5)$	$logn(0,1)$	$logn(.69,.25)$	$logn(0,0.5)$	$logn(0,2/3)$
f_1	Best	2.92e+05	4.00e+05	3.46e+05	4.97e+05	3.72e+05	4.81e+05	5.12e+05	3.72e+05	7.01e+08	3.82e+05	3.54e+05
	Mean	6.46e+05	7.53e+05	7.45e+05	7.67e+05	7.16e+05	7.10e+05	7.40e+05	6.48e+05	6.57e+09	5.34e+05	6.32e+05
	Var	3.21e+10	2.56e+10	2.24e+10	2.82e+10	3.33e+10	2.79e+10	2.51e+10	2.15e+10	3.21e+19	6.96e+09	1.48e+10
f_7	Best	700	700	700	700	700	700	700	700	1965.7	700	700
	Mean	700	700	700	700	700	700	700	700	3159.2	700	700
	Var	6.65e-25	2.77e-24	1.75e-24	3.70e-22	5.03e-23	2.97e-22	6.64e-23	1.56e-24	2.63e+05	1.34e-22	5.55e-21
f_{13}	Best	1300.2	1300.3	1300.2	1300.2	1300.2	1300.2	1300.2	1300.2	1317.7	1300.2	1300.2
	Mean	1300.3	1300.3	1300.3	1300.3	1300.3	1300.3	1300.3	1300.3	1328.9	1300.3	1300.3
	Var	1.39e-03	8.42e-04	1.84e-03	1.42e-03	1.12e-03	1.56e-03	1.72e-03	7.30e-04	28.63	1.41e-03	9.28e-04
f_{18}	Best	1843.3	1830.3	1842.2	1834.6	1838.3	1832.9	1827.6	1833.4	2.91e+10	1830	1833.6
	Mean	1859.6	1850.9	1856.4	1851.4	1856.4	1853.5	1853.4	1854.7	7.62e+10	1850.3	1854.9
	Var	145.62	143.06	136.53	89.423	108.16	169.72	93.004	120.63	4.19e+20	104.59	94.003
f_{20}	Best	2033.5	2030.3	2036.6	2026	2032.7	2032.7	2034.2	2037.4	1.25e+05	2036.5	2033.7
	Mean	2043.8	2042.7	2046.1	2043.1	2041.7	2042.8	2043.9	2045.1	1.54e+09	2043.8	2044.9
	Var	22.967	34.75	46.147	52.618	17.839	32.583	39.687	36.689	2.96e+18	33.398	30.237
f_{23}	Best	2615.2	2615.2	2615.2	2615.2	2615.2	2615.2	2615.2	2615.2	12419	2615.2	2615.2
	Mean	2615.2	2615.2	2615.2	2615.2	2615.2	2615.2	2615.2	2615.2	26820	2615.2	2615.2
	Var	2.18e-25	2.18e-25	2.18e-25	2.18e-25	2.18e-25	2.18e-25	2.18e-25	2.18e-25	5.50e+07	2.18e-25	2.18e-25
f_{25}	Best	2703.9	2703.6	2703.6	2703.8	2704.1	2703.8	2703.6	2703.7	3171.9	2703.9	2704
	Mean	2704.8	2704.4	2704.6	2704.6	2705.1	2704.8	2704.9	2705.1	4143	2705	2705.1
	Var	0.28	0.25	0.46	0.35	0.28	0.23	0.45	0.76	2.16e+05	0.39	0.38
F_8	Best	924.24	913.1	905.61	874.39	887.55	883.15	891.24	904.51	1593.5	865.49	888.58
	Mean	947.46	937.15	938.27	919.17	914.73	912.46	922.11	928.04	1943.7	916.34	918.49
	Var	287.24	272.62	251.48	341.25	213.17	108.03	255.46	287.42	34289	354.4	311.77
F_{14}	Best	1446.1	1451.1	1442.6	1447.1	1446.5	1442.6	1447	1446.2	1453	1446.9	1446.7
	Mean	1455.5	1459.5	1455.9	1456.7	1459.7	1456.5	1455.9	1457.2	8.427e+09	1456.2	1457
	Var	44.31	34.55	62.52	34.97	56.84	64.83	29.23	39.89	1.07e+20	38.85	55.49
F_{22}	Best	2300	2300	2300	2300	5509.9	2396	2300.1	2301.5	4151	2300.1	2301.1
	Mean	2300	2300	2677.2	2300.8	6055.2	5536.8	4460.7	4935.1	6349.5	5910.6	5578
	Var	1.96e-13	3.23e-13	1.34e+06	1.61	83776	1.82e+06	3.21e+06	3.07e+06	1.58e+06	7.54e+05	2.01e+06

Fun		$E(0.5)$	$E(0.1)$	$E(0.8)$	$Rayl(0.4)$	$Rayl(0.8)$	$Rayl(0.1)$	$Weib(1,1.5)$	$Weib(1.5,1)$	$Weib(1,1)$	$random$	LHS
f_1	Best	4.36e+05	6.13e+05	4.44e+05	4.05e+05	3.06e+05	4.20e+05	3.45e+05	4.01e+05	4.80e+05	5.17e+05	5.67e+05
	Mean	7.74e+05	8.59e+05	7.08e+05	7.17e+05	5.86e+05	7.04e+05	6.44e+05	6.88e+05	7.00e+05	7.96e+05	8.06e+05
	Var	3.21e+10	2.56e+10	2.24e+10	2.82e+10	3.33e+10	2.79e+10	2.51e+10	2.15e+10	3.21e+19	6.96e+09	1.48e+10
f_7	Best	700	700	700	700	700	700	700	700	700	700	700
	Mean	700	700	700	700	700	700	700	700	700	700	700
	Var	1.93e-24	4.76e-23	8.45e-23	1.50e-22	3.28e-21	6.39e-23	1.25e-21	4.44e-23	1.53e-22	1.13e-20	1.84e-22
f_{13}	Best	1300.2	1300.2	1300.2	1300.2	1300.2	1300.2	1300.2	1300.2	1300.2	1300.2	1300.2
	Mean	1300.3	1300.3	1300.3	1300.3	1300.3	1300.3	1300.3	1300.3	1300.3	1300.3	1300.3
	Var	1.30e-03	1.25e-03	1.71e-03	1.17e-03	9.58e-04	8.29e-04	2.46e-03	1.64e-03	2.04e-03	1.18e-03	1.05e-03
f_{18}	Best	1832.2	1832.9	1840.3	1837.6	1838.6	1836	1836.5	1828.7	1838.8	1840.1	1835.5
	Mean	1852.7	1851.6	1851.7	1859.6	1855.6	1859.7	1855.9	1851.6	1855.5	1856.9	1851.4
	Var	168.69	84.948	79.543	193.97	144.25	127.88	152.01	140.67	139.09	124.63	96.454
f_{20}	Best	2034.2	2032.3	2033.2	2031.4	2034.7	2034.4	2034.4	2026.8	2028.9	2030.7	2034.8
	Mean	2041.5	2043.7	2042.9	2044.9	2044.3	2041.8	2044.3	2042.4	2043.6	2044	2043.5
	Var	26.93	27.444	40.999	32.61	57.805	27.786	37.643	42.141	32.321	51.707	39.462
f_{23}	Best	2615.2	2615.2	2615.2	2615.2	2615.2	2615.2	2615.2	2615.2	2615.2	2615.2	2615.2
	Mean	2615.2	2615.2	2615.2	2615.2	2615.2	2615.2	2615.2	2615.2	2615.2	2615.2	2615.2
	Var	2.18e-25	2.07e-25	2.18e-25	2.18e-25	2.18e-25	2.18e-25	2.18e-25	2.18e-25	2.18e-25	2.18e-25	2.18e-25
f_{25}	Best	2703.8	2703.4	2704.1	2703.9	2704.3	2703.2	2703.9	2704.1	2703.5	2703.5	2703.8
	Mean	2704.7	2703.9	2704.9	2704.8	2705	2703.9	2704.7	2705	2704.8	2704.8	2704.7
	Var	0.2876	0.1534	0.3632	0.3239	0.2161	0.0997	0.2584	0.2986	0.2694	0.4495	0.2606
F_8	Best	886.82	884.55	881.53	902.53	888.96	871.05	896.85	865.81	892.52	892.77	883.74
	Mean	917.25	925.93	915.66	926.21	923.64	914.05	919.95	913.91	915.83	920.02	919.88
	Var	232.06	406.49	368.09	171.7	391.17	330.79	363.33	342.47	173.41	140.17	353.47
F_{14}	Best	1445.9	1443.5	1445.8	1448	1439.1	1447.7	1442.7	1448.4	1442.7	1448.1	1449.5
	Mean	1457	1458.6	1454.9	1459.3	1455.8	1456.3	1452.4	1457.5	1455.8	1455.8	1457
	Var	34.19	40.95	38.71	58.42	84.53	39.29	37.57	32.53	44.76	25.117	18.55
F_{22}	Best	2300.2	5476.3	2301	2300	2309.9	5289.8	2302.5	2303.1	2302.5	2300	2300
	Mean	4251	5851.4	4626.8	2493.3	5714	5744.1	4728.8	5884	5435.1	3045.9	2304.2
	Var	3.13e+06	44081	3.05e+06	7.47e+05	1.40e+06	45164	3.28e+06	7.72e+05	1.86e+06	2.26e+06	112.71

Table 20: Friedman ranks of different initialization methods for three algorithms.

DE	p	$Be(3,2)$	$Be(2.5,2.5)$	$Be(2,3)$	$U(0,1)$	$N(0,1)$	$N(0.5,1)$	$N(0.5,0.5)$	$logn(0,1)$	$logn(.69,.25)$	$logn(0,0.5)$	$logn(0,2/3)$
	0.30	9	5	11	22	8	12	21	2	20	1	14
		$E(0.5)$	$E(0.1)$	$E(0.8)$	$Rayl(0.4)$	$Rayl(0.8)$	$Rayl(0.1)$	$Weib(1,1.5)$	$Weib(1.5,1)$	$Weib(1,1)$	$random$	LHS
		15	4	16	7	18	17	3	6	13	10	19
PSO-w	p	$Be(3,2)$	$Be(2.5,2.5)$	$Be(2,3)$	$U(0,1)$	$N(0,1)$	$N(0.5,1)$	$N(0.5,0.5)$	$logn(0,1)$	$logn(.69,.25)$	$logn(0,0.5)$	$logn(0,2/3)$
	0.04	3	16	1	5	22	21	10	18	14	15	17
		$E(0.5)$	$E(0.1)$	$E(0.8)$	$Rayl(0.4)$	$Rayl(0.8)$	$Rayl(0.1)$	$Weib(1,1.5)$	$Weib(1.5,1)$	$Weib(1,1)$	$random$	LHS
		8	4	13	12	19	9	11	20	6	2	7
CS	p	$Be(3,2)$	$Be(2.5,2.5)$	$Be(2,3)$	$U(0,1)$	$N(0,1)$	$N(0.5,1)$	$N(0.5,0.5)$	$logn(0,1)$	$logn(.69,.25)$	$logn(0,0.5)$	$logn(0,2/3)$
	0.01	7	4	18	10	14	12	13	15	22	5	17
		$E(0.5)$	$E(0.1)$	$E(0.8)$	$Rayl(0.4)$	$Rayl(0.8)$	$Rayl(0.1)$	$Weib(1,1.5)$	$Weib(1.5,1)$	$Weib(1,1)$	$random$	LHS
		1	3	11	20	19	2	21	8	6	16	9

5. Discussions

Based on the above extensive simulations and tests, we can now investigate any possible correlation between the average distance of the initial population from the true optimal solution and the performance of final solutions found by algorithms. We also discuss their implications. In addition, we will give the initialization suggestions for the other two algorithms: ABC and GA.

5.1. Correlations and discussions

We now conduct some further analyses and discussions to demonstrate the influence of the initial population with different distributions on the results of algorithms. To begin with, we first define the average distance Δ between the initial population and the corresponding real optimal solution as

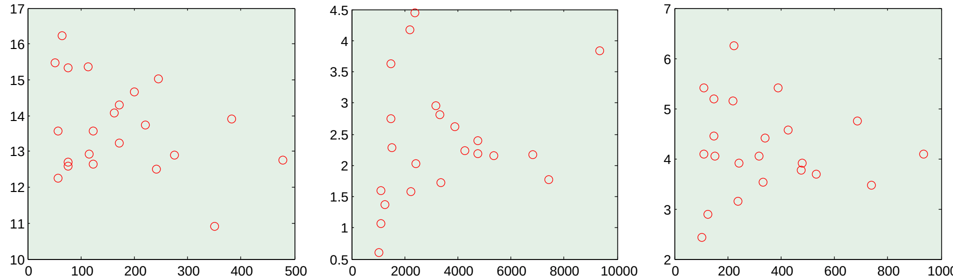
$$\Delta = \frac{\sum_{i=1}^{NP} \sum_{j=1}^{D} \left| x_{i,j} - {x_j}^{opt} \right|}{NP} \tag{22}$$

where $x_{i,j}$ indicates the j-th dimension of the i-th individual, NP represents the total population size, and ${x_j}^{opt}$ represents the j-th dimension of the true optimal solution.

In order to explore the relationship between the initial distribution and the performance of an algorithm (in terms of the final solution obtained), we investigate if the average distance between the generated initial population and the true optimal solution (Δ) has any positive correlation to the distance between the obtained final solution from the true optimal solution. Each initialization method for each algorithm runs 20 times independently, so as to avoid any potential bias of the experiment. Let $\overline{\Delta}$ be the mean value of Δ in 20 experiments. The previously defined '*Dist*' represents the average distance between the optimal solution obtained by the algorithm and the real optimal solution in NT (here $NT = 20$) experiments. Then, we carry out some analyses to see if there is any connection between $\overline{\Delta}$ and '*Dist*'. The results are summarized in Table 21 to Table 23.

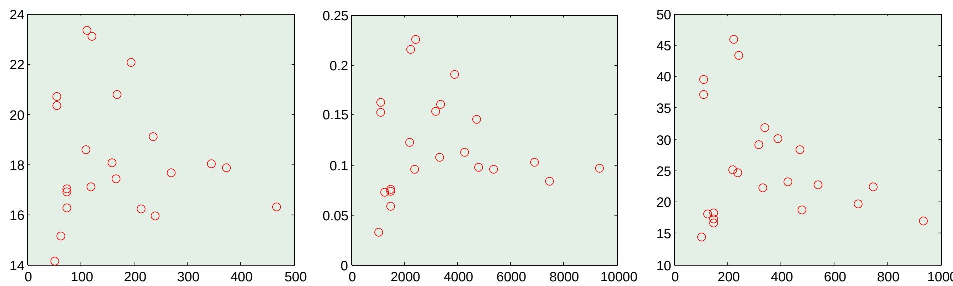
As seen from Tables 21 to 23, $\overline{\Delta}$ and 'Dist' have no significant correlation. For the ease of observation, the relationship between the $\overline{\Delta}$ and 'Dist' of the partial functions is given in Figs. 8 to 10. It is worth noting that, due to the larger value of $logn(0.69, 0.25)$ used in the CS algorithm, we can essentially consider such an extreme value as an outlier and thus remove it in the correlation test.

From a statistical perspective, the correlation tests have been undertaken, with the results shown in Fig. 24. Using a significance level of 0.01, when $p < 0.01$ means that the $\overline{\Delta}$ and '*Dist*' of the corresponding function has a significant correlation, mark with $**$. Another symbol $*$ means that the $\overline{\Delta}$ and '*Dist*' may have a weak correlation. The results show that except for one function f_9, there is no significant correlation for the DE-a algorithm. For the PSO-w, the $\overline{\Delta}$ and '*Dist*' of f_3 may have a weak correlation, and those of f_8 and f_9 have significant correlations between $\overline{\Delta}$ and '*Dist*'. For other functions, there



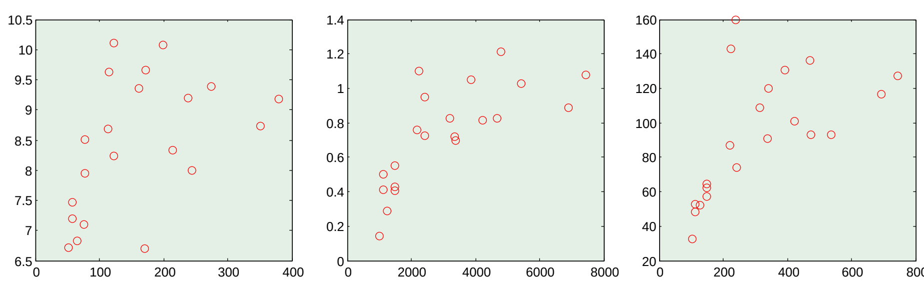
(a) (b) (c)

Figure 8: The relationship of $\overline{\Delta}$ and 'Dist': (a) f_4 of DE-a. (b)f_6 of DE-a. (c)f_7 of DE-a. The x-axis represents $\overline{\Delta}$: the average value of Δ (the average distance between the initial population and the real optimal solution) in 20 experiments, and the y-axis represents 'Dist' : average value of the distance between the best obtained solution and the real optimal solution in 20 experiments. It shows that there is no obvious correlation.



(a) (b) (c)

Figure 9: The relationship of $\overline{\Delta}$ and 'Dist': (a)f_4 of PSO-w. (b)f_6 of PSO-w. (c)f_7 of PSO-w. The x-axis represents $\overline{\Delta}$, and the y-axis represents 'Dist'. It can be seen that there is no obvious correlation.



(a) (b) (c)

Figure 10: The relationship of $\overline{\Delta}$ and 'Dist': (a)f_4 of CS. (b)f_6 of CS. (c)f_7 of CS. Among these three figures, f_4 has no significant correlation, while f_6 and f_7 may have some weak positive correlation.

Table 21: The result of $\overline{\Delta}$ and $Dist$ on DE-a.

Fun	Value	$Be(3,2)$	$Be(2.5,2.5)$	$Be(2,3)$	$U(0,1)$	$N(0,1)$	$N(0.5,1)$	$N(0.5,0.5)$	$logn(0,1)$	$logn(.69,.25)$	$logn(0,0.5)$	$logn(0,2/3)$
f_1	$\overline{\Delta}$	49.68	55.83	71.94	77.85	281.77	241.58	121.80	362.31	436.91	170.55	216.18
	Dist	0.20	0.20	0.60	0.50	0.30	0.60	0.80	0.20	0.20	0.20	0.20
f_2	$\overline{\Delta}$	112.27	101.99	112.42	150.39	536.64	478.01	237.95	751.80	933.73	389.20	473.33
	Dist	5.33e-14	5.32e-14	5.50e-14	0.08	0.08	5.19e-14	5.77e-14	5.61e-14	5.76e-14	5.64e-14	5.19e-14
f_3	$\overline{\Delta}$	56.40	50.77	55.97	75.09	267.53	240.01	119.68	369.47	466.27	194.37	238.51
	Dist	9.39e-94	9.69e-94	8.54e-94	5.78e-94	5.78e-94	2.18e-93	6.32e-94	1.09e-93	2.28e-93	1.36e-93	3.34e-94
f_4	$\overline{\Delta}$	57.35	52.26	57.83	76.71	275.66	245.93	122.35	382.70	478.07	199.97	242.47
	Dist	13.58	15.47	12.25	12.70	12.89	15.04	13.57	13.90	12.76	14.65	12.51
f_5	$\overline{\Delta}$	6.77e+03	6.11e+03	6.76e+03	8.96e+03	3.22e+04	2.89e+04	1.43e+04	4.44e+04	5.62e+04	2.33e+04	2.86e+04
	Dist	2.64	4.08	3.14	2.65	2.67	1.63	3.68	2.98	4.03	4.76	3.87
f_6	$\overline{\Delta}$	1.12e+03	1.02e+03	1.12e+03	1.50e+03	5.36e+03	4.77e+03	2.39e+03	7.44e+03	9.33e+03	3.88e+03	4.74e+03
	Dist	1.06	0.61	1.60	2.28	2.16	2.39	4.44	1.77	3.84	2.62	2.19
f_7	$\overline{\Delta}$	112.21	101.87	111.88	149.37	532.96	479.66	239.24	739.01	932.91	389.61	473.91
	Dist	5.40	2.43	4.10	4.44	3.69	3.91	3.15	3.47	4.09	5.41	3.78
f_8	$\overline{\Delta}$	1.09e+03	1.02e+03	1.16e+03	1.49e+03	5.43e+03	4.79e+03	2.41e+03	7.46e+03	9.25e+03	3.81e+03	4.64e+03
	Dist	1.43e+03	1.35e+03	1.37e+03	1.31e+03	1.36e+03	1.35e+03	1.33e+03	1.35e+03	1.35e+03	1.37e+03	1.36e+03
f_9	$\overline{\Delta}$	9.69e+03	1.26e+04	1.56e+04	1.28e+04	3.35e+04	2.60e+04	1.60e+04	3.43e+04	3.39e+04	1.32e+04	1.86e+04
	Dist	1.44e+03	1.59e+03	2.76e+03	1.77e+03	3.76e+04	2.77e+04	1.28e+03	5.79e+04	3.51e+04	1.20e+04	2.49e+04

Fun		$E(0.5)$	$E(0.1)$	$E(0.8)$	$Rayl(0.4)$	$Rayl(0.8)$	$Rayl(0.1)$	$Weib(1,1.5)$	$Weib(1.5,1)$	$Weib(1,1)$	$random$	LHS
f_1	$\overline{\Delta}$	120.89	150.23	166.66	69.82	151.70	142.42	149.66	334.02	211.59	77.96	77.99
	Dist	0.30	0.50	0.30	0.40	0.40	0.30	0.40	0.20	0.50	0.20	2.42e-15
f_2	$\overline{\Delta}$	220.39	241.23	335.92	126.67	337.15	224.77	318.54	691.83	429.84	150.48	150.01
	Dist	5.41e-14	5.46e-14	4.94e-14	5.75e-14	5.35e-14	5.17e-14	5.62e-14	5.46e-14	5.46e-14	5.77e-14	5.30e-14
f_3	$\overline{\Delta}$	110.39	120.58	166.38	62.85	169.67	112.37	158.61	347.37	213.49	74.80	75.00
	Dist	5.82e-94	6.52e-94	1.31e-93	6.80e-94	1.23e-93	1.07e-93	8.57e-94	1.28e-94	6.02e-94	4.70e-94	7.76e-94
f_4	$\overline{\Delta}$	113.31	123.09	172.12	64.60	172.07	115.14	163.03	351.33	221.10	77.04	76.79
	Dist	15.36	12.66	14.31	16.23	13.23	12.92	14.06	10.91	13.75	12.58	15.34
f_5	$\overline{\Delta}$	1.32e+04	1.44e+04	2.00e+04	7.58e+03	2.03e+04	1.35e+04	1.91e+04	4.10e+04	2.54e+04	8.98e+03	8.99e+03
	Dist	1.87	3.23	4.83	3.97	1.72	3.54	4.36	3.07	3.44	4.14	1.68
f_6	$\overline{\Delta}$	2.21e+03	2.41e+03	3.32e+03	1.26e+03	3.37e+03	2.25e+03	3.19e+03	6.81e+03	4.27e+03	1.49e+03	1.50e+03
	Dist	4.17	2.03	2.81	1.37	1.73	1.58	2.95	2.17	2.23	2.75	3.63
f_7	$\overline{\Delta}$	220.73	240.91	333.45	126.38	339.20	224.52	316.49	687.03	427.83	150.72	149.99
	Dist	5.15	3.91	3.53	2.88	4.40	6.25	4.04	4.74	4.56	4.04	5.20
f_8	$\overline{\Delta}$	2.23e+03	2.50e+03	3.35e+03	1.27e+03	3.31e+03	2.34e+03	3.12e+03	6.85e+03	4.29e+03	1.50e+03	1.50e+03
	Dist	1.34e+03	1.37e+03	1.37e+03	1.30e+03	1.37e+03	1.34e+03	1.35e+03	1.39e+03	1.31e+03	1.32e+03	1.37e+03
f_9	$\overline{\Delta}$	1.73e+04	2.46e+04	1.88e+04	1.33e+04	1.26e+04	2.39e+04	1.46e+04	3.15e+04	2.15e+04	1.28e+04	1.28e+04
	Dist	1.86e+03	2.71e+03	1.14e+04	2.47e+03	796.78	9.39e+03	5.82e+03	3.84e+04	2.44e+04	1.60e+03	1.74e+03

are no noticeable correlations. For the CS, there are six functions that may have some correlations between $\overline{\Delta}$ and '$Dist$'.

As demonstrated by our experiments, the relationship between $\overline{\Delta}$ and '$Dist$' for the DE-a algorithm is not very significant. The difference between the final solutions obtained by different initialization methods is due to the distribution characteristics of the initial population. In most cases, the PSO-w algorithm is not sensitive to the distribution range of the initial population. The average distance between the initial population and the real optimal has no positive correlation with the final solutions. The $\overline{\Delta}$ has a great influence on the performance of the CS algorithm. The closer the initial points are to the real optimal solution, the better the algorithm result. This may be part of the reason why the CS algorithm is sensitive to initialization. This is also in good agreement with the previous experimental results. On the influence of initialization on the performance of the three algorithms, we can conclude that CS > PSO > DE.

Table 22: The result of $\overline{\Delta}$ and $Dist$ on PSO-w.

Fun	Value	$Be(3,2)$	$Be(2.5,2.5)$	$Be(2,3)$	$U(0,1)$	$N(0,1)$	$N(0.5,1)$	$N(0.5,0.5)$	$logn(0,1)$	$logn(.69,.25)$	$logn(0,0.5)$	$logn(0,2/3)$
f_1	$\overline{\Delta}$	49.747	56.00	71.59	77.96	281.06	240.68	121.98	359.49	437.16	171.58	217.81
	Dist	2.55	19.89	20.26	18.88	20.69	19.93	19.38	18.62	20.02	17.10	15.12
f_2	$\overline{\Delta}$	120.28	150.15	167.03	69.78	151.88	142.40	149.09	333.41	209.28	78.02	78.00
	Dist	20.40	19.59	19.87	19.63	17.26	19.96	16.49	17.79	20.01	20.02	20.15
f_3	$\overline{\Delta}$	56.23	50.89	56.29	74.98	268.60	239.30	119.66	373.32	467.00	194.48	236.53
	Dist	1.90e-06	1.58e-06	1.70e-06	1.73e-06	1.71e-06	1.61e-06	1.80e-06	1.90e-06	1.83e-06	1.73e-06	1.87e-06
f_4	$\overline{\Delta}$	56.25	50.92	56.29	75.03	268.84	239.34	119.67	372.52	467.11	194.41	236.46
	Dist	20.35	14.14	20.71	17.02	17.67	15.93	17.12	17.87	16.28	22.05	19.11
f_5	$\overline{\Delta}$	6.75e+03	6.11e+03	6.75e+03	9.00e+03	3.22e+04	2.87e+04	1.44e+04	4.48e+04	5.60e+04	2.34e+04	2.84e+04
	Dist	6.25	5.78	5.22	6.87	5.15	7.94	4.67	5.47	7.24	8.22	6.36
f_6	$\overline{\Delta}$	1.12e+03	1.02e+03	1.12e+03	1.50e+03	5.37e+03	4.79e+03	2.39e+03	7.46e+03	9.34e+03	3.89e+03	4.73e+03
	Dist	0.15	0.03	0.16	0.08	0.10	0.10	0.09	0.08	0.09	0.19	0.15
f_7	$\overline{\Delta}$	112.52	101.83	112.55	150.02	537.84	478.52	239.50	748.06	934.13	389.29	472.24
	Dist	39.44	14.34	37.13	17.29	22.75	18.77	24.64	22.36	16.92	30.09	28.27
f_8	$\overline{\Delta}$	1.09e+03	1.02e+03	1.16e+03	1.50e+03	5.41e+03	4.79e+03	2.39e+03	7.41e+03	9.25e+03	3.82e+03	4.67e+03
	Dist	1.10e+03	1.02e+03	1.14e+03	1.47e+03	5.44e+03	4.87e+03	2.45e+03	7.09e+03	9.22e+03	3.74e+03	4.41e+03
f_9	$\overline{\Delta}$	9.68e+03	1.26e+04	1.56e+04	1.28e+04	3.34e+04	2.60e+04	1.60e+04	3.38e+04	3.41e+04	1.32e+04	1.87e+04
	Dist	6.22e+03	9.97e+03	1.39e+04	8.69e+03	4.01e+04	2.98e+04	1.84e+04	1.03e+05	3.65e+04	1.83e+04	3.03e+04

Fun		$E(0.5)$	$E(0.1)$	$E(0.8)$	$Rayl(0.4)$	$Rayl(0.8)$	$Rayl(0.1)$	$Weib(1,1.5)$	$Weib(1.5,1)$	$Weib(1,1)$	$random$	LHS
f_1	$\overline{\Delta}$	120.28	150.15	167.03	69.78	151.88	142.40	149.09	333.41	209.28	78.02	78.00
	Dist	20.40	19.59	19.87	19.63	17.26	19.96	16.49	17.79	20.01	20.02	20.15
f_2	$\overline{\Delta}$	220.66	240.79	333.92	126.32	338.74	224.82	317.97	690.47	427.24	149.94	150.00
	Dist	3.36e-04	0.36	0.26	0.23	0.56	0.43	0.11	0.32	0.50	0.09	0.17
f_3	$\overline{\Delta}$	110.38	120.43	166.65	63.18	169.11	112.39	158.93	344.79	213.83	74.99	75.00
	Dist	1.58e-06	1.65e-06	1.77e-06	1.47e-06	1.85e-06	1.79e-06	1.73e-06	1.86e-06	1.76e-06	1.73e-06	1.69e-06
f_4	$\overline{\Delta}$	110.39	120.39	167.03	63.18	169.16	112.39	158.89	345.13	214.05	75.02	75.00
	Dist	18.56	23.09	17.42	15.13	20.80	23.34	18.07	18.01	16.22	16.27	16.92
f_5	$\overline{\Delta}$	1.32e+04	1.44e+04	2.00e+04	7.58e+03	2.03e+04	1.35e+04	1.91e+04	4.14e+04	2.57e+04	9.01e+03	9.00e+03
	Dist	7.27	7.29	5.73	8.25	9.39	8.74	7.24	5.19	7.47	7.51	6.13
f_6	$\overline{\Delta}$	2.21e+03	2.41e+03	3.34e+03	1.26e+03	3.38e+03	2.25e+03	3.18e+03	6.89e+03	4.28e+03	1.50e+03	1.50e+03
	Dist	0.12	0.23	0.11	0.07	0.16	0.22	0.15	0.10	0.11	0.07	0.06
f_7	$\overline{\Delta}$	220.64	240.79	333.92	126.25	338.46	224.74	317.27	690.00	427.88	149.96	149.99
	Dist	25.05	43.34	22.18	18.09	31.77	45.85	29.09	19.63	23.20	16.61	18.17
f_8	$\overline{\Delta}$	2.23e+03	2.50e+03	3.34e+03	1.27e+03	3.32e+03	2.34e+03	3.15e+03	6.86e+03	4.26e+03	1.50e+03	1.50e+03
	Dist	2.19e+03	2.51e+03	3.80e+03	1.26e+03	3.18e+03	2.33e+03	3.11e+03	6.87e+03	4.18e+03	1.55e+03	1.51e+03
f_9	$\overline{\Delta}$	1.74e+04	2.46e+04	1.88e+04	1.32e+04	1.26e+04	2.39e+04	1.46e+04	3.13e+04	2.15e+04	1.28e+04	1.28e+04
	Dist	1.98e+04	1.19e+04	2.69e+04	1.01e+04	1.48e+04	1.62e+04	1.77e+04	4.58e+04	3.00e+04	9.07e+03	8.05e+03

5.2. Experiments on ABC and GA

The above experiments and analyses have focused on the three algorithms, and the conclusions have been drawn accordingly. In order to see if these conclusions are still valid for other algorithms, we have carried out more tests on two other algorithms: the artificial bee colony (ABC) algorithm [48] and the genetic algorithm (GA) [45].

By using the same 22 initialization methods mentioned above, we have carried out some numerical experiments on the original ABC algorithm for all 19 test functions with the dimensionality of $D = 30$. To make a fair comparison, the parameters of NP and $limit$ are set to 50 and $D \cdot NP$ respectively [63, 64]. Experiments of each initialization method have been executed independently for 20 times, and the maximum number of function evaluations (FEs) is set to 150000. In our experimental studies, the 'Best', 'Mean', 'Var' and 'Dist' values were recorded for the 9 basic functions to measuring the performance of the algorithm for each initialization method. In addition, the 'Best', 'Mean', 'Var' values were also recorded for the 10 CEC functions. Then, the experimental results are sorted out and analyzed in the same ways as discussed in the previous section, and the results of Friedman rank test of different initialization methods are given in Table 25.

Table 23: The result of $\overline{\Delta}$ and $Dist$ on CS.

Fun	Value	$Be(3,2)$	$Be(2.5,2.5)$	$Be(2,3)$	$U(0,1)$	$N(0,1)$	$N(0.5,1)$	$N(0.5,0.5)$	$logn(0,1)$	$logn(.69,.25)$	$logn(0,0.5)$	$logn(0,2/3)$
f_1	$\overline{\Delta}$	49.73	56.26	71.94	78.38	281.48	241.61	122.37	353.43	437.41	172.19	218.41
	Dist	9.86e-04	1.22e-05	1.87e-05	3.16e-05	0.20	0.10	0.10	0.15	114.56	0.21	0.02
f_2	$\overline{\Delta}$	113.18	101.92	112.73	149.55	537.84	478.92	240.10	751.52	929.21	392.43	476.58
	Dist	0.29	0.08	3.19e-14	0.0821	0.1307	0.08	3.23e-14	3.17e-14	276.49	0.18	0.08
f_3	$\overline{\Delta}$	56.28	50.79	56.12	75.29	270.07	242.34	120.46	361.59	469.61	193.96	239.33
	Dist	1.68e-33	9.69e-34	1.07e-33	1.23e-33	1.94e-33	1.38e-33	1.71e-33	3.11e-33	138.75	1.45e-33	2.11e-33
f_4	$\overline{\Delta}$	57.77	52.27	57.58	77.14	275.17	244.98	122.82	379.29	478.37	199.52	238.07
	Dist	7.47	6.71	7.20	7.94	9.39	7.99	8.24	9.19	145.21	10.08	9.19
f_5	$\overline{\Delta}$	6.79e+03	6.07e+03	6.75e+03	9.01e+03	3.18e+04	2.88e+04	1.44e+04	4.53e+04	5.57e+04	2.34e+04	2.85e+04
	Dist	5.49e-07	0.47	5.58e-07	5.28e-07	5.39e-07	5.63e-07	5.44e-07	5.56e-07	1.66e+04	5.56e-07	5.65e-07
f_6	$\overline{\Delta}$	1.13e+03	1.02e+03	1.13e+03	1.50e+03	5.42e+03	4.80e+03	2.41e+03	7.45e+03	9.34e+03	3.88e+03	4.68e+03
	Dist	0.41	0.14	0.50	0.55	1.03	1.21	0.72	1.08	3000	1.05	0.83
f_7	$\overline{\Delta}$	112.33	102.37	114.10	150.21	535.35	473.63	240.64	745.48	935.03	392.70	470.15
	Dist	52.99	32.62	48.38	62.53	92.89	93.18	74.23	127.28	297.82	130.38	136.33
f_8	$\overline{\Delta}$	1089.3	1019.1	1165.5	1510.4	5379.6	4753	2398.2	7335.7	9233.2	3834.6	4611.2
	Dist	1652.9	1484	1744.2	1771	1790.2	1913.5	1928	1859.1	1918.7	1913.9	1654.5
f_9	$\overline{\Delta}$	9669.3	12671	15598	12716	33576	26275	16028	33901	33986	13126	18727
	Dist	1486.8	4068.6	6337.3	3297.6	11811	5084	3071.5	48085	23146	415.86	9495.9

Fun		$E(0.5)$	$E(0.1)$	$E(0.8)$	$Rayl(0.4)$	$Rayl(0.8)$	$Rayl(0.1)$	$Weib(1,1.5)$	$Weib(1.5,1)$	$Weib(1,1)$	$random$	LHS
f_1	$\overline{\Delta}$	119.31	150.81	166.93	70.51	151.75	142.37	149.90	330.09	208.52	78.20	77.98
	Dist	0.10	0.20	0.10	1.87e-05	0.02	0.20	1.01	0.11	0.10	1.59e-05	4.34e-05
f_2	$\overline{\Delta}$	221.56	241.44	332.23	126.70	339.83	224.47	313.59	693.58	425.49	149.70	150.08
	Dist	0.13	0.08	3.18e-14	0.08	3.21e-14	3.09e-14	3.18e-14	0.08	3.25e-14	3.09e-14	3.15e-14
f_3	$\overline{\Delta}$	109.79	120.15	165.66	63.07	169.69	112.01	157.64	348.43	211.76	75.20	74.97
	Dist	1.68e-33	2.45e-33	1.01e-33	1.20e-33	1.77e-33	3.33e-33	1.71e-33	1.73e-33	1.61e-33	1.29e-33	1.66e-33
f_4	$\overline{\Delta}$	112.79	123.01	170.93	64.89	172.29	114.89	162.25	351.19	214.81	76.44	76.77
	Dist	8.68	10.10	6.69	6.83	9.65	9.63	9.35	8.73	8.33	7.11	8.51
f_5	$\overline{\Delta}$	1.33e+04	1.44e+04	2.01e+04	7.59e+03	2.03e+04	1.35e+04	1.89e+04	4.12e+04	2.57e+04	9.04e+03	9.00e+03
	Dist	5.48e-07	0.31	5.42e-07	5.31e-07	5.23e-07	5.37e-07	5.57e-07	0.76	5.50e-07	5.60e-07	0.38
f_6	$\overline{\Delta}$	2.19e+03	2.41e+03	3.35e+03	1.25e+03	3.39e+03	2.25e+03	3.21e+03	6.89e+03	4.23e+03	1.50e+03	1.50e+03
	Dist	0.76	0.95	0.72	0.29	0.69	1.09	0.83	0.89	0.82	0.41	0.43
f_7	$\overline{\Delta}$	221.64	240.06	339.18	127.31	340.17	224.67	315.12	691.94	422.38	150.08	150.06
	Dist	87.03	159.94	91.07	51.97	119.77	142.70	108.97	116.65	100.92	64.26	57.20
f_8	$\overline{\Delta}$	2215.8	2498.2	3377.4	1272.6	3337.9	2343.3	3107.9	6838.5	4289.5	1504.8	1501.4
	Dist	1863.4	1882.5	1793.1	1721.3	1826.3	1784.9	1864.2	1802.4	1869.5	1745.2	1783
f_9	$\overline{\Delta}$	17351	24632	18814	13227	12749	23862	14486	31290	21511	12858	12815
	Dist	4601	9859.8	3021.2	3756.6	697.04	10847	970.16	32208	9003.2	3756.6	2992.8

From Table 25, we can see that, for all 9 basic functions or CEC functions, the experimental results are basically the similar as before. Both the p-values are far less than 0.05, so the null hypothesis can be rejected, which indicates that ABC is greatly affected by initialization. For the ABC, the initialization methods: $Be(2.5, 2.5)$, $Rayl(0.4)$, LHS, $Be(3, 2)$ seem to lead to better performance.

Similarly, we have carried out the same numerical experiments on the GA. It is worth pointing out that there are many GA variants, and the variant used in this paper is to keep half of the population with better fitness to be passed onto the next generation. The mutation probability has been set to 0.1. The maximum number of iterations and the population size are 100 and 3000, respectively. Each initialization method has been executed independently for 20 times for all 19 test functions with $D = 30$. The experimental results are summarized in Table 26.

As indicated by the results in Table 26, the Friedman rank test shown that for the 9 basic functions, the p-value is 0.878, which is greater than 0.05. Thus, we can essentially conclude that, for these problems, different initialization methods have little influence on the GA algorithm. For the complex CEC functions, the p value is far less than 0.05.

Table 24: Correlation test.

Algorithm		f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9
DE-a	r	-0.163	0.003	0.368	-0.328	0.050	0.214	-0.054	0.117	0.872**
	p	0.47	0.988	0.092	0.136	0.825	0.338	0.813	0.604	0.000
PSO-w	r	-0.184	-0.179	0.469*	-0.104	-0.108	-0.056	-0.243	0.998**	0.752**
	p	0.411	0.425	0.028	0.646	0.633	0.803	0.275	0.000	0.000
CS	r	0.774**	-0.071	0.588**	-0.466*	0.126	0.727**	0.598**	0.397	0.774**
	p	0.000	0.759	0.004	0.033	0.586	0.000	0.004	0.074	0.000

Table 25: Friedman ranks of different initialization methods for the ABC algorithm.

basic	p	$Be(3,2)$	$Be(2.5,2.5)$	$Be(2,3)$	$U(0,1)$	$N(0,1)$	$N(0.5,1)$	$N(0.5,0.5)$	$logn(0,1)$	$logn(.69,.25)$	$logn(0,0.5)$	$logn(0,2/3)$
	0.00	1	3	5	7	17	12	9	16	21	15	18
		$E(0.5)$	$E(0.1)$	$E(0.8)$	$Rayl(0.4)$	$Rayl(0.8)$	$Rayl(0.1)$	$Weib(1,1.5)$	$Weib(1.5,1)$	$Weib(1,1)$	$random$	LHS
		13	22	8	2	14	20	10	19	11	6	4
CEC	p	$Be(3,2)$	$Be(2.5,2.5)$	$Be(2,3)$	$U(0,1)$	$N(0,1)$	$N(0.5,1)$	$N(0.5,0.5)$	$logn(0,1)$	$logn(.69,.25)$	$logn(0,0.5)$	$logn(0,2/3)$
	0.00	4	1	7	6	21	20	9	16	22	11	17
		$E(0.5)$	$E(0.1)$	$E(0.8)$	$Rayl(0.4)$	$Rayl(0.8)$	$Rayl(0.1)$	$Weib(1,1.5)$	$Weib(1.5,1)$	$Weib(1,1)$	$random$	LHS
		12	18	8	2	10	15	14	19	13	5	3

This shows that the GA is affected by initialization methods when problems are complex. However, the most appropriate initialization methods for the GA are Bate distribution, LHS, $random$, Uniform distribution.

5.3. Findings and implications

The main contribution of this work is to study systematically the influence of different initialization approaches on algorithms, so as to gain some insight on this topic. Based on the extensive simulations and statistical tests, we can now discuss our findings and their implications.

One surprise finding is that some algorithms are more sensitive to initialization than others. However, such sensitivity can also be problem-dependent. For example, differential evolution is not quite sensitive to initialization, while particle swarm optimization, cuckoo search and artificial bee colony algorithm are greatly affected by initialization. In addition, the genetic algorithm is less sensitive to initialization for many easy and smooth functions, but it becomes more sensitive to initialization for highly complex functions. Another surprise finding is that the commonly used technique in terms of uniform distributions for initialization is not necessarily the best approach. For example, for the PSO, our recommendation is to use the random, beta distribution and LHS as the main initialization methods. But for the CS, the preferred initialization methods are the beta distribution, exponential distribution, and Rayleigh distribution.

In addition, the population size can also have a significant effect. For the PSO, a larger population size is usually required, while a smaller population with more iterations can give better results for the DE. However, only a very small population size is sufficient for the CS. Furthermore, the above conclusions may also depend on the objective landscapes and thus may be problem-dependent, the correlation between the initialization methods and the premature convergence is very weak. Consequently, as long as the diversity of the

Table 26: Friedman ranks of different initialization methods for the GA algorithm.

basic	p	$Be(3,2)$	$Be(2.5,2.5)$	$Be(2,3)$	$U(0,1)$	$N(0,1)$	$N(0.5,1)$	$N(0.5,0.5)$	$logn(0,1)$	$logn(.69,.25)$	$logn(0,0.5)$	$logn(0,2/3)$
	0.88	2	12	9	17	19	22	1	8	6	4	7
		$E(0.5)$	$E(0.1)$	$E(0.8)$	$Rayl(0.4)$	$Rayl(0.8)$	$Rayl(0.1)$	$Weib(1,1.5)$	$Weib(1.5,1)$	$Weib(1,1)$	$random$	LHS
		5	10	20	21	3	14	15	18	11	13	16
CEC	p	$Be(3,2)$	$Be(2.5,2.5)$	$Be(2,3)$	$U(0,1)$	$N(0,1)$	$N(0.5,1)$	$N(0.5,0.5)$	$logn(0,1)$	$logn(.69,.25)$	$logn(0,0.5)$	$logn(0,2/3)$
	0.00	3	5	4	2	22	21	8	18	20	17	16
		$E(0.5)$	$E(0.1)$	$E(0.8)$	$Rayl(0.4)$	$Rayl(0.8)$	$Rayl(0.1)$	$Weib(1,1.5)$	$Weib(1.5,1)$	$Weib(1,1)$	$random$	LHS
		9	15	11	7	10	14	12	19	13	1	6

population is high enough and the iterations are long enough, the optimal solutions can be found by all the algorithms.

Though these findings are preliminary, they do have some interesting implications. Firstly, for a given new algorithm, some parametric study is always needed to see if it is sensitive to initialization and its population size. Secondly, different initialization methods, especially a combination of uniform distributions and long-tailed distributions such as the exponential distribution and Rayleigh distribution should be explored. Finally, different types of benchmark problems with different properties should be used to validate new algorithms, especially those with multimodal and optima-shifted functions.

Despite the above findings, we have not focused on how the selection mechanism of an algorithm may influence the diversity of the population in later iterations. In addition, almost all real-world problems have nonlinear constraints. We have not considered if the handling of constraints may affect the above findings. These will form the topics for further research.

6. Conclusions

Initialization has some significant influence on the performance of an algorithm for a given set of problems. In the current literature, the widely used initialization methods are the random methods, uniform distributions and LHS. However, there is no systematic comparison for different initialization techniques. In this paper, we have compared 22 different initialization methods based on different probability distributions for five algorithms over a set of 19 diverse benchmark functions. Based on our simulations and analyses, we can draw some conclusions:

- The accuracy of the metaheuristic algorithm is closely related to two parameters: the population size and the maximum number of iterations. However, the dependence of different algorithms on their population sizes is different. Under the same conditions (the same total number of function evaluations), the DE-a algorithm usually requires a small population size with a larger number of iterations. A clear advantage of the CS is that it requires even a smaller population size, typically less than 100. On the other hand, the population size for the PSO-w should be larger, in comparison with both the DE and CS.

- The DE-a algorithm is not particularly sensitive to initialization, which means that DE is more robust. On the other hand, the PSO-w performs differently for different initialization methods, and the most appropriate initialization methods are random, beta distribution, and LHS. This may explain why many PSO variants performed well for uniform random initialization in the literature. Similarly, the CS is also sensitive to different initialization methods, and the most suitable methods for general problems are the beta distribution, followed by the Rayleigh and uniform distributions. For more complex problems, the most suitable initialization methods of the CS algorithm are the exponential distribution and Rayleigh distribution. Similarly, the ABC algorithm is sensitive to initialization, while the GA is less sensitive. However, such sensitivity can be problem-dependent. For example, the sensitivity of the GA is low for easy and smooth functions, while this sensitivity increases significantly for highly complex functions.

- The average distance between the initial population and the real optimal solution does not have any significant correlation with the quality of the final solution for the algorithms. That is to say, the final solutions obtained are not usually affected by the locality of the optimal solutions. Thus, as long as the diversity of the population is high and the number of iterations is large, all these algorithms are capable of finding the optimal solutions.

The above conclusions are preliminary, and different algorithms can have different performance for different initialization methods and for different problems. Thus, there is a strong need to figure out what is the best initialization method for a given algorithm for a given set of problems. The present work paves a way for further investigation for a vast number of algorithms exist in the current literature. However, there are still some issues that need to be addressed so as to gain further insight into different algorithms and different initialization strategies. These can form the topics for further research.

- For more complex problems, two or more distribution methods can be used as a hybrid, so as to enhance the overall diversity of the population. This should be tested using rigorous statistical techniques to see if they can indeed affect the statistical properties of the population of an algorithm at different stages of iterations.

- In all our tests, we have used the benchmark problems with simple bounds on regular domains. It would be useful to test more complicated problems with nonlinear constraints on irregular search domains to see if the same conclusions still hold. Such problems can be drawn from real-world applications.

- An automatic and self-adaptive method can be developed to automatically find the most suitable initialization method(s) for a given type of problems with a given algorithm. This may be attempted by following a similar approach as the self-tuning algorithm framework [65]. It is hoped that this work can inspire more studies concerning algorithm performance, robustness and different initialization techniques.

Acknowledgement

This work was supported by the National Natural Science Foundation of China (Grant Nos.61877046 and 61877047), Shaanxi Provincial Natural Science Foundation of China (Grant No.2017JM1001).

Declaration of interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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