# A Review of Hough Transform and Line Segment Detection Approaches

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Keywords: Hough Transform, Line Detection, Line Segmentation.

Abstract: In a wide range of image processing and computer vision problems, line segment detection is one of the most critical challenges. For more than three decades researchers have contributed to build more robust and accurate algorithms with faster performance. In this paper we review the main approaches and in particular the Hough transform and its extensions, which are among the most well-known techniques for the detection of straight lines in a digital image. This paper is based on extensive practical research and is organised into two main parts. In the first part, the HT and its major research directions and limitations are discussed. In the second part of the paper, state-of-the-art line segmentation techniques are reviewed and categorized into three main groups with fundamentally distinctive characteristics. Their relative advantages and disadvantages are compared and summarised in a table.



## 1 INTRODUCTION

Straight-line detection and segmentation are the techniques that allow a machine to extract linear features from a digital image or sequence of images. Extracted features, can later be used either by human or machine to make decisions. Extensive applications of line detection can be found in object recognition (Olson, 2001), shape detection (Ayala-Ramirez et al., 2006), power line detection (Zhang et al., 2012), and road or lane detection (Borkar et al., 2012; S.Rahmdel et al., 2013a). However, despite the significant progress, there is still a rising demand for methods that are swift in computation and precise in segment detection in real-world applications. In fact, there is always a trade-off between these two parameters, i.e. speed and accuracy, as they are two sides of the same coin.

The Hough transform (HT) (Hough, 1962; Duda and Hart, 1972) is indeed one of the most popular methods for the detection of linear and curvilinear structures. It has demonstrated a robust behaviour to variant noise and degraded environment. This paper consists of two main parts. In the first part, the HT and its major research directions and limitations have been discussed in Section 2 and 3 respectively. In the second part, state-of-the-art line segmentation techniques have been reviewed and categorized in three main groups in Section 4. This provides an unique insight to the advantages and disadvantages of each approach. Section 5 concludes the paper.

## 2 HT RESEARCH DIRECTIONS

In general, the HT maps every pixel in a binary image from its 2D Cartesian coordinates $(x,y)$ to a new 2D coordinate space $(\rho,\theta)$ using the mapping function

$$\rho = x\cos\theta + y\sin\theta \tag{1}$$

The new coordinate space is called Hough space, also known as parameter space, and the process is called mapping from image or feature space to data or parameter space. We classify major research directions of the HT into four group as discussed in the following.

### 2.1 Determine Optimal $\rho-\theta$ Resolution

It is important for both the accuracy and computational efficiency to determine a sufficient resolution for discretising the Hough space. The higher the resolution the slower the process. Numerous works have tackled the problem such as using gradient direction (van Veen and Groen, 1981), interpolation in the Hough space (Niblack and Petkovic, 1988), dynamically quantised spaces (O'Rourke, 1981), trial and error (Niblack and Petkovic, 1988), sensitivity function (Svalbe, 1989), using information from the discretisation of image space (Zhang, 1996), error propagation (Ji and Haralick, 2001), weight of accumulation (Nguyen et al., 2008), and non-uniform discretisation (Duan et al., 2007).

S. Rahmdel P., Comley R., Shi D. and McElduff S..
A Review of Hough Transform and Line Segment Detection Approaches.
DOI: 10.5220/0005268904110418
In *Proceedings of the 10th International Conference on Computer Vision Theory and Applications* (VISAPP-2015), pages 411-418
ISBN: 978-989-758-089-5

### 2.2 Enhance Computational and Memory Efficiency

A large number of distinct variations of the HT were introduced to advance its performance, mainly focusing on computational complexity and memory efficiency of the standard Hough transform (Kiryati and Bruckstein, 1991; Walsh and Raftery, 2002; Shapiro, 2006; Fernandes and Oliveira, 2008). Representative works that appear in most of the related literature include Stephens's probabilistic Hough transform (PHT) (Stephens, 1991), progressive probabilistic Hough transform (PPHT) (Matas et al., 2000), randomized Hough transform (RHT) (Xu et al., 1990; Xu and Oja, 1993), hierarchical Hough transform (HHT) (Princen et al., 1990), elliptical Gaussian kernel-based Hough transform (KHT) (Fernandes and Oliveira, 2008), and regularized Hough transform (Aggarwal and Karl, 2006).

### 2.3 HT Peak Detection

A peak isolation algorithm can simply detect the local maxima in an $N \times N$ neighbourhood in the Hough space (Thrift and Dunn, 1983), given an odd number for $N$. But determining the optimal value of $N$ can be problematic. The larger the value of $N$ the higher the probability of missing adjacent lines, and the smaller the value of $N$ the higher the probability of repetitious detection. (Princen et al., 1990) used an iterative global peak detection approach where the globally highest peak is detected first and its corresponding feature points get eliminated. A HT is then applied and the process repeats iteratively until all the peaks are found. Peak detection using self-organising maps were suggested to reduce the memory requirement of the HT (Choy et al., 1995). More recent works focused on generating solid and distinguishable peaks and the ways to search and identify the true peaks with minimal false detection (Aggarwal and Karl, 2006; Shapiro, 2006; Fernandes and Oliveira, 2008; Chao et al., 2009). These have major problems, in particular memory inefficiency and computational complexity. Detection accuracy is directly proportional to the memory capacity.

### 2.4 HT Butterfly Analysis

HT butterfly analysis considers the peak and the area surrounding the peak. The term butterfly refers to the shape of the peak and its associated sinusoidal curves. The combination of sinusoids that represent the line segment form a butterfly shape around the peak. In fact, such butterflies contain valuable information about a segment's length, endpoint and thickness. Because every single pixel of a line contributes to create its HT butterfly in a one to one mapping process, the resulting butterfly is capable of providing highly accurate line-segment parameters. Efforts have been made to parameterise the HT butterfly. Representative works include line segment descriptors (Atiquzzaman and Akhtar, 1995), multi-segmentation (Kamat-Sadekar and Ganesan, 1998), peak enhancement using butterfly features (Ji et al., 2011; Furukawa and Shinagawa, 2003), HT neighbourhood (Du et al., 2010; Du et al., 2011), butterfly self-similarity (Tu et al., 2011), and butterfly symmetry (Du et al., 2012).

## 3 HOUGH TRANSFORM LIMITATIONS

In general, there are three main problems associated with the existing HT-based line detection methods:

1. **Computational Complexity.** Despite the efforts to improve the efficiency of the HT, its heavy computational cost is still a major concern in real-time applications.
2. **Edge Detection.** The HT is applicable only to binary images, hence accurate edge detection is an important prerequisite. In addition to imposing extra computational burden, edge detection may increase the false detection error; when the noise-level is high, some of the true feature points in an image can be ignored as noisy pixels and also some false points may be recognised as true feature points.
3. **Segment Detection.** The HT is incapable of specifying the endpoints of a line segment. It can identify only line that pass through the entire image. Therefore segmentation procedures have to be adapted in order for the HT to identify the endpoints of a segment.

A number of works addressed the edge detection problem of the HT using Fourier-based HT techniques (Ho et al., 2000; Shi et al., 2010; Zheng and Shi, 2011). One of the recent advances of this approach is line detection through the high-resolution parameter space of multi-layer fractional Fourier transform (MLFRFT) that results in higher accuracy of the HT in comparison with the representative techniques (Shi et al., 2010). Multiple instances of fractional Fourier transform of the same image provide more frequency samples that lead to higher accuracy and better performance. Higher accuracy can be achieved by extending the method to a more customizable Fourier spectrum for each $x$ and $y$ axes using the generalised interpolated Fourier transform (GIFT) (Zheng and Shi, 2011). Although the GIFT suggests a more flexible frequency grid than the MLFRFT, it does not increase the number of frequency samples. Furthermore, without an adaptable parameter tuning method, the GIFT suffers from tedious trial and error tuning for parameter adjustment.

As mentioned earlier, the HT and its extension are powerful tools to detect straight lines but not line segments. These techniques are incapable of detecting line segments and their endpoints. They can identify only the location and orientation of the straight lines passing through the segments. Therefore, an additional segmentation procedure needs to be performed to extract the segments from the detected lines. In the following section, which forms the second part of this paper, we classify the existing line segmentation methods, including non-HT-based methods, through an extensive survey of various approaches.

# 4 LINE SEGMENTATION APPROACHES

Popular line segmentation methods used in practical applications can be classified into three major groups, namely, *bottom-up*, *top-down*, and *domain-crossing* approaches. The bottom-up approach starts with single pixels, which grow to segments; whereas the top-down approach extracts straight lines followed by segmentation. Domain-crossing methods take into account both local features, from a bottom-up, and global features, from a top-down approach. In the following, different characteristics of these approaches will be surveyed.

## 4.1 Bottom-up Approach

A bottom-up approach, also known as local approach, typically begins from the pixel level and the line grows pixel by pixel to reach the requirement of a true line segment defined by the algorithm. These approaches mainly use gradient information to draw the line. State-of-the-art methods include line segment detector (LSD) (von Gioi et al., 2010), Edge Drawing (EDLines) (Akinlar and Topal, 2011), and two-orthogonal direction image scanning (TODIS) (Yang et al., 2011). Other works use a small matrix of eigenvalues (Guru et al., 2004; Koeck and Zhang, 2002).

One of the first bottom-up frameworks was introduced in (Nevatia and Babu, 1980). The algorithm starts with a convolution-based edge detection followed by a line thinning and threshold. Afterwards, a linear approximation is used to link the edge points based on their gradient orientations. Khan et al. extended the concept using a connected component algorithm (CCA) (Kahn et al., 1990). A CCA groups together adjacent feature points with similar gradient orientation into line support regions. The algorithm was simple and fast, and hence later became the core of some of the well-known bottom-up algorithms such as (Burns et al., 1986) and (von Gioi et al., 2010).

The LSD utilises iterative *region-growing* process (Burns et al., 1986). Using image gradient magnitude and angle, each region starts from a pixel by setting the region's angle to the pixel's gradient direction. In the next iteration the algorithm compares the gradient direction of adjacent pixels (in an eight-pixel neighbourhood) with the region angle. If any neighbouring pixel shares a similar angle within a certain tolerance it will be added to the growing region. Pixels belonging to a particular region will be labelled to avoid revisiting. In the second phase of the LSD, Desolneux et al.'s (Desolneux et al., 2000) approach was used to validate the result. LSD has proven to be a breakthrough in line segmentation. However, there are two major problems involved with the LSD's performance: 1) it is highly sensitive to noise that misleads the region-growing process in connecting the true segments; 2) it loses accuracy when facing dense intersecting straight lines.

The TODIS partially addressed the LSD's shortcomings but pays the price of computational time. Unlike the LSD, TODIS works with binary images, thus edge detection has to be applied in the first place. An image is examined in both horizontal and vertical directions. A multi-scale scanning approach was taken to label each candidate line segment. There are some issues due to the multi-scale nature of TODIS. In smaller scales a long segment will mistakenly appear as a number of short segments. Similarly, in larger scales a number of collinear short segments may appear as one continuous long segment. In addition, unlike the LSD, in TODIS each pixel may be visited more than once and that has a negative effect on the computational time.

The EDLines suggests a faster algorithm for line segment detection. In terms of segmentation accuracy, the results of EDLines are similar to the LSD's; however, the EDLines processes the same image ten times faster than the LSD. That makes the EDLines a perfect candidate for real-time applications. In their algorithm Akinlar and Topal used the concept of *Edge Drawing* (ED) (Topal and Akinlar, 2012) to produce an accurate edge map. The underlying idea is to use image gradient information to connect the edge pixels that belong to the same segment. Unlike the other

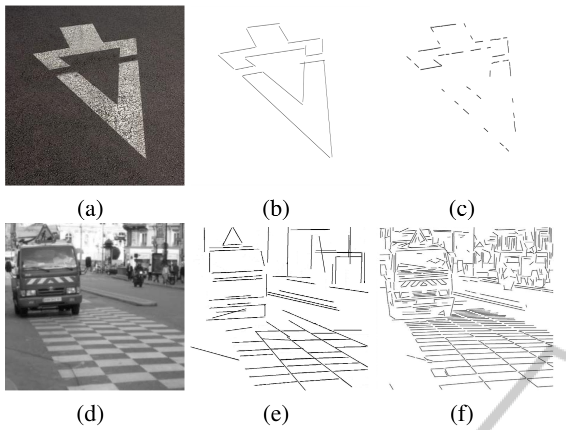


Figure 1: Performance of the bottom-up versus top-down method. (a) Original image of arrow (512×512). (b) Result of the EHT where long segments are correctly detected. (c) Result of the LSD shows disconnections in the segments due to background noise. (d) Original image of truck (911×850). (e) Result of the EHT when it fails to detect many of the short segments. (f) The LSD successfully detects segments as small as a few pixels length.



edge detection approaches such as the Canny edge detector that generates a binary edge map consisting of arbitrary pixels, ED results in a number of related and joint edge pixels in a shape of edge segments. This pre-processing step plays a key role in EDLines' success by reducing the error caused by broken segments. However, for the next step, i.e. line segment extraction they applied a Least Square Line Fitting approach that causes inaccuracy in noisy situations and breaks down the long segments.

In general, bottom-up approaches are computationally simple and easy to implement. Plus, their local nature is well-suited for taking short line segments into account. However, their local characteristic fails to maintain the robustness in challenging situations such as when line segments intersect or when there are rather long segments in an image. Due to the effect of noise and image resolution such long segments appear as a series of disconnected short segments. This sensitive behaviour in response to noise appears to be the main drawback of the bottom-up approaches. Using global information can help solve such problems. The top-down approach looks at the problem from a rather different perspective.

## 4.2 Top-down Approach

In the top-down approach, also known as the global approach, the true straight-lines are firstly extracted before they are broken down into segments. Most of the proposed HT-based line segmentation methods use the parameter space information during and after the HT voting process. For instance, the connective HT (CHT) utilises a probabilistic approach to investigate the connectively of the feature points (Yuen et al., 1993). After applying the HT, a fixation point is selected using the information obtained during the accumulation to vote for two 1D accumulators. The CHT suggest a faster computation than the SHT. To capture segment coordinates using the fast Hough transform (FHT), Guil et al. (Guil et al., 1995) kept aside those feature points that collaborate during the voting process. By arranging the image points belonging to the same straight-line in a respective order, coordinates of the segment endpoint can be captured by selecting the points that are further away from each other. Nevertheless, the method is not perfectly designed for line segments with a negative slope smaller than one or line segments that are collinear.

The PPHT outperforms the SHT in terms of speed but results in lower accuracy and a large number of missing lines (i.e. false negative). Moreover, it requires a large set of parameter adjustments such as threshold that have to be delicately tuned. The problem with the accuracy of the PPHT was addressed later in (Nguyen et al., 2008) at the expense of a high computation and memory requirement.

The extended Hough transform (EHT) optimized the traditional 2D HT with a third parameter (Cha et al., 2006). In this 3D representation of the HT, each individual column or row of the image space is plotted to a unique dual 2D HT. Later in (Chung et al., 2009), an optimized algorithm was proposed to reduce the execution time during the voting process. However, similar to all other HT-based methods, it requires prior edge detection to enhance the linear features that brings additional computational time. In addition, edge detection may cause inaccuracy in segmentation performance by neglecting true segments due to inappropriate threshold selection.

Global searching algorithms embedded in the HT-based approaches showed a better robustness to variant noise. Furthermore, in the case of long segments, a global view helps to detect long lines as a single continuous segment, unlike the local approaches that break them down. As a more tangible example, this is similar to looking through a window with scratched glass from a distance without noticing the blurriness caused by the scratches. However if you get closer to the glass, the scratches will become more noticeable. Let us compare the performance of the bottom-up and top-down approaches using natural images to better understand their differences. As a significant limitation, due to its global nature, the HT usually fails to detect short segments. Because short lines generate smaller clusters in the accumulator array and, therefore, it is more difficult to distinguish these small peaks from the surrounding peaks.

### Bottom-up Vs. Top-down

Performance of the above two approaches can be better explained in Figure 1. The first row shows an example where the global approach outperforms the local method. The image of an arrow degraded with Gaussian noise has been tested with the EHT, as a representative of the top-down approach, and the LSD, as a representative of the bottom-up approach in Figure 1(b) and (c), respectively. Although not flawless, the EHT detects the segments, especially the long lines, with an acceptable accuracy. In contrast, due to the background noise the LSD fails to detect the true long segments despite the simplicity of the image. Instead, it produces a number of collinear and disjointed segments. The additive noise stops the growing region from expansion before reaching the true endpoint. However, in an image with rather complex texture, without additive noise, in the second row, i.e. Figure 1(d), the LSD performs better by detecting most of the segments with a greater detail, see Figure 1(f). In contrast, the global nature of the HT-based approach is incapable of detecting many of the short segments as shown in Figure 1(e). This is when the LSD outperforms the HT-based approach.

From the machine learning viewpoint, the top-down method is a model selection technique from parameter space to data or feature space, whereas the bottom-up method is a regularisation technique from data or feature space to parameter space. The optimal solution can be obtained using a two pathway learning process, in which both top-down model selection and bottom-up regularisation are considered. This leads us to the third category of the methods which is the domain-crossing approach.

## 4.3 Domain-crossing Approach

The domain-crossing approach utilises the information from both the image and parameter space. It starts by mapping the feature points from the image to the parameter domain and again using the image-domain data combined with the parameter domain data to capture the line segments.

Song and Lyu speed up the HT voting process using image gradient prediction (Song and Lyu, 2005). After peak detection in Hough space, a unique line verification method is used by crossing from Hough space over to the image space. In this way, line thickness can also be distinguished as well as achieving a reduction in false detection. Despite its accuracy the method is computationally expensive.

Bandera et al. suggest a more efficient algorithm in (Bandera et al., 2006). A random window randomised Hough transform is used in the global phase to construct the Hough space and capture the line parameters, i.e. the peaks. A unique mean shift clustering technique is utilised afterwards to highlight the peaks and find the potential straight-lines. Crossing over to the image space, edge pixels that are aligned with the detected lines are projected onto the lines to approximate the true line segments in a local merging procedure. The idea was further improved in slice sampling weighted mean shift (SSWMS) analysis (Nieto et al., 2011). Sequential sampling of the Hough space was suggested to enhance the random sampling approach as well as a new likelihood function for the local clustering. The SSWMS shows a better accuracy than the PPHT with faster execution time.

In (Berlemont and Olivo-Marin, 2010), the beamlet transform has been viewed as a special case of the Radon transform (RT). The underlying idea is to apply a feature-adapted RT in a quadtree decomposition manner to obtain a feature-adapted beamlet transform (FABT). The term *feature-adapted* comes from the convolution of basis filters (i.e. second- or third-order edge and ridge detectors) with the corresponding image scale prior to RT computation. This filtering highlights linear structures of the image and enhances the accuracy and performance of the task. However, the FABT is also computationally expensive because all of the convolutions must be applied in the time domain due to the indefinability of the convolution theorem for the pseudo-polar Fourier transform.

Unite-and-divide (UND) approach to the Radon transform also uses the unique property of Fourier slice theorem, however, in a more efficient way (Shi et al., 2013). Unlike the FABT, which uses pseudo-polar Fourier transform, the UND uses multi-layer fractional Fourier transform to compute a higher resolution Radon sinogram. The peaks of the sinogram will later be divided into a number of sub-spaces for segment detection. A unique windowing approach was suggested in (S.Rahmdel et al., 2013b) to isolate the target segment and decompose the Radon butterfly of each segment separately. The proposed segmentation approach outperformed the LSD and EHT in terms of the robustness to noise.

A new definition of the HT neighbourhood was introduced in (Du et al., 2010; Du et al., 2011). The neighbourhood of a line segment in the parameter domain is defined in a lozenge-like quadrangle to approximate the neighbourhood of the segment in the image domain. Instead of two endpoints, authors used

Table 1: An overview of the different approaches to line segment detection.

| Methods | Description | Advantages | Disadvantages |
|---|---|---|---|
| Bottom-up | Region growth from pixel to segment. Representative works include (Nevatia and Babu, 1980), (Kahn et al., 1990), (Burns et al., 1986), (von Gioi et al., 2010), (Akinlar and Topal, 2011), (Yang et al., 2011) | Fast and simple, appropriate for real-time applications | Sensitive to noise, breaks down lengthy segments |
| Top-Down | Straight-line detection, followed by segment extraction. Representative works include Hough transform family, (Yuen et al., 1993), (Guil et al., 1995), (Matas et al., 2000), (Nguyen et al., 2008), (Cha et al., 2006), (Chung et al., 2009) | Robust to noise, suitable for detecting large objects | Slow, inaccuracy in extracting short lines |
| Domain-Crossing | Decomposition of lines through transformed space. Representative works include (Song and Lyu, 2005), (Bandera et al., 2006), (Nieto et al., 2011), (Berlemont and Olivo-Marin, 2010),(Du et al., 2010; Du et al., 2011), (S.Rahmdel et al., 2014; S.Rahmdel et al., 2013b), (Shi et al., 2013) | Robust to noise, more accurate for short segments | slow |

the centre point of a segment. However, this method suffers from the traditional problem of the HT-based techniques, i.e. detecting short line segments, where we face a trade-off in the neighbourhood radius selection. That means segments with shorter length have bigger approximation errors in comparison to those with longer length. In addition, the neighbourhood approach can result in wrong segment endpoint when the number of intersecting lines increase in the image. A noise elimination algorithm was proposed in (S.Rahmdel et al., 2014) as a vital extension to the neighbourhood approach to remove the negative effect of non-collinear segments. First, the algorithm identifies those segments that have a common intersection point within the image space. Second, it filters out the pixels that are not collinear with the target segment to significantly increase the detection accuracy in comparison with the neighbourhood approach. Nevertheless, due to its computational cost, this method cannot be seen as an appropriate candidate for real-time video processing.

# 5 CONCLUSION

Table 1 gives an overview of the different approaches to line segment detection discussed in this paper.

The traditional point-to-segment extraction methods are computationally efficient and simple to implement. That makes such methods suitable for real-time object detection and tracking applications. Nevertheless, their focus is on local information such as gradient magnitude and angle at pixel level. That makes noise an effective parameter in degrading their performance. In addition, failing to consider global information results in discontinuity in the case of elongated segments.

The traditional line-to-segment extraction methods work the other way around. Their global nature reduces the effect of noise and increases their robustness. That is also the reason that global approaches can detect large objects more accurately. However, neglecting the local gradient information in an image causes inaccuracy in extracting short line segments. In addition to that, the computational burden of the global methods is considerable and such methods are more appropriate for off-line image processing.

The domain-crossing approaches benefit from different properties in both image space and transformed space to identify the line segments. They are more robust to noise and occlusion due to their global nature as well as having higher accuracy for short line segmentation because of their image-domain line segmentation algorithms. However, they require a delicate design of algorithm to speed up the line extraction process, and it is not well-suited for real-time applications.

In conclusion, It is important that the user thoroughly understands the problem in order to make the best use of the available algorithms. Authors hope that this research could highlight some of the key features of existing detection algorithms and encourage the continuation of the work in this open research question.

# ACKNOWLEDGEMENTS

Authors would like to acknowledge MITACS for providing the support needed for this research under the Elevate Fellowship.

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