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Discrete-Time Analysis of a Cyclic Service System with Gated Limited Service

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Abstract
This study deals with cyclic service systems with gated limited service, where each queue has a fixed, but individual limit. An approximate discrete-time analysis for the cycle-length and waiting-time distribution for such systems is presented, considering general renewal input traffic, service time and switch-over time distributions. To catch the correlation between the state processes of the queues, the weighted sum of the cycle-time distribution of an uncorrelated and a perfectly correlated system are used, as in [6]. Thus whole distributions are calculated, higher moments and quantiles can be obtained. Numerical results are given to show the accuracy of the approximation.

1 Introduction

Cyclic service systems have been widely used as models for computer communication systems. A comprehensive survey can be found in [8]. In the following we review some recent literature dealing with the gated limited service policy:

In [9] a 1-limited cyclic service system with general service and switch-over time distributions has been investigated. Job arrivals follow a Poisson process and the queues are assumed to be of finite capacity. By using the method of embedding a Markov chain, approximate results which are also valid for non-symmetrical load conditions have been derived for blocking probabilities and mean waiting times. The assumption that job arrivals must follow a Poisson process was relaxed and renewal processes have been taken into account in [10]. By using discrete convolution operations based on the fast Fourier transform, approximations for blocking probability, cycle time distribution, and mean waiting time have been derived. Under the same modelling assumptions as in [9] the exhaustive k-limited cyclic service system has been investigated in [5]. In [1] pseudo-conservation laws for cyclic service systems with gated/exhaustive limited service and non-zero switch-over times have been derived. It was assumed that jobs arrive as a Poisson process and that the queues are unlimited. A probabilistically-limited service policy for a cyclic service system with infinite queues, Poisson arrival process, and general service and switch-over time distributions was analyzed in [7]. The limits for the number of served jobs at each queue within one service cycle are given by an arbitrary distribution. Therefore, the service policies exhaustive and exhaustive k-limited are included as special cases. By using a numerical technique based on Fourier transforms the queue-length and waiting time distributions have been obtained approximately. The mean waiting times of jobs for the exhaustive limited and gated limited service policies have been approximated in [2] and [3] under the assumptions of Poisson arrivals of jobs, infinite queues, and general distributions for the service and switch-over time distributions. In [6] a cyclic service system with limited service (the number of jobs that shall be served in the next service cycle is prereserved and bounded), Poisson arrival process, and deterministic service time has been investigated by an iterative approximation. The queue-length and sojourn-time distributions are given as approximations.

In this paper we present a discrete-time analysis under more general assumptions as made for the investigations described above. The results are approximate but comparison with simulation results shows that they are extremely accurate.

The paper is organized as follows. In Section 2 we describe the investigated queueing system. The iterative algorithm for determining performance measures like the cycle time distribution and waiting time distribution is presented in Section 3. Numerical results are presented in Section 4 and Section 5 concludes the paper and contains a short outlook.

2 Queueing system

We consider a cyclic queueing system with N infinite capacity queues in discrete-time domain. After a specific queue is served, the server takes a non-zero switch-over time before it serves the next queue. A service cycle is over, if all N queues have been served once. Time is slotted and it is assumed that jobs arrive at slot boundaries. Jobs which arrive at a specific queue are stored until this queue is served. We take into account the gated limited service policy. Only a limited number of jobs per queue are served within a service cycle. Jobs arriving to a specific queue after start of service at this queue are not served within the current service cycle. The following discrete random variables (RV) are used:

$\boldsymbol{S}_i$ switch-over time between queue i and $(i+1)$ MOD N, distribution $\boldsymbol{s}_i(\boldsymbol{k})$

$\boldsymbol{A}_i$ job inter-arrival time for queue i, distribution $\boldsymbol{a}_i(\boldsymbol{k})$

$\boldsymbol{B}_i$ service time for one job of queue i, distribution $\boldsymbol{b}_i(\boldsymbol{k})$

$\boldsymbol{X}_i$ system size for queue i, distribution $\boldsymbol{x}_i(\boldsymbol{k})$

$\boldsymbol{L}_i$ constant limit for number of served jobs from queue i within a service cycle

$\boldsymbol{C}$ cycle length, distribution $\boldsymbol{c}(\boldsymbol{k})$

The queueing system and its notations are illustrated in Fig.1.

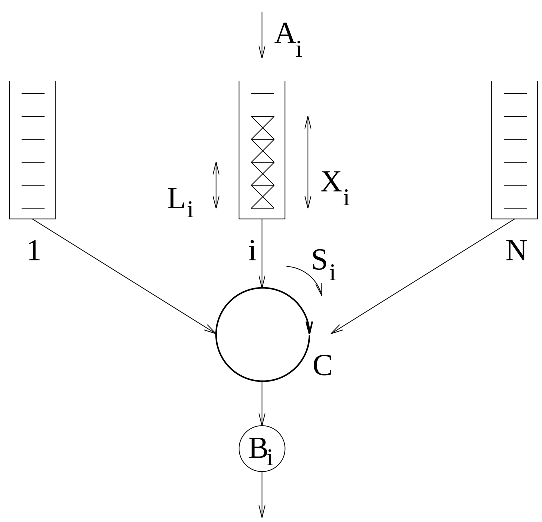

Figure 1: *Basic queueing model.*

The traffic intensity at queue i is given as $\rho_i = \mathrm{E}[B_i]/\mathrm{E}[A_i]$. The overall traffic intensity is $\rho = \sum\limits_{i=1}^{N} \rho_i$. The queueing system is stable, if the mean number of arrivals at queue i during a cycle is smaller than the limit L_i (cf. [1], [4]):

$$\frac{\sum\limits_{i=1}^{N} \mathrm{E}[S_i]}{\mathrm{E}[A_i](1-\rho)} < L_i \quad \text{for } i = 1, 2, \ldots, N \text{ and } \rho < 1. \tag{1}$$

3 Discrete-Time Analysis

In this analysis we consider discrete-time RVs, i.e the time axis is divided into intervals of unit length Δt. Thus the distribution of e.g. S_i is given by $s_i(k) = P(S_i = k \cdot \Delta t)$ for $k = 0, 1, 2, \ldots$. The RVs X_i and L_i are discrete RVs representing a number of jobs. We observe the system always immediately before slot boundaries.

We denote deterministic distributions by $\delta(k-t)$, which is defined as

$$\delta(k-t) = \begin{cases} 1 & \text{if } k = t \\ 0 & \text{otherwise} \end{cases} . \tag{2}$$

To calculate the distribution of a RV U with a lower bound m or an upper bound M, we use the operators π_m and π^M, which are defined as follows:

$$\pi_m[u(t)] := \begin{cases} 0 & t < m \\ \sum\limits_{\nu=-\infty}^{m} u(\nu) & t = m \\ u(t) & t > m \end{cases} \quad \text{and} \quad \pi^M[u(t)] := \begin{cases} u(t) & t < M \\ \sum\limits_{\nu=M}^{\infty} u(\nu) & t = M \\ 0 & t > M \end{cases} . \tag{3}$$

The distribution of the sum of two independent RVs, which is the discrete convolution of their distributions, denoted by operator $\circledast$, is defined as

$$(y \circledast z)(k) := \sum_{i=-\infty}^{\infty} y(i) \cdot z(k-i). \tag{4}$$

Furthermore, $u^{(j)}(k)$ denotes the j-fold convolution of the distribution $u(k)$ with itself and $u^{(0)}(k) := \delta(k)$.

3.1 Iterative Algorithm

We observe the state process of each queue, denoting the number of waiting jobs X_i, and the cycle length C. On the one hand, the number of the waiting jobs X_i determines the cycle length, but on the other hand, the number of new job arrivals during a cycle depends on its length. Thus, we use the following iterative algorithm similar to that used in [10]:

0) Initialize the state processes of the queues $X_{i,0}$, e.g. by setting the system size to zero, and the cycle time C_0, e.g. deterministically.

1) Calculate $X_{i,n+1}$ and C_{n+1} from $X_{i,n}$ and C_n for $n = 0, 1, 2, \ldots$ as described in Section 3.2 and 3.3.

2) Repeat step 1) until a convergence criteria is fulfilled.

In the further sections we assume for step 1) that the switch-over times S_i and the service times B_i are independent from the queue states $X_{i,n}$ and the cycle length C_n, [1] which is fulfilled in many applications. ¿From the calculated distributions $c(k)$ and $x_i(k)$ we can derive other performance measures, as e.g. waiting time distribution.

3.2 State Process of the Queues

In this section we calculate $X_{i,n+1}$ from $X_{i,n}$ and C_n for each queue i. To simplify the notation we omit the index i. The number of jobs waiting in queue i when cycle $n+1$ starts is the sum of the number of jobs beyond the limit at the start of cycle n, denoted by X_n^+ and the new arrivals during cycle n denoted by G_n with distribution $g_n(k)$:

$$X_{n+1} = X_n^+ + G_n \tag{5}$$

The number of jobs served during cycle n is $\min(X_n, L)$. Thus, we get:

$$X_n^+ = X_n - \min(X_n, L) = \max(0, X_n - L) \tag{6}$$

The difference of a discrete RV and a constant value L means in terms of distributions a shift by L units:

$$\widetilde{x_n^+}(k) = x_n(k+L), \qquad \text{for } k = -L, -L+1, \ldots \tag{7}$$

$\widetilde{X_n^+}$ might become negative, thus we get the distribution x_n^+ by lower bounding it to 0, i.e. applying the operator π_0 (cf. eq. (3)) on $\widetilde{x_n^+}(k)$:

$$x_n^+(k) = \pi_0[\widetilde{x_n^+}(k)] = \pi_L[x_n(k+L)] \tag{8}$$

The number of arrivals during the cycle n can be calculated from $a_i(k)$ and $c_n(k)$ [10]. First, we determine the RV $F^{(j)}$ for the time until the jth job will arrive after an observed time instance (here the start of cycle n):

$$F^{(j)} = A^r + \underbrace{A + \cdots + A}_{(j-1)\text{ times}} \tag{9}$$

where A^r denotes the forward recurrence time of the inter-arrival time A, which is distributed according to

$$a^r(k) = \frac{1}{EA}\left(1 - \sum_{\nu=0}^{k} a(\nu)\right), \quad k = 0, 1, 2, \ldots \tag{10}$$

[1] To investigate systems with dependencies we additionally would have to calculate $S_{i,n}$ and $B_{i,n}$ for each iteration step n.

Thus, we can obtain the distribution of $F^{(j)}$:

$$f^{(j)}(k) = (a^r \circledast a^{(j-1)})(k) \quad \text{for } j = 1, 2, \ldots \tag{11}$$

If we consider that no job arrives in zero time units and a cycle of length $m > 0$, we get the following conditional distributions [10]:

$$\begin{aligned} (g|C_n = 0)(k) &= \delta(k) \\ (g|C_n = m)(k) &= \sum_{i=0}^{m-1} (f^{(k)}(i) - f^{(k+1)}(i)) \quad \text{for } m = 1, 2, \ldots \end{aligned} \tag{12}$$

Finally, we can obtain the distribution of the number of new arrivals during the last cycle by applying the law of total probability:

$$g_n(k) = c(0)\delta(j) + \sum_{m=1}^{\infty} c(m) \sum_{i=0}^{m-1} (f^{(k)}(i) - f^{(k+1)}(i)) \quad \text{for } k = 0, 1, \ldots \tag{13}$$

Thus, we can calculate the conditional distributions $(x|C_n = m)_{n+1}(k)$:

$$(x|C_n = m)_{n+1}(k) = \left(x_n^+ \circledast (g|C_n = m)_n\right)(k) \tag{14}$$

Finally, we get:

$$\begin{aligned} x_{n+1} &= \sum_{m=0}^{\infty} c_n(m) \cdot (x_n^+ \circledast (g|C_n = m)_n)(k) \\ &= \left(\sum_{m=0}^{\infty} c_n(m) \cdot x_n^+ \circledast \sum_{m=0}^{\infty} c_n(m) \cdot (g|C_n = m)_n\right)(k) = (x_n^+ \circledast g_n)(k) \end{aligned} \tag{15}$$

3.3 Distribution of the cycle-length

After calculating the state distribution for all queues, we are able to derive the length of cycle $n+1$. Since all queues have seen the same realization of C_n, we have to use the conditional distributions $(x_{n+1}|C_n = m)_i(k)$ (cf. eq. (14)). To simplify the notation in the following we omit the index $n+1$ except for C due to the dependency of C_{n+1} on C_n.

At most L_i of the waiting jobs in queue i $(X|C_n = m)_i$ are removed. Thus, queue i contributes $(\overline{X}|C_n = m)_i := \min((X|C_n = m)_i, L_i)$ jobs. Since L_i is deterministic, we can use π^{L_i} to calculate the distribution of the minimum:

$$(\overline{x}|C_n = m_i)(k) = \pi^{L_i}[(x|C_n = m)_i)(k)] \tag{16}$$

The service time distribution for j jobs is $b_i^{(j)}$. Thus, we get the conditional length of the cycle segment contributed by queue i, denoted by $(CS|C_n = m)_i$, with its distribution:

$$(cs|C_n = m)_i(k) = \left(\sum_{j=0}^{\infty} (\overline{x}|C_n = m)_i(j) \cdot b_i^{(j)} \right)(k) \tag{17}$$

Under the assumption that the cycle-segments $(CS|C_n = m)_i$ and the switch-over times S_i are independent, we get the conditional cycle length distribution $(c_{n+1}|C_n = m)$(k):

$$\begin{aligned}(c_{n+1}|C_n = m)(k) &= \Big(\big((cs|C_n = m)_1 \circledast s_1\big) \circledast \cdots \circledast \big((cs|C_n = m)_N \circledast s_N\big) \Big)(k) \\ &= \left(\left(\bigcircledast_{i=1}^{N} (cs|C_n = m)_i \right) \circledast \left(\bigcircledast_{i=1}^{N} s_i \right) \right)(k)\end{aligned} \tag{18}$$

Finally, we get for the distribution of C_{n+1}:

$$c_{n+1}(k) = \Big(\sum_{m=0}^{\infty} c_n(m) \cdot (c_{n+1}|C_n = m) \Big)(k) \tag{19}$$

Unfortunately, the states of the queues and consequently the cycle segments are correlated. Therefore we use, as in [6], the weighted sum of the cycle-length distributions $c_{n+1}(k)$ (cf. eq. (19)) for an uncorrelated system and for a perfectly correlated system. I.e. for a symmetric system each RV CS_i has the same realization, denoted by $\widetilde{c_{n+1}}(k)$. A generalization for non symmetrical systems can be found in [6]. Using this weighted sum, denoted by $\widehat{c_{n+1}}(k)$, instead of $c_{n+1}(k)$, we can improve the accuracy of our results (cf. Section 4).

$$\widehat{c_{n+1}}(k) := (1-p) \cdot c_{n+1}(k) + p \cdot \widetilde{c_{n+1}}(k) \tag{20}$$

In [6] some requirements for p are discussed; for our work we have chosen:

$$p = \rho^{\frac{1}{N-1}} \left(\rho^N (1 - \frac{\rho}{N}) + \frac{\rho}{N} \right) \tag{21}$$

3.4 Waiting-time distribution

The waiting-time of a particular job in queue i is the time interval from its arrival until it is removed from queue i. To simplify the notation we omit the index i. We assume that the observed job arrives T time units after the start of the current cycle of length $m \geq T$. Due to the gated service every job has to wait until the next cycle starts. Then it has to wait until all jobs are served, which are still in front of it. These consist of two kinds of jobs:

1. jobs that arrived before the start of the current cycle but are not served within it due to the limit L. The distribution of their number x^+ can be obtained according to eq. (8).

2. jobs that arrived within the current cycle before the observed job. The distribution of their number $(G|T = t)$ can be calculated by using eqs. (10), (11) and (12).

Thus the number of jobs, which has to been served prior to the observed job, $(\widehat{X}|T = t)$ is distributed according to $(\widehat{x}|T = t)(k) = \big(x^+ \circledast (g|T = t)\big)(k)$.

Obviously, it lasts $(\widetilde{W}|T = t) = \lfloor(\widehat{X}|T = t)/L\rfloor \cdot C + (\widehat{X}|T = t)(\mathrm{mod}L) \cdot B$ time units to serve these jobs. We can calculate the distribution of the number of cycles and service times as follows:

$$\lfloor(\widehat{x}|T = t)/L\rfloor(k) = \sum_{j=k\cdot L}^{(k+1)\cdot L-1} (\widehat{x}|T = t)(j) \quad \text{for } k = 0, 1, 2, \ldots \tag{22}$$

$$(\widehat{x}|T = t)(\mathrm{mod}L)(k) = \sum_{j=0}^{\infty}(\widehat{x}|T = t)(k + j \cdot L) \quad \text{for } k = 0, 1, \ldots, L - 1 \tag{23}$$

Thus, we obtain the distribution of $(\widetilde{W}|T = t)$:

$$(\widetilde{w}|T = t)(k) = \left(\left(\sum_{j=0}^{\infty}\lfloor(\widehat{x}|T = t)/L\rfloor(j)c^{(j)}\right)\circledast\left(\sum_{j=0}^{\infty}(\widehat{x}|T = t)(\mathrm{mod}L)(j)b^{(j)}\right)\right)(k) \tag{24}$$

Obviously, $(\widetilde{w}|T = t)$ is independent of C. The conditional waiting-time distribution, denoted by $(w|T = t \wedge C = m)$, is the sum of $(\widetilde{W}|T = t)$ and the time to the start of the next cycle, which is distributed according to $\delta(k - m + t)$. Since a convolution with $\delta(k - m + t)$ results just in a shift of $m - t$ time units, we get:

$$(w|T = t \wedge C = m)(k) = \begin{cases} 0 & \text{for } k < m - t \\ (\widetilde{w}|T = t)(k - m + t), & \text{for } k \geq m - t \end{cases} \tag{25}$$

The probability to observe a system t time units after the start of the current cycle is given by the backward recurrence time distribution of C denoted by $c^r(t)$:

$$c^r(t) = \frac{1}{E[C]}\left(1 - \sum_{\nu=0}^{k-1} c(\nu)\right) \quad \text{for } t = 1, 2, \ldots \tag{26}$$

The probability, that a cycle has a length of at least t time units, is:

$$P\{C \geq t\} = \sum_{\nu=t}^{\infty} c(\nu) = 1 - \sum_{\nu=0}^{t-1} c(\nu) = \mathrm{E}[C]c^r(t) \tag{27}$$

Now, we can obtain the waiting-time distribution by applying the law of total probability:

$$\begin{aligned} w(k) &= \sum_{t=1}^{\infty} c^r(t) \cdot \sum_{m=t}^{\infty} c(m) \cdot \left(\sum_{\nu=t}^{\infty} c(\nu) \right)^{-1} \cdot (w|T = t \wedge C = m)(k) \\ &= \mathrm{E}[C]^{-1} \sum_{t=1}^{\infty} \sum_{m=t}^{\infty} c(m) \cdot (w|T = t \wedge C = m)(k) \end{aligned} \tag{28}$$

4 Numerical results

For the implemention of the iterative algorithm we have used FFT standard routines to calculate convolutions of distributions in a more efficient way. The convergence speed depends strongly on the local utilization of the queues. We need about 20 to 40 iterations until the algorithm converges. Only for a few configurations we have to run more than 100 iteration steps (e.g. the configuration with limit $L = 1$ in Fig. 2 with $\rho_{local} = 0.75$). The most time consuming procedure is to calculate the number of arrivals in a given interval. If we can use a known distribution, as the Poisson distribution for negative exponential interarrival times, the calculation times reduce considerably. In most cases the discrete-time algorithm runs more than 10 times faster as the simulation for validating. For all simulation results depicted in the following the 95% confidence intervals' width are smaller than 10% of the estimated mean values.

We have chosen the following configuration to compare the accuracy of the algorithm according to eq. (20) with $p = 0$ and p chosen as in eq. (21): A system with 8 queues, a mean service time of $\mathrm{E}[B] = 1$, a total switch-over time of $S = 2\mathrm{E}[B] = 2$ and an offered load of $\rho = 0.75$. All queues have the same limit L. The arrival process is a discretized Poisson process, thus we can calculate the diagrams in a reasonable time. The service time is deterministic, since this shows the effects of the calculation according to eqs. (20) and (21) most clearly.

Fig. 2 shows the improvement of the accuracy, if we calculate the cycle-length distribution according to eq. (20). With the chosen p (cf. eq. (21)) we slightly overestimate the second moment, whereas the algorithm clearly underestimates it for $p = 0$. We can also observe that the second moment increases and approaches asymptotically the value for a gated system without limits.

Fig. 3 illustrates the influence of the perfectly correlated system. Since all cycle-segments has the same realization, the components $c(S + i \cdot N)$ are increased, which can be observed as a steep slope of the complimentary cycle-time distribution function at $t = S + i \cdot N\Delta t$ (marked by vertical lines). On the other hand, all other components of $c(k)$ are decreased. Thus, the complementary distribution function of $\widehat{c}(k)$ decreases slower than the simulation results for other values of t. Therefore, we can observe several cross-overs of this curves. For a wide range of parameters we get quite accurate approximations for the moments and quantiles of the cycle-time, although the calculated distribution follows only roughly the real distribution (cf. Fig. 3).

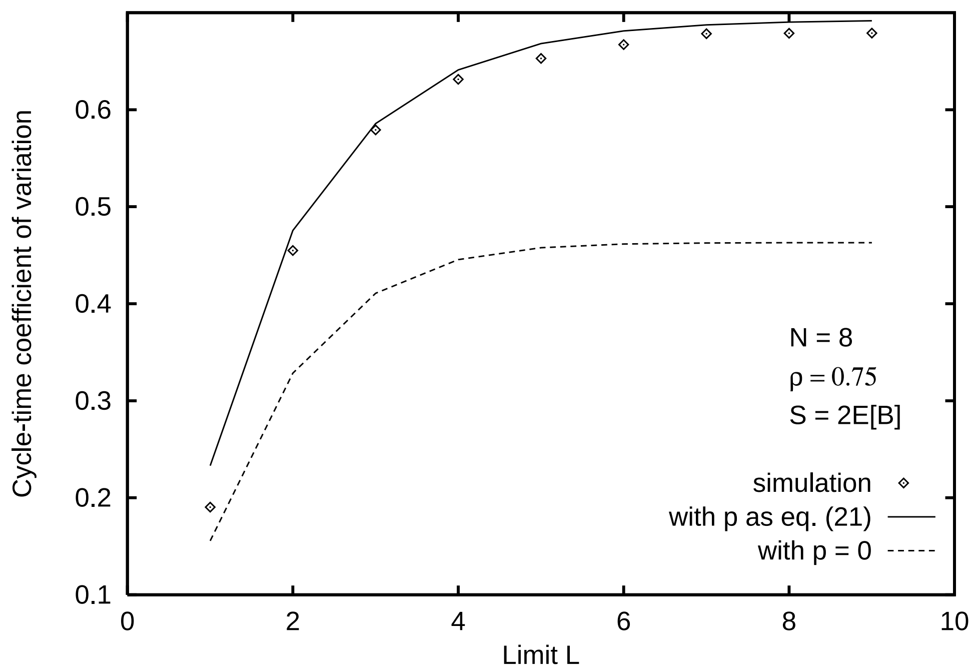

Figure 2: *Influence of the limit on the cycle-time coefficient of variation.*

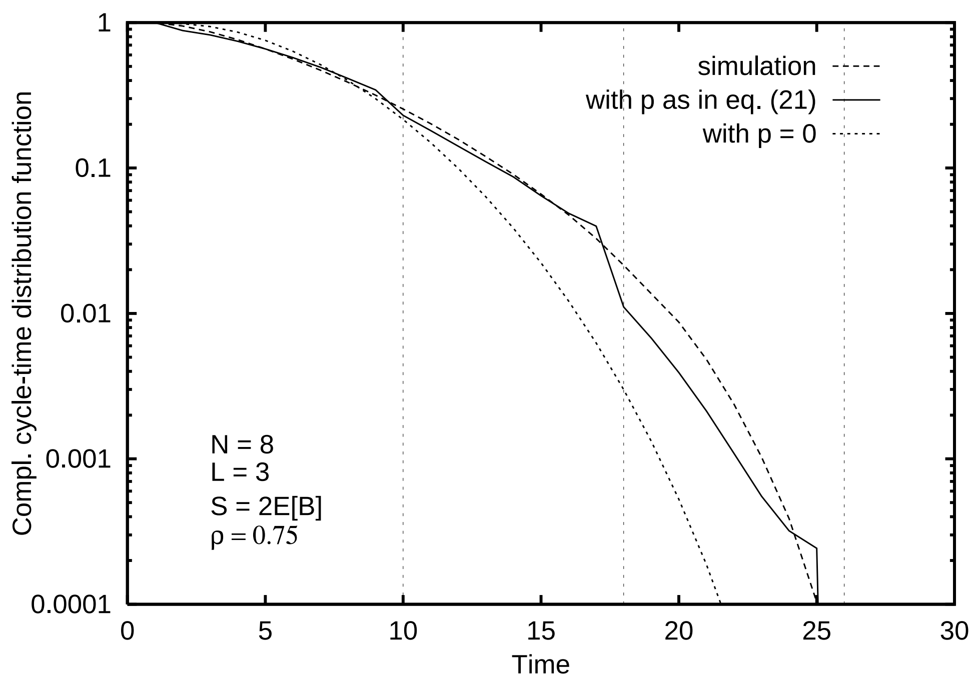

Figure 3: *Cycle-time distribution.*

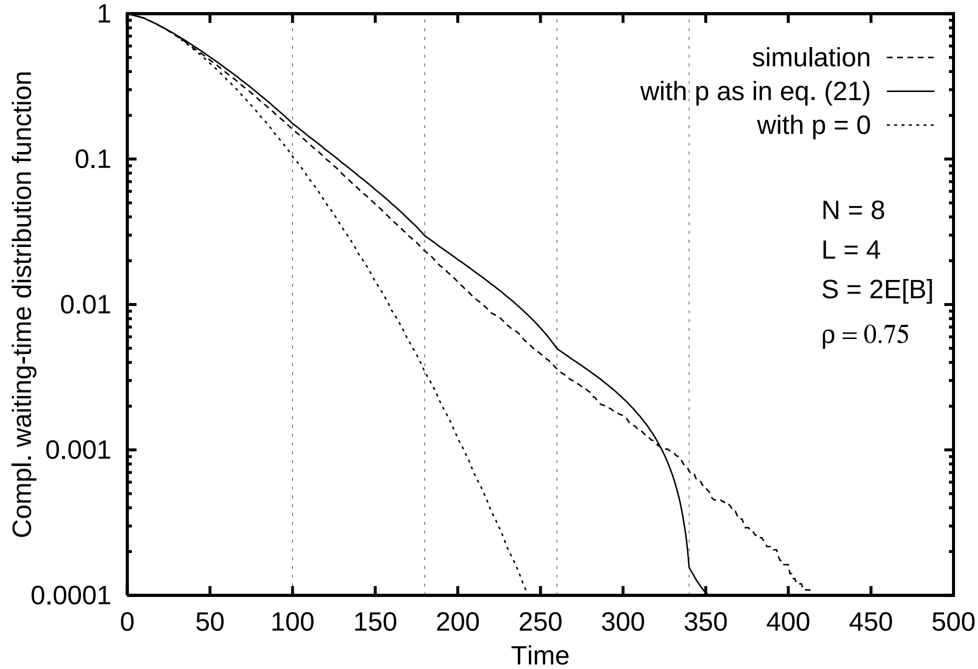

Figure 4: *Waiting-time distribution.*

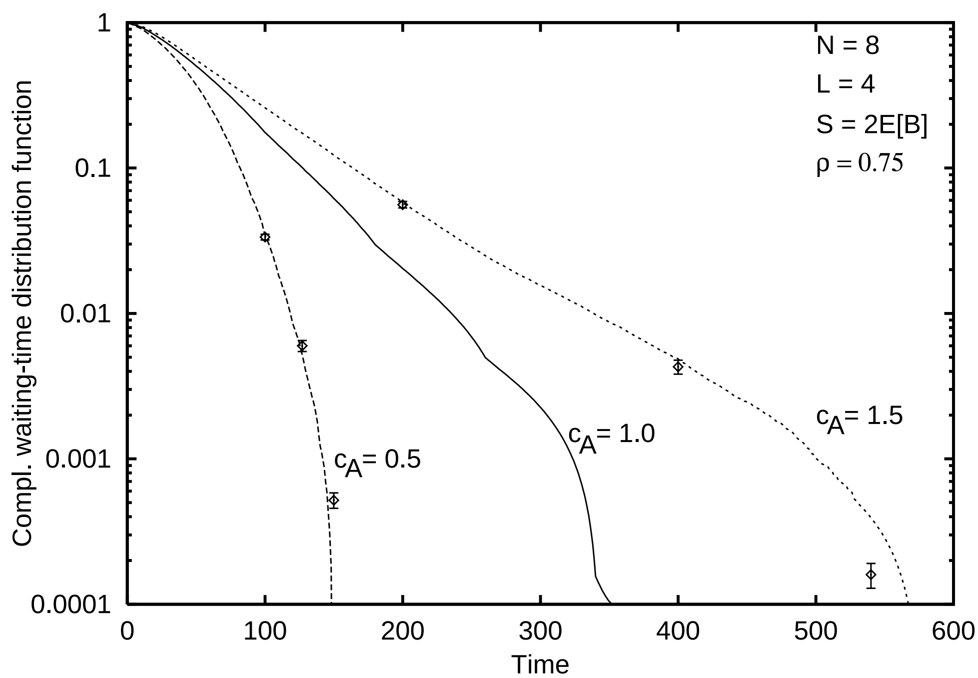

Figure 5: *Influence of arrival process on the waiting-time distribution.*

An example for the waiting-time distribution is given in Fig. 4. Here the time was normalized to $\Delta t = \mathrm{E}[B]/10$. The influence of the perfectly correlated system can be observed as a change of the slope at $t = (S + i \cdot N) \cdot 10\Delta t$ (marked by vertical lines). Since the waiting-time depends on the cycle-time, the approximation according to eqs. (20) and (21) shows again the better results. We get rather accurate results for the moments and quantiles of the waiting-time distribution, which might be very helpful for the dimensioning of the server capacity.

The influence of the arrival process is depicted in Fig. 5. We use a discretized Erlang distribution with 4 phases and hyperexponentiel distribution with two phases for the input traffic with the coefficient of variation $c_A = 0.5$ and $c_A = 1.5$, respectively. As expected the waiting time increases with c_A. We get very accurate approximations for the 99%-quantile and the first two moments of the waiting-time distributions.

5 Conclusion and outlook

An approximate algorithm for cyclic service systems with gated limited service strategy, where each queue has a fixed but individual limit, is presented. The discrete-time analysis bases on the evaluation of discrete convolutions, which can be efficiently calculated using the fast Fourier transform. General renewal input traffic and service times are considered. Thus the whole cycle-time and waiting-time distributions are derived, higher moments and quantiles can be approximated. To cope with the correlation between the queues, the weighted sum of the cycle-time for an uncorrelated and a perfectly correlated system are used [6]. Numerical results, validated by simulation, show the accuracy of the approximation.

The possibility to generalize the algorithm for distributed limits and for finite capacity queues is currently investigated. Furthermore the choice of p in eq. (20) is under study.

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