CASE STUDY

Authentic by design: developing mathematicians for the talent economy

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Abstract

The graduate skills expected of mathematics students from employers has changed over the past decade. Traditionally, mathematics graduates are recognised for their logical approaches, critical thinking and analytical skills as well as their ability to solve complex problems. The nature of the employment market is also changing with many industries increasingly seeking digital and technology-driven employees. Digitally fluent graduates with a broad skill set are highly sought after. For mathematics programme teams this mean that these skills need to be updated for the modern economy. Previously Middlesex University's mathematics programmes have embedded problem solving and communication skills in two modules. In this case-study we will outline how the programme team has developed our approach to teaching these skills to enhance students' skills.

Keywords: Problem solving, authentic assessment, Moore's method, students as partners, best of old and new.

1. Introduction

The landscape of both the employability skills expected from mathematics students and university education in general has changed drastically since the BSc Mathematics and BSc Mathematics with Computing programmes at Middlesex University were first validated in 2013. Increasingly mathematics graduates have pursued careers in areas related to data science and financial technology (Prospects, 2021). These careers place a greater emphasis on the requirement for applicants to be technologically literate, so they can disseminate technical material to a non-technical audience, as well as the ability to collaborate and be creative.

While traditional mathematics students are viewed as being highly skilled in critical thinking and problem solving, graduates often find it difficult to demonstrate and evidence their creativity and ability to communicate complex ideas and concepts. Authentic assessment has been defined by Guliker et al. (2004) as 'an assessment requiring students to use the same competencies, or combinations of knowledge, skills, and attitudes that they need to apply in the criterion situation in professional life'. This raises questions relating to what authenticity means for mathematics assessment and how programme teams can design schemes that provide tangible opportunities for students to evidence these skills. The use of authentic assessment is a key institutional priority and it is noted that there is an increased emphasis on authentic assessment within the sector (Pitt and Quinlan, 2022).

Emerging from the pandemic, universities are embracing blended and hybrid approaches, and enhancing learning, teaching and assessment using technology and digital tools. These competing demands have placed undergraduate mathematics programmes in a challenging position. Space must be found in already crowded curriculums for new material to ensure that graduates are suitably equipped for this new employment landscape. However, ways must be found to make assessment among even the purest mathematics modules, such as analysis and algebra, more authentic.

In this case study we will discuss the approaches we employed to teaching undergraduate mathematics students to support the development of skills that are valued and needed for employment within the talent economy. This approach draws on the best of the old and new: retaining mathematical rigour, creative problem solving, and construction and communication of arguments, whilst considering how each of these can be evidenced and demonstrated for a graduate mathematician in the current employment context.

Key elements of the approach include:

- An intellectually demanding mathematics curriculum;
- Problem solving and communication themes;
- A learning, teaching and assessment strategy that is flexible, inclusive and supported by technology;
- The use of portfolios to evidence professional skills.

This has resulted in the team articulating what authentic now means for mathematics and incorporating innovative teaching and assessment methods supported by technology to address these demands.

Sections 2 and 3 will outline the structure of the Problem Solving Methods and Communicating Mathematics modules, which were stand-alone core modules embedded within the original undergraduate mathematics programmes at Middlesex in 2013. These modules were created to support the development of employment skills in an authentic mathematical context. The associated module learning outcomes include ‘effectively work in a group to find solutions to problems’ and ‘demonstrate knowledge of how to communicate and motivate advanced mathematical topics through a variety of mediums’, respectively.

In section 4 we will discuss how the approaches evolved from the original design and validation in 2013 to the revalidation in 2020, including how we adapted assessment on the programmes and, more generally, based on what we learned from our approaches to problem solving and communication. This approach combines the best of the old and new: retaining mathematical rigour while incorporating innovative teaching and assessment methods to address these demands.

2. Problem Solving

When the undergraduate mathematics programmes were first validated in 2013, they were designed to explicitly incorporate elements of problem solving and communication. Both skillsets had their own dedicated modules. The module Problem Solving Methods at level 5 is discussed in detail in Jones and Megeney (2019). The module does not introduce new mathematical content, instead students apply mathematical and quantitative knowledge developed in other modules and from their broader experiences to solve mathematical problems. The teaching is inspired by Freudenthal’s ‘Realistic Mathematics Education’, see Freudenthal (1968, 1973) which emphasise the usability of mathematics as a focus for its development in teaching. Freudenthal talks about ‘mathematizing’ problems to solve them. We take a broader view of ‘usability’ to include common themes in mathematical arguments as well as the usual notions of applicable and authentic mathematics. At the start of workshops students are given a problem to work on in groups, sometimes specific mathematical problems, sometimes word problems that students need to mathematize. The tutor facilitates discussions with minimal but judicial input. When students have solved the problem, the class reflects on the approaches used with the aim of developing an understanding of the cognitive process that they use to understand and solve problems more generally. Stepwise approaches that are developed by students are linked to classic work by Polya (1957) as well as more modern approaches (Mason et al., 2010), (Bransford and Stein, 1993) and we use these texts to formalise students with their own internal understanding of the process of problem solving. The problem, solution, reflection cycle then starts again. Workshops are designed so that problems discussed have similarities and commonalities and students are encouraged to make links between these in class.

For example, students might be asked to explain why a number is divisible by 3 (or 9) if and only if the sum of its digits are divisible by 3 (or 9) – this requires mathematizing the problem. Or they might be asked to revisit examples from their first-year modules such as showing that $8^n + 13$ is always divisible by 7, or $5^n - 1$ is always divisible by 4. These latter problems are given in the first year and students solve them using mathematical induction – however there is a deeper reason they are true, for example $5^n - 1$ modulo 4 is equivalent to $1^n - 1 = 0$. The students then discuss - and the reader is encouraged to do the same - the relationship between these problems and the first problem in this paragraph. Commonalities such as the reduction to modulo arithmetic then become part of the students’ problem-solving arsenal. These problems are authentic from the point of view of solving abstract mathematics problems, but other workshops include problems that have real-life interpretations.

The group coursework consists of an open-ended question, with some parts that hint at directions to study (as shown in Figure 1), and some that are entirely open. Students must formulate these problems mathematically and demonstrate a creative and critical understanding of the topic of the coursework; they then develop strategies to study their problems and attempt to solve them. Since the module does not include new mathematical content, the students are assessed entirely on their creativity and engagement with the problem-solving process. Mathematical calculations and arguments are given some credit, of course, but the emphasis is on originality, creativity, and reflecting on problem solving skills rather than just solving the problem. One advantage of this approach to the programme design is that module learning outcomes deal specifically with problem solving skills and so students can explicitly evidence where these problem-solving skills have been developed and assessed in the degree. Furthermore, it gives the programme team the opportunity to focus specifically on developing these skills in in an authentic way. More details of how the type of problems are chosen can be found in Jones and Megeney (2019) where the authors introduce themes to discuss common approaches to mathematical problem solving. Students find this approach very helpful with one graduate commenting recently on how it has contributed to their professional development:

> “*Being forced to think more creatively when trying to formulate a solution and becoming more patient when doing so has helped me not only in a professional setting but it has also helped me in a day-to-day basis also. The variety of complex topics I came across throughout the degree helped me become more adaptable when being introduced to new topics professionally*”.

2. From 1998 onwards a number of control measures have been introduced. Your group needs to develop and study models that capture the following

   (a) Trapping and removing grey squirrels — two scenarios are proposed:

      i. a quota is fixed and grey squirrels are removed throughout the year at a rate to meet this quota;

      ii. traps are laid throughout the year and all grey squirrels are removed that are caught;

   (b) Red squirrels are re-introduced from habitats in Scotland — for a given number of years a quota is fixed and red squirrels are introduced throughout the year at a rate to meet this quota.

   Your aim is to develop models and to study the effect of changing the values of the parameters on the long-term sustainability of the red-squirrel population.

Figure 1: Sample question from Problem Solving coursework

3. Communication

The ability to effectively communicate and disseminate mathematics to a non-technical audience is becoming increasingly sought after by employers. In recognition of this we embedded and developed communication related learning, teaching and assessment activities throughout the undergraduate programme as well as in a core Communicating Mathematics module at level 6.

Traditionally communication related assessment may take the form of formal presentations, proofs and reports. When designing the Communicating Mathematics module, we expanded on these traditional communication mediums to include assessment activities which allow students to create mathematical videos, blogs, vlogs, flyers, and activity sheets.

Student feedback indicated that they valued having elements of choice within assessment tasks, but some opted for more traditional forms of assessment as their access to technology was limited when at home. Some students indicated that they only had access to suitable technology to complete these different forms of assessment on campus. This is particularly important for Middlesex given the demographics of our students. For example, our TEF (Teaching Excellence Framework) 4-year aggregate data for all modes of study, shows that 59.7% are from households that are located in neighbourhoods in the first or second quintile of the Index of Multiple Deprivation (IMD), an aggregate index used by the UK government to measure deprivation, which is 20 percentage points higher than the sector average. Furthermore, 43.6% were eligible for free school meals, compared to an average of 18% across all registered higher education institutions which is a 25.6 percentage points difference. This means that many of our students are affected by digital poverty.

Prior to the pandemic mathematics lecturing staff were equipped with iPads as part of departmental technology enhanced learning project called iF (iPads for feedback). The aim of the project was to enable staff to provide quicker more useful feedback on mathematical or notation heavy technical assessment. In addition, the equipment supported an enhanced approach to session capture.

During the pandemic students were loaned iPads which students retained for the duration of their degree. This has allowed the maths team to reflect on the design of learning, teaching and assessment strategies and provided opportunities for the team make use of digital tools. More important though is the knowledge that each of the students have access to identical hardware and software.

This has also allowed a more inclusive approach to assessment by providing more flexibility. Teams can confidently design flexibility into schemes around a common set of Apps. For example, the following is an assessment brief from for the Communicating Mathematics module:

> "*This communication brief requires you to develop an activity or resource that could be used to promote, or engage people with, mathematics.*
>
> *This can be done either by*
> - *a short mathematics activity with feedback sheet,*
> - *a blog (1000-2000 words or multimedia equivalent) or*
> - *a short video such as a screencast (2.5-4 minutes).*
>
> *The activity or video must link clearly with a mathematical concept or problem and be suitable to be used to engage people with mathematics."*

The assessment brief requires students to create an artefact (activity, blog or video) which forms part of their portfolio of evidence. Upon graduation students use these portfolios as evidence of their skill development and have shared examples with potential employers. One recent graduate using their communication project in a successful job interview as a data scientist, saying: "*Specifically, the panel were impressed with my communicating mathematics project."*

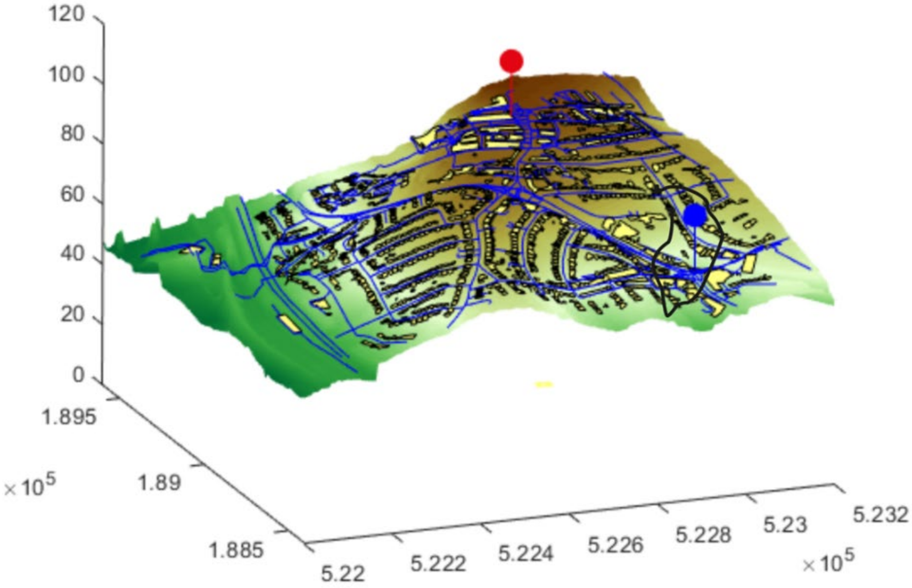
As part of the module assessment students are required to reflect on the work produced and the skills they used to create it. Here students are encouraged to recognise the skills they have developed whilst creating the work and align skills to those required for professional employment focusing on creativity, critical thinking, problem solving, collaboration, and use of technology. Feedback from graduates supports the approach with one commenting that their

> "*Communicating mathematics project expanded my communication skills, working on a project that had real world application bridged that gap for me*."

4. Authentic and Accessible Assessment

When the revalidation process began for the undergraduate mathematics at Middlesex University in 2019, we sought to build on the innovative learning, teaching and assessment approaches core to the Problem Solving and Communicating Mathematics modules and integrated these techniques more broadly into even the most pure of our mathematics modules. Our goal was to move beyond the view that problem-solving and communication were additional skills but rather as core skills that are important to their development as mathematicians.

As part of the revalidation process it was decided that exams would be removed from the programmes and replaced with more authentic assessment, such as projects, portfolios of work, and presentations, requiring students to use the same competencies, or combinations of knowledge, skills, and attitudes that they need to apply in professional life (Guliker et al. 2004). There was much discussion about what authenticity meant for mathematics especially for the purest forms, making it particularly important that the assessment made clear how it supported the development of skills needed for employment, in addition to assessing learning outcomes. For example, Figure 2 shows an excerpt from a level 6 analysis module where students ultimately are applying techniques from multivariable calculus. However, the context of the problem (in which the students must interpret their results) is the familiar topography of the university together with a discussion of Ordnance Survey co-ordinate systems, LIDAR (Light Detection and Ranging) altitude measuring from DEFRA (Department for Environment, Food and Rural Affairs), the analysis of open data, and polynomial approximation (see Sharples, 2021). This embeds a practical and current government-funded project into an otherwise abstract assessment, thereby making it more authentic. Core principles to our revised approach were to ensure all students can communicate mathematical ideas and concepts, collaborate on mathematical problems, demonstrate their learning in creative ways and have equitable access to technology to support their mathematical learning. By the end of their programme students will have developed a portfolio of authentic evidence to demonstrate their mathematical knowledge and skills in creative ways through authentic assessment.



4. From first principles (i.e. working directly from Definition 3.3 and without using other theorems) prove that f is differentiable at the point **p**. — 5 marks

5. In fact **all** multivariable polynomials are differentiable at every point in the domain. Sketch an argument to show this. Either
   - write a formal argument,
   - draw an illustration,
   - record an audio explanation or
   - record a video explanation.

   Imagine you're trying to convince a mathematician of this fact.

   Pro-tip:
   You may recall last year we demonstrated all single variable polynomials $f : \mathbb{R} \to \mathbb{R}$ were differentiable.

   3 marks

6. We are using a polynomial to approximate the altitude of terrain near Middlesex University. In general would you expect the true altitude of terrain to be differentiable? If not, provide a mathematical description of a non-differentiable geographic feature. Either
   - write an explanation,
   - draw an illustration,
   - record an audio explanation or
   - record a video explanation

   to justify your answer. — 4 marks

Figure 2: Sample questions from Real Analysis coursework

This required us to revise the programme wide learning, teaching and assessment strategy which would support the development of the skills needed for the talent economy in a mathematical context. The strategy promotes the use of enquiry based methods for learning, collaborative problem-solving approaches, and assessment schemes that are varied, inclusive, accessible, authentic, future focused, and designed around common hardware and software.

Key elements within the revised overall programme design were:

- Balance of mathematical theory and practice within the overall programme design;
- Communication and creative problem solving embedded across the programme;
- Communicate mathematical ideas and concepts ;
- Providing choice of assessment activities;
- Collaboration and learning supported via online learning communities ;
- Reflection is embedded within modules and skill recognition is promoted;
- All activities are supported and designed around iPads and agreed Apps;
- The students develop a portfolio to evidence their skills.

To support this across the programmes the team sought to ground the assessment in real world application and/or clearly align to an employability skill. For example, the level 6 analysis coursework referenced above (see Figure 2) rigorously tested the learning outcomes while also allowing students to choose the medium of assessment allowing for personalisation and a more inclusive approach. These options allow students to build a varied portfolio of work which they can easily use to evidence the skills required by the talent economy. It is more inclusive and accessible and reduces the need for reasonable adjustments to be made.

5. Conclusion

This case study has outlined the journey the maths team has taken when designing its programmes to support the development of mathematical knowledge and skills.

Building flexibility into assessment encourages students to think creatively about how to best approach the problem while negating the need to make reasonable adjustments. The foundation of our approach is that students have access to identical hardware and software. This means we can write multi-modal assessment with the knowledge of what resources the students have access to.

It is noteworthy that many students who completed the real analysis coursework, see for example Figure 2, still elected to complete the assessment with formal written mathematical arguments. In focus groups students stated that the reason for this was related to familiarity with the various forms of assessment. Students stated that they felt a formal written mathematical argument was easiest because historically that is the form most of their previous assessments had taken whereas they believed a video submission, for example, would take a great deal more work to get up to the standard they would be happy to submit as part of summative assessment.

Students did appreciate being given an option of a different form of assessment within schemes with one saying:

> "*Mathematics as a subject is one that demands patience and creativity when trying to find solutions. I see being able to develop my level of patience when approaching new issues and understanding how to use the tools I am provided with more creativity is an invaluable skill.*"

The response from students has been positive, with recent graduates specifically citing this approach as having a positive influence on their professional career. The feedback from students indicate that it would be beneficial to introduce student to these alternative methods for completing assessment at an earlier stage, so they gain more familiarity with them.

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