

A Robust Transmission Strategy for Multi-cell Interference Networks

Tuan Anh Le[†], Mohammad Reza Nakhai[‡], and Keivan Navaie[§]

[†]School of Science and Technology, Middlesex University, London, NW4 4BT, UK

[‡]Centre for Telecommunications Research, King's College London, Strand, WC2R 2LS, UK

[§]School of Computing and Communications, Lancaster University, Lancaster, LA1 4WA, UK

E-mails: t.le@mdx.ac.uk, reza.nakhai@kcl.ac.uk, k.navaie@lancaster.ac.uk

Abstract—In this paper, we propose a robust transmission strategy for multi-cell networks equipped with multiple-antenna base stations (BSs) under universal frequency reuse and in the presence of channel estimation error. We propose a distributed optimization scheme, where each BS individually minimizes a combination of its total transmit power and its resulting overall interference inflicted on the users of the adjacent cells, subject to maintaining a desired quality of service at its local users. We transform the proposed scheme to a robust optimization problem for the worst case of errors and derive a semidefinite programming (SDP) using rank-relaxation. We prove that the derived SDP always yields exact rank-one optimal solutions. This is in contrast to the standard rank-relaxed SDP technique that requires an additionally high computational complexity to approximate the solutions with sufficient accuracies, required for an effective beamforming. A comparison of simulation results show that the proposed transmission strategy can expand the signal-to-interference-plus-noise-ratio operational range with significantly reduced power consumption levels at BSs and perform closely to its centralized counterpart.

I. INTRODUCTION

Scarcity of practical radio frequency resources and increasing demands on mobile data streams forces mobile network operators to operate their networks with frequency reuse of one. This type of networks, also known as multi-cell interference networks, face severe inter-cell-interference (ICI) problems. Coordinated beamforming (CBF) is an effective approach for tackling ICI [1]–[3] in such networks. In CBF the beamforming vectors for all users of coordinating base stations (BSs) are jointly designed in a way that each user only receives signal from its corresponding BS. In [1], [2], [4]–[6], CBF is considered to minimize the total transmit power at BSs subject to satisfying users' signal-to-interference-plus-noise-ratio (SINR) constraints. The work in [2] assumes perfect channel state information (CSI), while the works in [1] and [4]–[6] account for the imperfect CSI. Distributed solutions to this problem are obtained using various methods, e.g., uplink-downlink duality in [2], dual/primal decomposition, and alternating direction method of multipliers, see, e.g., [4].

To efficiently utilize the available resources, transmission strategies are usually proposed in forms of optimization problems. Unfortunately, those problems are normally non-convex. In such cases, semidefinite relaxation (SDR) technique [7] is usually adopted to transform a non-convex problem into a semidefinite programming (SDP) [8] problem by changing the

optimization variables of the original problem from a vector to an equivalent form of symmetric positive semidefinite matrix, see e.g., [9]. In order to guarantee that the original and the transformed problems have the same optimal solution, the newly introduced matrix must be rank-one [10]. If solving the transformed SDP problem does not result in rank-one solutions, randomization techniques, such as [10], [11] are adopted to generate an approximated rank-one solution. In such cases however, only a sub-optimal solution is obtained. Moreover, the randomization process is computationally expensive and can inflict an additional high complexity overhead to the SDP solver, in particular, when more accurate multi-cell beamforming mandates more accurate rank-one approximations.

In this paper, we introduce a new robust downlink beamforming algorithm that is amenable to decentralized implementation in multi-cell networks with frequency reuse of one. This is due to minimizing the intercell interference of cross-links as an integral part of the cost function of a proposed optimization process that designs the beamforming vectors for sum-power minimization at individual BSs, subject to satisfying a set of SINR constraints at the local users of each BS. In this way, while the SINR demands of all users are satisfied with transmit power efficiency, the transmission strategies at individual BSs of the multi-cell interference network are also brought into a balance and stabilized at an equilibrium point agreed by all BSs.

The proposed downlink beamforming requires direct as well as the interfering channel knowledge at BSs. Estimating CSI at BS transmitter (CSIT) is usually associated with an estimation error due to various reasons such as channel estimation error, feedback channel bandwidth limitations, etc. Therefore, in practice any design based on assuming perfect CSIT may not function as expected. To address this issue, we propose a robust design against CSIT uncertainties. Confining the uncertainties in the captured CSIT in hyper-spherical sets, we first find a robust counterpart for the proposed optimization problem for the worst case of estimation error, and then recast the robust counterpart in SDP form using the standard rank relaxation. We further prove that the resulting SDP problem always yields the rank-one solutions. Hence, the proposed algorithm does not need to execute the standard randomization procedure to approximate the feasible rank-one solutions to the SDP problem.

Simulation results indicate that our approach outperforms the conventional beamforming method by providing a larger SINR operation range while closely following its centralized counterpart. Simulation results also confirm that the proposed scheme provides robustness against channel estimation error in return of an increase in the transmission power.

Throughout the paper, we use notations $\|\cdot\|$, and $\mathbb{E}(\cdot)$ to denote the Frobenius norm operator, and the expectation of a random variable, respectively. Further, $\mathbf{W} \succeq 0$ is used to denote that \mathbf{W} is a positive semi definite matrix while $\mathbf{w} \succcurlyeq 0$ designates that all elements of \mathbf{w} are non-negative.

II. SYSTEM MODEL

We consider a cellular system comprising of K BSs, where each BS has M antennas and simultaneously supports U active single-antenna users. It is assumed that each BS can measure the channel gains for all users located in its cell boundaries, irrespective of which BS these users are assigned to. Let $\mathbf{h}_{imn} = \tilde{\mathbf{h}}_{imn} + \mathbf{e}_{imn}$ be the actual channel for user i in cell m as seen by the BS of cell n , where $\tilde{\mathbf{h}}_{imn} \in \mathbb{C}^{M \times 1}$, and $\mathbf{e}_{imn} \in \mathbb{C}^{M \times 1}$ are the estimated channel, and its corresponding estimation error, respectively. Let $\mathbf{w}_{im} \in \mathbb{C}^{M \times 1}$, and s_{im} denote the beamforming vector and the data symbol for the local user i of cell m , respectively. The signal received by user i in cell m is

$$y_{im} = \sum_{j=1}^U \mathbf{h}_{imj} \mathbf{w}_{jm} s_{jm} + n_{im} + v_{im}, \quad (1)$$

where n_{im} is a zero mean circularly symmetric complex Gaussian noise at user i in cell m with the variance of σ_{im}^2 and v_{im} is the overall inter-cell interference received by user i in cell m . Let $\mathbf{R}_{imn} = \mathbb{E}_{\mathbf{h}_{imn}} (\mathbf{h}_{imn} \mathbf{h}_{imn}^H)$, then $\mathbf{R}_{imn} = \tilde{\mathbf{R}}_{imn} + \mathbf{\Delta}_{imn}$ where $\tilde{\mathbf{R}}_{imn} = \mathbb{E}_{\tilde{\mathbf{h}}_{imn}} (\tilde{\mathbf{h}}_{imn} \tilde{\mathbf{h}}_{imn}^H)$, $\mathbf{\Delta}_{imn} = \mathbb{E}_{\mathbf{e}_{imn}} (\mathbf{e}_{imn} \mathbf{e}_{imn}^H)$. Denoting $\xi_{im} = \mathbb{E}_{v_{im}} (v_{im} v_{im}^*)$ and letting $\mathbb{E}_{s_{im}} (|s_{im}|^2) = 1$, the signal power at user i in cell m is

$$\mathbb{E}_{y_{im}} (|y_{im}|^2) = \sum_{j=1}^U \mathbf{w}_{jm}^H \mathbf{R}_{imj} \mathbf{w}_{jm} + \sigma_{im}^2 + \xi_{im}. \quad (2)$$

Hence, the SINR at any local user i of cell m is

$$g_{im}(\{\mathbf{w}_{im}\}) = \frac{\mathbf{w}_{im}^H \mathbf{R}_{imm} \mathbf{w}_{im}}{\sum_{j=1, j \neq i}^U \mathbf{w}_{jm}^H \mathbf{R}_{imj} \mathbf{w}_{jm} + \xi_{im} + \sigma_{im}^2}, \quad (3)$$

where $\{\mathbf{w}_{im}\} = \{\mathbf{w}_{1m}, \mathbf{w}_{2m}, \dots, \mathbf{w}_{Um}\}$ denotes the set of all beamforming vectors for cell m . Note that, in LTE networks, mobile users are capable of estimating their received inter-cell interference power, i.e., ξ_{im} , based on available physical-layer measurements [12, Sec. 5.1.3].

III. ROBUST TRANSMISSION OPTIMIZATION

In this paper we formulate the optimal beamforming problem by considering a cost function, $f_m(\{\mathbf{w}_{im}\})$, which is a linear combination of the total signal power transmitted by

BS m to its locally active users, and the aggregated inter-cell interference power imposed on the other users by this BS:

$$f_m(\{\mathbf{w}_{im}\}) = \sum_{i=1}^U \mathbf{w}_{im}^H \mathbf{w}_{im} + \sum_{n=1, n \neq m}^K \sum_{t=1}^U \sum_{i=1}^U \mu_{tn} \mathbf{w}_{im}^H \mathbf{R}_{tnm} \mathbf{w}_{im}, \quad (4)$$

where μ_{tn} is the pricing factor for user t in cell $n \neq m$.

The optimal downlink beamforming problem of cell m is then formulated as the following minimization:

$$\begin{aligned} \min_{\{\mathbf{w}_{im}\}} \quad & f_m(\{\mathbf{w}_{im}\}) \\ \text{s. t.} \quad & g_{im}(\{\mathbf{w}_{im}\}) \geq \gamma_{im}, \quad \forall i, \end{aligned} \quad (5)$$

where γ_{im} is the SINR level required by an active local user i in cell m .

To incorporate the impact of inaccurate channel estimation, here we assume that the uncertainty in the estimation of channel covariance matrix is confined within a hyper-spherical set \mathcal{E}_{imn} with radius δ_{imn} defined as

$$\mathcal{E}_{imn} = \{\mathbf{\Delta}_{imn} \in \mathbb{C}^{M \times M} : \|\mathbf{\Delta}_{imn}\| \leq \delta_{imn}\}, \forall i, m, n.$$

Furthermore, for any $M \times M$ Hermitian positive semidefinite matrix, \mathbf{Y} , $\|\mathbf{Y}\| \leq \delta$, and a $M \times 1$ arbitrary vector \mathbf{x} , we have

$$\mathbf{x}^H \mathbf{Y} \mathbf{x} \leq \mathbf{x}^H \delta \mathbf{I} \mathbf{x}. \quad (6)$$

The proof for (6) is presented in Appendix A.

Utilizing (6), we then evaluate the worst case effect of the channel estimation error on the second term of (5):

$$\begin{aligned} & \max_{\|\mathbf{\Delta}_{tnm}\| \leq \delta_{tnm}} \sum_{n=1, n \neq m}^K \sum_{t=1}^U \sum_{i=1}^U \mu_{tn} \mathbf{w}_{im}^H \mathbf{R}_{tnm} \mathbf{w}_{im} \\ &= \sum_{n=1, n \neq m}^K \sum_{t=1}^U \sum_{i=1}^U \mu_{tn} \mathbf{w}_{im}^H (\tilde{\mathbf{R}}_{tnm} + \delta_{tnm} \mathbf{I}) \mathbf{w}_{im}. \end{aligned} \quad (7)$$

Similarly, utilizing (6) we then write the worst case of error on $g_{im}(\{\mathbf{w}_{im}\})$ as (8)¹ given at the top of next page. Hence, in the worst case (5) can be cast as (9) given at the top of next page.

To map (9) to a SDP form, we define $\mathbf{F}_{im} = \mathbf{w}_{im} \mathbf{w}_{im}^H$, where \mathbf{F}_{im} is rank one Hermitian positive semidefinite matrix. We then rewrite (9) in a SDP form as (10) given at the top of next page, where $\{\mathbf{F}_{im}\} = \{\mathbf{F}_{1m}, \mathbf{F}_{2m}, \dots, \mathbf{F}_{Um}\}$ is the set of beamforming matrices for all users of cell m . The SDP problem in (10) can be then solved by adopting optimization packages, e.g., CVX [13], to obtain $\{\mathbf{F}_{im}\}$.

In transforming (9) into (10), shown at the top of next page, we have relaxed the constraint that $\text{rank}(\mathbf{F}_{im}) = 1$, $\forall i$. However, we prove in the sequel that relaxing the rank-one condition preserves the optimality since problem (10) always yields rank-one optimal solutions.

Proposition 1: An $M \times M$ Hermitian matrix \mathbf{F}_{im} can be decomposed as $\mathbf{F}_{im} = \sum_{j=1}^N \epsilon_{im,j} \mathbf{v}_{im,j} \mathbf{v}_{im,j}^H$, where $N \leq M$

¹This type of worst-case evaluation for SINR was first introduced in [10].

$$\min_{\|\mathbf{\Delta}_{imm}\| \leq \delta_{imm}} g_{im}(\{\mathbf{w}_{im}\}) = \frac{\mathbf{w}_{im}^H (\tilde{\mathbf{R}}_{imm} - \delta_{imm} \mathbf{I}) \mathbf{w}_{im}}{\sum_{j=1, j \neq i}^U \mathbf{w}_{jm}^H (\tilde{\mathbf{R}}_{imm} + \delta_{imm} \mathbf{I}) \mathbf{w}_{jm} + \xi_{im} + \sigma_{im}^2} \quad (8)$$

$$\begin{aligned} \min_{\{\mathbf{w}_{im}\}} & \sum_{i=1}^U \mathbf{w}_{im}^H \mathbf{w}_{im} + \sum_{n=1, n \neq m}^K \sum_{t=1}^U \sum_{i=1}^U \mu_{tn} \mathbf{w}_{im}^H (\tilde{\mathbf{R}}_{tnm} + \delta_{tnm} \mathbf{I}) \mathbf{w}_{im} \\ \text{s. t.} & \frac{\mathbf{w}_{im}^H (\tilde{\mathbf{R}}_{imm} - \delta_{imm} \mathbf{I}) \mathbf{w}_{im}}{\sum_{j=1, j \neq i}^U \mathbf{w}_{jm}^H (\tilde{\mathbf{R}}_{imm} + \delta_{imm} \mathbf{I}) \mathbf{w}_{jm} + \xi_{im} + \sigma_{im}^2} \geq \gamma_{im}, \forall i \end{aligned} \quad (9)$$

$$\begin{aligned} \min_{\{\mathbf{F}_{im}\}} & \sum_{n=1, n \neq m}^K \sum_{t=1}^U \sum_{i=1}^U \mu_{tn} \text{Tr}(\tilde{\mathbf{R}}_{tnm} \mathbf{F}_{im}) + \sum_{i=1}^U \text{Tr}(\mathbf{F}_{im}) \left(\sum_{n=1, n \neq m}^K \sum_{t=1}^U \mu_{tn} \delta_{tnm} + 1 \right) \\ \text{s. t.} & \left(1 + \frac{1}{\gamma_{im}} \right) \text{Tr}(\tilde{\mathbf{R}}_{imm} \mathbf{F}_{im}) - \sum_{j=1}^U \text{Tr}(\tilde{\mathbf{R}}_{imm} \mathbf{F}_{jm}) - \sum_{j=1, j \neq i}^U \delta_{imm} \text{Tr}(\mathbf{F}_{jm}) - \frac{\delta_{imm}}{\gamma_{im}} \text{Tr}(\mathbf{F}_{im}) - (\xi_{im} + \sigma_{im}^2) \geq 0, \\ & \mathbf{F}_{im} = \mathbf{F}_{im}^H \succeq 0, \forall i, \end{aligned} \quad (10)$$

is the rank, $\epsilon_{im,j}$ is the j th eigenvalue, and $\mathbf{v}_{im,j}$ is the j th eigenvector of \mathbf{F}_{im} , respectively².

Next, we introduce the following theorem.

Theorem 1: The optimization problem (10) always yields rank-one optimal solutions $\mathbf{F}_{im}^* \forall i$.

Proof: Since SDPs are convex, the optimization problem (10) and its corresponding dual problem have zero-duality gap [8]. The Lagrangian of (10) is

$$\mathcal{L}(\{\mathbf{F}_{im}\}, \vec{\alpha}, \{\mathbf{Z}_i\}) = \text{Tr} \sum_{i=1}^U \mathbf{Q}_{im} \mathbf{F}_{im} + \sum_{i=1}^U \alpha_i (\xi_{im} + \sigma_{im}^2), \quad (11)$$

where α_i , and \mathbf{Z}_i are Lagrange multipliers associated with the corresponding constraints of (10), $\vec{\alpha} = [\alpha_1, \dots, \alpha_U]^T$, $\{\mathbf{Z}_i\} = \{\mathbf{Z}_1, \dots, \mathbf{Z}_U\}$ and \mathbf{Q}_{im} is given in (12) at the top of next page. The sketch of mathematical manipulations to arrive at (11) is given in Appendix B. The dual function of (10) is

$$t(\vec{\alpha}, \{\mathbf{Z}_i\}) = \min_{\{\mathbf{F}_{im}\}} \mathcal{L}(\{\mathbf{F}_{im}\}, \vec{\alpha}, \{\mathbf{Z}_i\}), \quad (13)$$

and the dual problem of (10) is

$$\begin{aligned} \max_{\vec{\alpha}, \{\mathbf{Z}_i\}} & t(\vec{\alpha}, \{\mathbf{Z}_i\}) \\ \text{s. t.} & \vec{\alpha} \succcurlyeq 0, \mathbf{Z}_i \succeq 0, \forall i. \end{aligned} \quad (14)$$

Let $\vec{\alpha}^*$, $\{\mathbf{Z}_i^*\}$ represent the optimal solutions to (14). The optimal solution $\{\mathbf{F}_{im}^*\}$ to (10) can be attained by solving

$$\min_{\{\mathbf{F}_{im}\}} \mathcal{L}(\{\mathbf{F}_{im}\}, \vec{\alpha}^*, \{\mathbf{Z}_i^*\}) = \min_{\{\mathbf{F}_{im}\}} \text{Tr} \sum_{i=1}^U \mathbf{Q}_{im} \mathbf{F}_{im}. \quad (15)$$

²This is due to the facts that the Hermitian matrix \mathbf{F}_{im} has N real eigenvalues and N orthogonal eigenvectors.

Since the optimal solution to (10) exists, \mathbf{Q}_{im} must be positive semidefinite for all i so that the Lagrangian dual is bounded below on \mathbf{F}_{im} [8].

Let us assume that the optimal solution to (10), i.e., \mathbf{F}_{im}^* , has a rank of N with $N > 1$, $\forall i$. According to Proposition 1, $\mathbf{F}_{im}^* = \sum_{j=1}^N \epsilon_{im,j} \mathbf{v}_{im,j} \mathbf{v}_{im,j}^H$. Then, we can find $\hat{\mathbf{F}}_{im}^* = \epsilon_{im,p} \mathbf{v}_{im,p} \mathbf{v}_{im,p}^H$, where $p = \arg \min_{j \in \{1, \dots, N\}} (\epsilon_{im,j} \mathbf{v}_{im,j}^H \mathbf{Q}_{im} \mathbf{v}_{im,j})$. This points to the conclusion that $\text{Tr} \sum_{i=1}^U \mathbf{Q}_{im} \hat{\mathbf{F}}_{im}^* < \text{Tr} \sum_{i=1}^U \mathbf{Q}_{im} \mathbf{F}_{im}^*$. This contradicts the assumption that \mathbf{F}_{im}^* is the optimal solution. Therefore, the rank of \mathbf{F}_{im}^* must be one for all i . ■

The proposed approach is summarized in the Algorithm 1, given on the next page, where l and τ are the updating cycle at BSs and the stopping criteria, respectively. Regarding step 6 of the algorithm, the authors of [14] show that inter-cell interference at a given user can be computed whenever the channels between the user and BSs in the network are available. Details on MMSE inter-cell interference estimation approaches, which also account for uncertainties in channel measurements in LTE networks, are presented in [14].

In our formulations, ξ_{im} represents the cross-link interference estimated at user i in cell m . The estimated cross-link interference levels at the intra-cell users are used by the local BS of the corresponding cell to adjust its transmission strategy so that its aggregate cross-link interference on the users of the other cells is further minimized after each iteration of Algorithm 1. Since, this transmission strategy is followed by all BSs individually and simultaneously at each iteration, the convergence of Algorithm 1 to a consensus amongst the BSs, where the transmit sum-power at each BS is minimized, is

$$\begin{aligned} \mathbf{Q}_{im} = & \sum_{n=1, n \neq m}^K \sum_{t=1}^U \mu_{tn} \tilde{\mathbf{R}}_{tnm} + \left(\sum_{n=1, n \neq m}^K \sum_{t=1}^U \mu_{tn} \delta_{tnm} + 1 + \alpha_i \left(\frac{\delta_{imm}}{\gamma_{im}} - \delta_{imm} \right) \right) \mathbf{I} \\ & - \mathbf{Z}_i - \left(1 + \frac{1}{\gamma_{im}} \right) \tilde{\mathbf{R}}_{imm} + \sum_{j=1}^U \alpha_j \left(\tilde{\mathbf{R}}_{jmm} + \delta_{jmm} \mathbf{I} \right) \end{aligned} \quad (12)$$

Algorithm 1 The Proposed Algorithm Implemented at BS m

- 1: **Inputs:** τ , $\tilde{\mathbf{R}}_{tnm}$, $\tilde{\mathbf{R}}_{imm}$, δ_{tnm} , δ_{imm} , γ_{im} , μ_{tn} , $\sigma_{im}^2 \forall i, t, n$;
 - 2: **Initialize:** $l = 1$, $\xi_{im}(l) \geq 0$, $\forall i$;
 - 3: **Repeat**
 - 4: Solve (10) $\rightarrow \{\mathbf{F}_{im}(l)\} \rightarrow \{\mathbf{w}_{im}(l)\}$, i.e., obtaining from rank-one matrix $\mathbf{F}_{im}(l)$ as $\sqrt{\epsilon_{im,1}(l)} \mathbf{v}_{im,1}(l)$, $\forall i$;
 - 5: Transmit with $\{\mathbf{w}_{im}(l)\}$;
 - 6: Each user estimates its received inter-cell interference, e.g., using an MMSE approach in [14], and feeds back to its serving BS as $\xi_{im}(l+1)$;
 - 7: $l = l + 1$;
 - 8: **Until** $\left| \sum_{i=1}^U \text{Tr}[\mathbf{F}_{im}(l-1) - \mathbf{F}_{im}(l-2)] \right| \leq \tau$;
 - 9: **Outputs:** $\{\mathbf{w}_{im}(l-1)\}$.
-

guaranteed.

IV. SIMULATION RESULTS

We evaluate the performance of the proposed scheme and compare it against its centralized counterpart [5] and the conventional method [15] under cell-edge-user scenario where ICI problem is severe. We consider 3 adjacent sectors of 3 neighboring cells and randomly locate users near the mutual border of cells with one user per sector. Monte-Carlo is carried out over user distributions. We consider 6 antenna elements per sector. Any two neighboring BSs are located 3 km apart from one another. The channel covariance matrix \mathbf{R}_{imn} is obtained using

$$\mathbf{R}_{imn} = \beta_{imn} \mathbf{R}(\theta_{imn}, \sigma_a), \quad (16)$$

where β_{imn} captures the channel-gain coefficient, θ_{imn} is the angle of departure, σ_a is the standard deviation of the angular spread, and the (p, q) th entry of $\mathbf{R}(\theta, \sigma_a)$ is, [15], [16]:

$$e^{\frac{j2\pi\Delta}{\lambda}[(q-p)\sin\theta]} e^{-2\left[\frac{\pi\Delta\sigma_a}{\lambda}\{(q-p)\cos\theta}\right]^2}. \quad (17)$$

In (16) β_{imn} represents the impact of: i) the distance-dependent path-loss following $128.1 + 37.6\log_{10}(d)$, where d is in kilometers; ii) a log-normal shadow fading with the standard deviation of 10dB; and iii) a Rayleigh fading for the multi-path reception. In (17), $\sigma_a = 2^\circ$ and the antenna spacing at the BS $\Delta = \lambda/2$, where λ is the carrier wavelength. The subcarrier bandwidth, the noise figure at each user receiver, the noise power spectral density and antenna gain are set at 15kHz, 5dB, -174dBm/Hz and 15dBi, respectively.

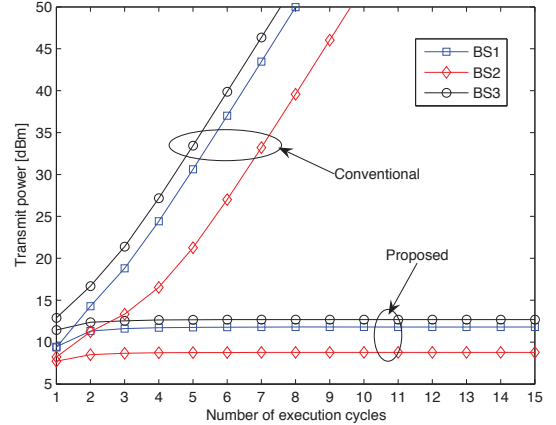


Fig. 1. Transmit power at different BSs using the proposed and conventional schemes [15] vs. the number of updates in beamforming vectors at BSs, for SINR target of 18dB per user with perfect CSI, i.e., $\delta_{imn} = 0, \forall i, m, n$.

We let the BSs simultaneously execute Algorithm 1 to obtain their beamforming vectors. Step 6 of the algorithm accounts for the coupling amongst the BSs. The algorithm converges to an equilibrium point amongst the BSs, due to the fact that each BS primarily tries to minimize the overall interference to unintended users in other cells, i.e., the first term in the cost function of (10). Furthermore, the weighting factor in the second term in the cost function of (10) directly depends on the radius of the uncertainty region. Hence, an increase in the uncertainty region encourages the optimization problem to further decrease the overall transmit power at BSs and this, in turn, leads to further reduction in ICI. Although at each one of these updating cycles the SINR constraints are held, the overall transmit power across the multiple cells is minimized at the equilibrium. Here we formulate the conventional scheme [15] as a special case of the proposed optimization problem in (10), where there is no control on ICI, i.e., $\mu_{tn} = 0 \forall t, n$.

The results in Fig. 1 indicate a drastic increase in the required transmit power level of all BSs in the conventional scheme within the first 10 updating cycles of beamforming vectors, due to the so-called ping-pong effect. Whereas in the proposed scheme, the transmit power levels of all BSs gradually increase and stabilize at some finite levels after 4 updating cycles. This confirms the effectiveness of the second term of the cost function in the proposed problem (5) in avoiding the ping-pong effect and stabilizing the multi-cell

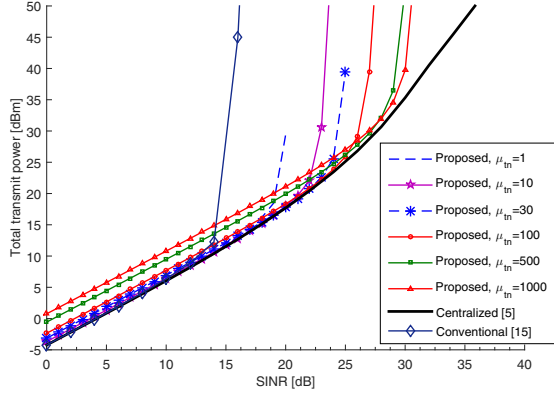


Fig. 2. Total transmit power of all BSs vs. various SINR targets for the proposed, with perfect CSI and different values of μ_{tn} , conventional approach [15] and centralized beamforming scheme [5].

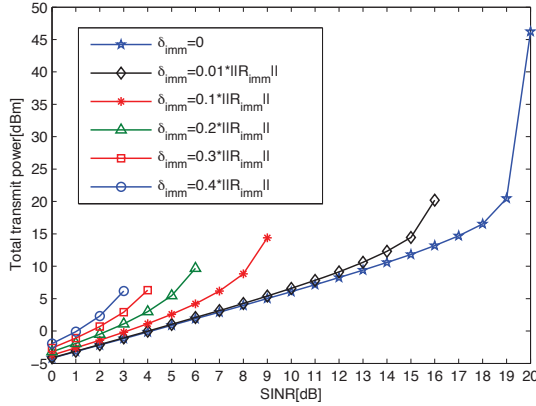


Fig. 3. The total transmit power of the BSs in the proposed scheme vs. the target SINR at each user with different values of δ_{imm} while δ_{tmn} is fixed at 10% of $\|R_{tmn}\|$, $\forall t, n \neq m$ and $\mu_{tn} = 1$, $\forall t, n$.

network at an equilibrium.

In Fig. 2, the centralized coordinated beamforming (CBF) approach proposed in [5] and the conventional scheme introduced in [15] are used for comparison. The results in Fig. 2 confirm that the proposed scheme outperforms the conventional approach at intermediate and higher SINR levels. Furthermore, they show the impact of the pricing factor μ_{tn} , that adjusts the degree of emphasize on the altruistic behavior of the cost function in problem (10), on the BSs' overall power consumption. The results reveal that the power-efficient range of operation of the proposed scheme can be expanded from 20 to 30 dB of SINR target, where μ_{tn} is allowed to vary from 1 to 1000. This is due to the first term of the cost function in (10) that controls the inflicted ICI by each BS on the users of the other cells and stabilizes the selfish dynamic of the conventional network in an equilibrium point, agreed by all the BSs. As a result of this stabilization, while closely following the behavior of its centralized counterpart CBF the proposed scheme extends the operational range of SINR target.

Fig. 3 illustrates the effect of inaccuracy in estimation of

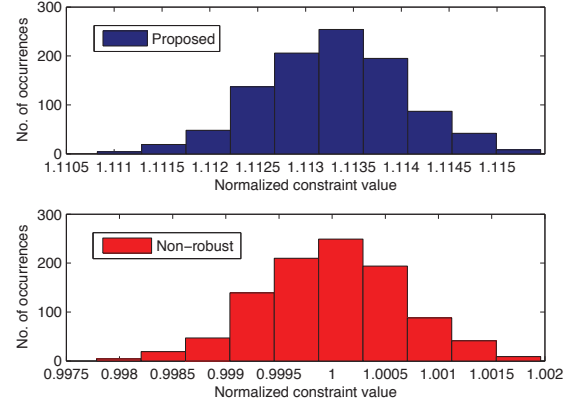


Fig. 4. Histogram of normalized SINR constraints for $\delta_{tmn} = 0.01 \|R_{tmn}\|$, $\mu_{tn} = 1 \forall t, m, n$, where target SINR is 10dB.

channel covariance matrices of the local users, i.e., represented by the value of δ_{imm} , on the performance of the proposed distributed scheme. A comparison of the results in Fig. 3 shows that achieving robustness comes at the cost of an excessive transmit power, which increases, even further, if robustness against larger error radii is required. This price, in terms of increasing transmit power, is paid for guaranteeing the SINR targets for all local users under uncertainties in channel estimations.

Let $\psi_{im} = \frac{g_{im}(\{\mathbf{w}_{im}\})}{\gamma_{im}}$ be the normalized SINR constraint value of user i in cell m . The SINR constraint of user i in cell m is satisfied if $\psi_{im} \geq 1$. It is not satisfied if $\psi_{im} < 1$. Fig. 4 illustrates the histograms of ψ_{im} for an SINR target of $\gamma_{im} = 10$ dB. It is observed that the proposed robust design always guarantees the satisfaction of the set SINR target, whereas the non-robust design fails at about half of the trials in satisfying the desired SINR target.

V. CONCLUSION

We have formulated and solved an optimization approach where mitigating inter-cell interference is the primary concern alongside the transmit power minimization at each BS in a multi-BS wireless network with full frequency reuse and channel estimation error. The resulting solution leads to a distributed robust algorithm with adjustable pricing factors capable of expanding the achievable range of SINR targets at affordable transmit power levels.

APPENDIX A PROOF OF (6)

Proof: \mathbf{Y} is positive semidefinite therefore, $\mathbf{x}^H \mathbf{Y} \mathbf{x} = \text{Tr}(\mathbf{Y} \mathbf{x} \mathbf{x}^H) \geq 0$. Using trace inequality, we also have $\text{Tr}(\mathbf{Y} \mathbf{x} \mathbf{x}^H) \leq \sqrt{\text{Tr}(\mathbf{Y} \mathbf{Y}^H) \text{Tr}(\mathbf{x} \mathbf{x}^H \mathbf{x} \mathbf{x}^H)} = \|\mathbf{Y}\| \|\mathbf{x}\|^2$. Using $\|\mathbf{Y}\| \leq \delta$, completes the proof: $\text{Tr}(\mathbf{Y} \mathbf{x} \mathbf{x}^H) \leq \|\mathbf{Y}\| \|\mathbf{x}\|^2 \leq \delta \|\mathbf{x}\|^2 = \mathbf{x}^H \delta \mathbf{I} \mathbf{x}$. ■

$$\begin{aligned}
\mathcal{L}(\{\mathbf{F}_{im}\}, \vec{\alpha}, \{\mathbf{Z}_i\}) &= \sum_{n=1, n \neq m}^K \sum_{t=1}^U \sum_{i=1}^U \mu_{tn} \text{Tr}(\tilde{\mathbf{R}}_{tnm} \mathbf{F}_{im}) + \sum_{i=1}^U \text{Tr}(\mathbf{F}_{im}) \left(\sum_{n=1, n \neq m}^K \sum_{t=1}^U \mu_{tn} \delta_{tnm} + 1 \right) \\
&- \sum_{i=1}^U \text{Tr}(\mathbf{F}_{im} \mathbf{Z}_i) - \sum_{i=1}^U \alpha_i \left(1 + \frac{1}{\gamma_{im}} \right) \text{Tr}(\tilde{\mathbf{R}}_{imm} \mathbf{F}_{im}) + \sum_{i=1}^U \alpha_i \sum_{j=1}^U \text{Tr}(\tilde{\mathbf{R}}_{imm} \mathbf{F}_{jm}) \\
&+ \sum_{i=1}^U \alpha_i \sum_{j=1, j \neq i}^U \delta_{imm} \text{Tr}(\mathbf{F}_{jm}) + \sum_{i=1}^U \alpha_i \frac{\delta_{imm}}{\gamma_{im}} \text{Tr}(\mathbf{F}_{im}) + \sum_{i=1}^U \alpha_i (\xi_{im} + \sigma_{im}^2) \tag{18}
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}(\{\mathbf{F}_{im}\}, \vec{\alpha}, \{\mathbf{Z}_i\}) &= \sum_{n=1, n \neq m}^K \sum_{t=1}^U \sum_{i=1}^U \mu_{tn} \text{Tr}(\tilde{\mathbf{R}}_{tnm} \mathbf{F}_{im}) + \sum_{i=1}^U \text{Tr}(\mathbf{F}_{im}) \left(\sum_{n=1, n \neq m}^K \sum_{t=1}^U \mu_{tn} \delta_{tnm} + 1 \right) \\
&- \sum_{i=1}^U \text{Tr}(\mathbf{F}_{im} \mathbf{Z}_i) - \sum_{i=1}^U \alpha_i \left(1 + \frac{1}{\gamma_{im}} \right) \text{Tr}(\tilde{\mathbf{R}}_{imm} \mathbf{F}_{im}) + \sum_{i=1}^U \alpha_i \sum_{j=1}^U \text{Tr}(\tilde{\mathbf{R}}_{imm} + \delta_{imm} \mathbf{I}) \mathbf{F}_{jm} \\
&+ \sum_{i=1}^U \alpha_i \left(\frac{\delta_{imm}}{\gamma_{im}} - \delta_{imm} \right) \text{Tr}(\mathbf{F}_{im}) + \sum_{i=1}^U \alpha_i (\xi_{im} + \sigma_{im}^2) \tag{19}
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}(\{\mathbf{F}_{im}\}, \vec{\alpha}, \{\mathbf{Z}_i\}) &= \sum_{n=1, n \neq m}^K \sum_{t=1}^U \sum_{i=1}^U \mu_{tn} \text{Tr}(\tilde{\mathbf{R}}_{tnm} \mathbf{F}_{im}) + \sum_{i=1}^U \text{Tr}(\mathbf{F}_{im}) \left(\sum_{n=1, n \neq m}^K \sum_{t=1}^U \mu_{tn} \delta_{tnm} + 1 \right) \\
&- \sum_{i=1}^U \text{Tr}(\mathbf{F}_{im} \mathbf{Z}_i) - \sum_{i=1}^U \alpha_i \left(1 + \frac{1}{\gamma_{im}} \right) \text{Tr}(\tilde{\mathbf{R}}_{imm} \mathbf{F}_{im}) + \sum_{i=1}^U \alpha_i \left(\frac{\delta_{imm}}{\gamma_{im}} - \delta_{imm} \right) \text{Tr}(\mathbf{F}_{im}) \\
&+ \sum_{j=1}^U \alpha_j \sum_{i=1}^U \text{Tr}(\tilde{\mathbf{R}}_{jmm} + \delta_{jmm} \mathbf{I}) \mathbf{F}_{im} + \sum_{i=1}^U \alpha_i (\xi_{im} + \sigma_{im}^2) \tag{20}
\end{aligned}$$

$$\mathcal{L}(\{\mathbf{F}_{im}\}, \vec{\alpha}, \{\mathbf{Z}_i\}) = \sum_{i=1}^U \text{Tr}(\mathbf{Q}_{im} \mathbf{F}_{im}) + \sum_{i=1}^U \alpha_i (\xi_{im} + \sigma_{im}^2) \tag{21}$$

APPENDIX B PROOF OF (11)

Proof: The Lagrangian of (10) is first written as $\mathcal{L}(\{\mathbf{F}_{im}\}, \vec{\alpha}, \{\mathbf{Z}_i\})$ given in (18) at the top of this page. Then, after some straightforward mathematical derivations, one can arrive at (21) which is (11). ■

REFERENCES

- [1] D. Ponukumati, F. Gao, and M. Bode, "Robust multicell downlink beamforming based on second-order statistics of channel state information," in *Proc. IEEE Global Telecommun. Conf.*, Dec. 2011, pp. 1–5.
- [2] H. Dahrouj and W. Yu, "Coordinated beamforming for the multicell multi-antenna wireless system," *IEEE Trans. Wireless Commun.*, vol. 9, no. 5, pp. 1748–1759, May 2010.
- [3] T. A. Le, S. Nasser, A. Zarrebini-Esfahani, A. Mills, and M. R. Nakhai, "Power-efficient downlink transmission in multicell networks with limited wireless backhaul," *IEEE Wireless Commun. Mag.*, vol. 18, no. 5, pp. 82–88, Oct. 2011.
- [4] C. Shen, T.-H. Chang, K.-Y. Wang, Z. Qiu, and C.-Y. Chi, "Distributed robust multicell coordinated beamforming with imperfect CSI: An ADMM approach," *IEEE Trans. Signal Process.*, vol. 60, no. 6, pp. 2988–3003, Jun. 2012.
- [5] T. A. Le and M. R. Nakhai, "Coordinated beamforming using semidefinite programming," in *Proc. IEEE Int. Conf. Commun.*, Jun. 2012, pp. 1–5.
- [6] A. Shaverdian and M. R. Nakhai, "Robust distributed beamforming with interference coordination in downlink cellular networks," *IEEE Trans. Commun.*, vol. 62, no. 7, pp. 2411–2421, Jul. 2014.
- [7] Z.-Q. Luo, W.-K. Ma, A. M.-C. So, Y. Ye, and S. Zhang, "Semidefinite relaxation of quadratic optimization problems," *IEEE Signal Process. Mag.*, vol. 27, no. 3, pp. 20–34, May 2010.
- [8] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.
- [9] Y. Huang and D. P. Palomar, "Rank-constrained separable semidefinite programming with applications to optimal beamforming," *IEEE Trans. Signal Process.*, vol. 58, no. 2, pp. 664–678, Feb. 2010.
- [10] M. Bengtsson and B. Ottersten, "Optimal and suboptimal transmit beamforming," in *Handbook of Antennas in Wireless Communications*, L. C. Godara, Ed. CRC Press, 2001.
- [11] H. Dahrouj and W. Yu, "Multicell interference mitigation with joint beamforming and common message decoding," *IEEE Trans. Commun.*, vol. 59, no. 8, pp. 2264–2273, Aug. 2011.
- [12] 3GPP, "Physical layer measurements (3GPP TS 36.214 version 10.1.0 Release 10)," *LTE Evolved Universal Terrestrial Radio Access (E-UTRA)*, 2011.
- [13] M. C. Grant and S. P. Boyd, *The CVX Users' Guide, Release 2.1.*, Mar. 2015. [Online]. Available: <http://web.cvxr.com/cvx/doc/CVX.pdf>.
- [14] F. Penna, S. Stanczak, Z. Ren, and P. Fertl, "MMSE interference estimation in LTE networks," in *Proc. IEEE Int. Conf. Commun.*, May 2014, pp. 4548–4552.
- [15] M. Bengtsson and B. Ottersten, "Optimal downlink beamforming using Semidefinite optimization," in *Proc. 37th Annu. Allerton Conf. Commun., Control, and Comput.*, 1999, pp. 987–996.
- [16] T. A. Le and M. R. Nakhai, "An iterative algorithm for downlink multicell beamforming," in *Proc. IEEE Global Telecommun. Conf.*, Dec. 2011, pp. 1–6.