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AFFECT IN MATHEMATICS EDUCATION:
AN INTRODUCTION

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Affect in Mathematics Education: An Introduction

Affect has been a focus of increasing interest in mathematics education research. However, affect has generally been seen as 'other' than mathematical thinking, as just not part of it. Indeed, throughout modern history, reasoning has normally seemed to require the suppression, or the control of, emotion (Walkerdine, 1988). Moreover, the term 'affect' has been interpreted in different ways and a need to increase cohesion and communication between different theoretical frameworks has become obvious. To increase cohesion within this field, four different theoretical frameworks were brought together into a discussion at a Research Forum at PME 28, Bergen (Hannula et al., 2004). This Special Issue is an extension and elaboration of that work.

Two different foci are apparent in 1960s and 1970s mathematics education research on affect: 'mathematics anxiety', and ‘attitude toward mathematics’ (ATM).

Studies of mathematics anxiety drew on methods and theories applied to test anxiety in psychology (Reyes, 1984). Most assumed a 'negative' relationship between test anxiety and performance: test anxiety inhibits cognitive processes, e.g. recall of prior learning, thereby reducing performance. Others considered test anxiety as the effect of repeated experiences of poor performance. Mathematics anxiety was typically measured by the *Mathematics Anxiety Rating Scale* (MARS: Richardson and Suinn, 1972), especially in programmes aiming to re-equip adults for further study.

Studies of attitude were based on two beliefs: attitude toward mathematics is related to achievement, and affective outcomes (such as liking mathematics) are significant *per se*. As with mathematics anxiety, the construct was borrowed from another field, namely from social psychology. Attitudes were measured by questionnaires, typically using Likert scale items. Some of these questionnaires, however, besides items about liking / disliking mathematics, included items on mathematics anxiety and beliefs about mathematics and self. The most widely used attitude measure has been the set of *Mathematics Attitude Scales* (Fennema and Sherman, 1976), which included separate scales for values (e.g. 'Attitude to Success in Math'), beliefs (e.g. 'Math as a Male Domain'), 'Confidence in Learning Math', 'Math Anxiety', and disposition towards active problem solving ('Effectance Motivation'). These scales were central to feminist research programmes aiming to improve girls' participation and performance.

Most of the critiques made of attitude research (e.g. Kulm, 1980; Leder, 1987) could be generalized to both strands of research on affect during that period:

1. The driving force in much research seemed to be "the statistical methodology rather than the theory" (McLeod, 1987); researchers rarely gave explicit definitions of their construct, often leaving the definition to be inferred from the type of instrument used. This lack of conceptual clarity was related to the borrowing of instruments and constructs from psychology, without specific theoretical elaboration for mathematics education.

2. The meta-analysis of Ma and Kishor (1997) clearly highlights the need to refine the measurement instruments.

3. Even if one rationale for research on affect in mathematics has been the assumption that improving affect would also improve achievement, the direction of influence is not clear. Although there is evidence that affect influences behaviour, there is also evidence that behaviour influences affect. In their meta-analyses, Ma and Kishor (1997) concluded that the causal relation was from attitude towards mathematics (liking mathematics) to achievement, but the effect size (.08) was too small for practical relevance.

4. Ma and Kishor's (1997) analyses also provide seemingly conflicting evidence about the relevance of gender: the correlation between liking mathematics and achievement is equal for both genders, but when both genders are pooled together, the correlation is weaker[1]. Other studies that looked also at self-concept as part of attitude indicate more clear gender differences. For example a meta-analysis of studies on gender differences and mathematics performance (Frost et al., 1994) confirmed that female students saw mathematics less as a male domain (than males), had lower self-confidence in mathematics (see also Leder, 1995) and were more

inclined to suffer from mathematics anxiety (see also Hembree, 1990). Arguably the most informative work in this field focused on the description of gender differences, and tended to use the same instrument, namely the *Mathematics Attitude Scale* of Fennema and Sherman (1976).

The lack of theoretical foundation and the consequent difficulty in interpreting and comparing different studies partly explains the minimal attention that these studies have received in cognitive research, curriculum development, and teacher training within mathematics education (McLeod, 1992).

The need to clarify theoretical foundations was felt keenly from the 1980s in the context of research on mathematical problem solving. The 'discovery' of the relationship between affect and cognition in problem solving (Norman, 1980; Silver, 1985) was supported by two complementary arguments. First, description of their activity, by important mathematicians, such as Hardy (1967), Hadamard, and Poincaré, is characterised by a strong interaction between cognitive, metacognitive, and emotional aspects. Second, the failure in problem solving of individuals apparently possessing the necessary cognitive resources suggests the importance of metacognition (Schoenfeld, 1985), and consequently of investigating factors influencing control processes.

The publication of *Affect and mathematical problem solving* (1989), edited by McLeod and Adams, represents a turning point in research on affect in mathematics education. Assuming the theory of emotions developed by the psychologist George Mandler, emotional factors are fully invoked to interpret the behaviour of students involved in mathematical problem solving. Given the importance ascribed to problem solving in mathematical activity (see e.g. Halmos, 1980), this change underlines the importance of affect for mathematics education in general.

Several contributions in the book (and, more generally, in that period) highlight the need to clarify concepts, to better analyse relationships among them, and to move beyond a methodology limited to quantitative data and statistical analysis (McLeod 1987; Hart, 1989; Fennema, 1989). Many contributors to the book adopted a ‘cognitive-constructivist’ (Mandler, 1989; McLeod, 1989) model, which describes the process of emotional experience as follows:

(i) a *discrepancy* between the individual's *expectations* and the demands of ongoing activity leads to visceral arousal;

(ii) the *physiological arousal*, on the one hand, and the person's *evaluation* of the situation, on the other, lead to the ‘construction’ of emotion; and

(iii) experiencing emotion may lead to a *reduction in conscious capacity* available for problem-solving (because the process of emotional construction itself, in this view, requires conscious capacity).

This particular description makes the experiencing of emotion seem somewhat negative or debilitating but we could describe a process similar to the above in order to account for the emergence of positive emotions, like pleasure or joy.

In general, emotion is more 'hot', here, more intense, than affect in previous models based on attitudes (Evans, 2000, Ch.7).

McLeod (1992) made an important contribution to conceptualising the field. He identified three concepts used in the research: *beliefs*, *attitudes* and *emotions*; and saw them as ranged along a dimension of increasing stability and decreasing intensity – with emotions as most intense / least stable, beliefs as most stable / least intense, and attitudes in between. Later DeBellis and Goldin (1997) added a fourth element, *values*, but argued that the four types could no longer be ordered on a single stability / intensity dimension.

McLeod's work in particular has ushered in a new period of research on affect in mathematics education. Much work refers to the McLeod or the DeBellis-Goldin categorisations, though in themselves they do not amount to a full theorisation of the area. The evidence about the interaction among cognition, metacognition, affect, given by research on problem solving, comes also from research in the context of neuroscience (Damasio, 1996; LeDoux, 1998), which highlights the deep relationship between emotions and decision-making processes. Currently, mathematics education researchers are considering applications of recent findings of neuroscience to inform our view of affect in mathematical thinking (Schlöglmann, 2002).

The main efforts of research on affect in mathematics education are therefore devoted to the construction of better-founded theoretical frameworks and a broader range of methodological instruments fit to interpret students' behaviour in mathematical activities.

Two broad directions of research have recently emerged. One has aimed to critique and revise McLeod's basic concepts, the other to break new ground. Concerning *attitude*, its significance in mathematics education, comparison of different definitions, and suggestions how to integrate quantitative and qualitative methodologies have recently been discussed (e.g. Ruffell et al., 1998; Di Martino and Zan, 2001; Hannula 2002). As for *beliefs*, continued interest in mathematics-related beliefs of teachers and students (e.g. Leder et al., 2002) has broadened to include a focus on 'self-efficacy' beliefs (e.g. Philippou and Christou, 2002), and on self-regulation (see Malmivuori and Hannula, this Issue).

Emotion has been used less in mathematics education research – so far – despite being arguably the most fundamental concept. Both the 'cognitive-constructivist' approach and insights from neuroscience (see above) suggest how repeated experience of emotion may be seen as the basis for more 'stable' attitudes and beliefs. Despite the different approaches in mathematics education, there is some measure of agreement. Emotions are seen to involve physiological reactions. Emotions also affect cognitive processing in several ways: they bias attention and memory and activate action tendencies. Moreover, emotions are seen to be functional, with a key role in human coping and adaptation (e.g. Evans, 2000; DeBellis and Goldin, this Issue; Hannula, 2002).

Value has probably been least studied of the four within mathematics education. However, discussions proceed on what a research focus on values in mathematics education can offer to concerns about affect (Bishop, 2001).

Of course, these four concepts do not cover the whole field of affect. Some investigation of 'new ground' focuses on constructs such as *motivation*, *mood* and *interest*. Among these, motivation has received the most attention among educational psychologists, but remained peripheral within mathematics education (see Hannula, this Issue; Mendick, 2002).

Another way of breaking new ground is in the use of innovative theoretical frameworks in mathematics education research. Arguably the most important problem for research on affect in mathematics is the understanding of the interrelationship between affect and cognition. It is interesting to note that the theoretical assumption about the interaction between affect and cognition is common to all the contributions presented in this Issue: it is assumed as a starting point, i.e. a working hypothesis, thus allowing researchers to investigate other aspects. This general hypothesis is further analyzed by proposing models to *explain* the nature of that interaction, for example identifying mediating factors between affect and cognition. In most cases the empirical data presented are used to test the model. Goldin and DeBellis interpret affect as a representational system – parallel to cognitive systems – that encodes important information regarding problem solving (DeBellis and Goldin, this Issue). 'Sociocultural' approaches emphasise the social basis and organisation of affective – and cognitive – experience. Thus, socio-constructivists see affect as primarily grounded in and defined by the social context (Op't Eynde et al., this Issue). And discursive approaches emphasise the social *practices* within which activity takes place, and the way that *positions* made available by these practices enable and constrain both thinking and the emotions experienced (Evans et al., this Issue). Another way of connecting affect and cognition is taking constructs and processes of the self and self-regulation as the combining feature of powerful affect and cognition (Malmivuori, this Issue). Alternatively, affect can also be seen as the object of self-regulation, which moves the focus onto processes that regulate the generation and development of affective constructs, such as motivation (Hannula, this Issue). Embodied cognition approaches see mind and body as inextricably linked (Brown and Reid, this Issue).

Though there has been intermittent attention to psychoanalytic approaches among mathematics education researchers (e.g. Tahta, 1993), this framework promises to provide distinctive resources for studying affect (see Evans, et al., this Issue).

The aim of this Special Issue is to consider the usefulness, for mathematics education research and practice, of a range of theoretical approaches, showing how they can shed light on the following research questions:

- Which dimensions of affect are most relevant to mathematics education?
- How is affect involved in, and part of, the development of mathematical thinking and behaviour?

A special feature of the Special Issue is that we aim to show how different frameworks can help in interpreting and intervening in students' learning processes, through the analysis of an empirical account of a particular student's solving of a mathematical problem in the classroom. The context and method used in the production of the transcripts of the efforts of a middle school student, 'Frank', is described here (Op't Eynde and Hannula, this Issue), and each of the articles then offers an initial analysis of this same case.

The six theoretical approaches to affect presented in this Special Issue vary in the major focus chosen by the researchers:

1. Valerie DeBellis and Gerald A. Goldin assume the notion of affect as one of a system of representations internal to the individual, introducing the concepts of meta-affect, mathematical intimacy and mathematical integrity.

2. Marja-Liisa Malmivuori proposes a model that focuses on ongoing self-evaluation and self-regulation processes, stressing the dynamic aspects of mathematics learning.

3. Markku S. Hannula conceptualises motivation as a potential to direct behaviour through the mechanisms that control emotion, structured through needs and goals. He also discusses aspects of motivation regulation.

4. Laurinda Brown and David A. Reid assume somatic markers as the emotional basis for decision making, consisting of interconnected emotions, feelings of emotions, and thoughts.

5. Peter Op't Eynde, Erik DeCorte and Lieven Verschaffel take a socio-constructivist perspective on learning and emotions, assuming a component systems approach that refers to emotions as constituted by the dynamic interplay of cognitive, physiological, and motivational processes in a specific context.

6. Jeff Evans, Candia Morgan and Anna Tsatsaroni give a central role to the notion of discourse in considering emotions as socially organised and shaped by power relations. They draw together strands from social semiotics, pedagogic discourse theories and psychoanalysis, so as to discuss how the positionings of individuals influence emotional experience and expression.

These six accounts will be followed by a reaction paper by Melissa Rodd, which discusses affect from a special needs perspective.

The key construct chosen by the authors of these theoretical frameworks (what Mason and Waywood, 1994, call their *weltanschauung*) suggests the categories that they need to classify their observations, i.e. the particular point of view that they choose. This point of view influences the kind of questions that they are capable of asking and answering, the range of phenomena that they cover, and also the way in which thay can inform practice.

Reflection leads us to conclude that the different approaches presented in this Special Issue are neither conflicting nor overlapping: they can be seen as complementary, as different lenses that allow researchers and teachers to assume different points of view, in order better to understand students' mathematical behavior.

Notes

1: Such an effect can be easily explained by males tending to have more positive attitudes than females with equal achievement levels. If we have two linear correlations where the graph for one gender lies higher on the y-axis than the other, the effect size will become smaller if the two populations are pooled together.

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