Relay Selection for Efficient HARQ-IR Protocols in Relay-Assisted Multisource Multicast Networks

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***Abstract*—This paper investigates relay selection for reliable data transmission in relay-assisted multisource multicast networks (RMMNs) where multiple source nodes distribute information to a set of destination nodes with the assistance of multiple relay nodes. Hybrid automatic repeat request with incremental redundancy (HARQ-IR) is used and supported by either a physical-layer network coding (PNC) or an analog network coding (ANC) technique employed at the relays. By deriving efficiency metrics of the HARQ-IR protocols, we propose relay selection schemes for RMMNs to minimize the transmission delay and energy consumption. Simulation results are provided to analyse each relay selection scheme.**

I. Introduction

Relay-assisted communications [1] is attracting an increasing interest in wireless communications with the aim of throughput enhancement and quality improvement by exploiting spatial diversity gains. Data transmission from source nodes to destination nodes is assisted by multiple relay nodes. Recently, network coding (NC) [2] has been applied at the relay nodes to improve network throughput [3], [4]. The relay nodes carry out either algebraic linear or logic operations on received packets from multiple source nodes, and then forward the newly generated packets to multiple destination nodes.

In wireless environments with deep fading and background noise, besides the design of high-throughput communication systems, the reliability of data transmission should also be taken into account. Dealing with this issue, hybrid automatic repeat request (HARQ) protocols were proposed to reliably deliver information over error-prone wireless channels [5]. Specifically, HARQ with incremental redundancy (HARQ-IR) was shown to achieve the ergodic capacity of fading and interference channels [6]. In general, within relay networks, data transmission from source nodes to destination nodes is carried out with the aid of one or multiple relays. The issue of relay selection (RS) is often considered so as to select the best relay for forwarding packets according to different selection criteria (e.g. minimizing bit error rate or maximizing throughput)[7], [8]. However, for NC-based relay-assisted multisource multicast networks (RMMNs), the RS for HARQ-IR protocols subject to either minimizing total transmission delay or minimizing total energy consumption has received little attention in the literature.

In this paper, we first formulate the two efficiency metrics, namely energy per bit (EB) and effective delay (ED), for HARQ-IR protocols[1] in a specific RMMN including two source nodes, N relay nodes and two destination nodes. In the considered RMMN, the relay nodes carry out either physical-layer network coding (PNC) [3] or analog network coding (ANC) [4] on the signals received from the two source nodes before forwarding to the two destination nodes. After deriving EB and ED of the HARQ-IR protocols with PNC and ANC schemes in RMMNs, we then propose two RS schemes subject to constraints on power allocation and location of the source and destination nodes. The first RS scheme is identified to minimize total transmission delay while the second scheme is designed to minimize total energy consumption in the system.

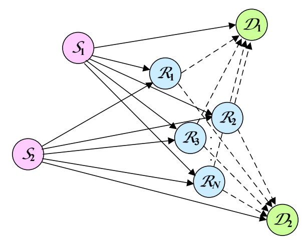


Fig. 1. System model of a relay-assisted multisource multicast network.

II. System Model

Fig. 1 illustrates a RMMN where data transmitted from two source nodes $\mathcal{S}_1$ and $\mathcal{S}_2$ to two destination nodes $\mathcal{D}_1$ and $\mathcal{D}_2$ is assisted by N relay nodes $\mathcal{R}^{(N)} = \{\mathcal{R}_1, \mathcal{R}_2, \ldots, \mathcal{R}_N\}$. A half-duplex system is considered where all nodes can either transmit or receive data, but not simultaneously. It is assumed to have no direct link between $\mathcal{S}_i$ and $\mathcal{D}_j$, $j = 1, 2$, $j \neq i$, due to either power limitation at $\mathcal{S}_i$ or distance between $\mathcal{S}_i$ and $\mathcal{D}_j$. In RMMN, the principle of NC is applied at $\mathcal{R}^{(N)}$ to help $\mathcal{S}_1$ and $\mathcal{S}_2$ simultaneously transmit two data packets $\mathbf{s}_1$ and $\mathbf{s}_2$ to $\mathcal{D}_1$ and $\mathcal{D}_2$ in two time slots. In the first time slot, $\mathcal{S}_1$ transmits $\mathbf{s}_1$ to $\mathcal{R}^{(N)}$ and $\mathcal{D}_1$ while $\mathcal{S}_2$ transmits $\mathbf{s}_2$ to $\mathcal{R}^{(N)}$ and $\mathcal{D}_2$. Then, $\mathcal{R}^{(N)}$ performs NC on the mixed signals received from $\mathcal{S}_1$ and $\mathcal{S}_2$ and broadcasts the network-coded signals to both $\mathcal{D}_1$ and $\mathcal{D}_2$ in the second time slot. Accordingly, $\mathcal{D}_1$ and $\mathcal{D}_2$ can extract the signals transmitted from $\mathcal{S}_2$ and $\mathcal{S}_1$, respectively. The data transmission in the first time slot

[1]For a wireless system using HARQ-IR, EB and ED can be tools for analysing energy efficiency [9].

consists of two direct (DR) phases ($\mathcal{S}_1 \to \mathcal{D}_1$ and $\mathcal{S}_2 \to \mathcal{D}_2$) and a multiple access (MA) phase ($\{\mathcal{S}_1\ \mathcal{S}_2\} \to \mathcal{R}^{(N)}$), while there is only a broadcast (BC) phase ($\mathcal{R}^{(N)} \to \{\mathcal{D}_1\ \mathcal{D}_2\}$) in the second time slot. Due to the broadcast nature of wireless medium, we assume that the DR and MA phases are carried out simultaneously using the same coding scheme.

The distances of links $\mathcal{S}_1 - \mathcal{D}_1$, $\mathcal{S}_1 - \mathcal{S}_2$, $\mathcal{S}_2 - \mathcal{D}_2$, $\mathcal{D}_1 - \mathcal{D}_2$, $\mathcal{S}_i - \mathcal{R}_n$ and $\mathcal{R}_n - \mathcal{D}_j$ are denoted by d_1, d_2, d_3, d_4, d_{iR_n} and d_{R_nj}, $i = 1, 2$, $j = 1, 2$, $n = 1, 2, \ldots, N$, respectively. The physical location angles $\widehat{\mathcal{D}_1\mathcal{S}_1\mathcal{S}_2}$, $\widehat{\mathcal{S}_1\mathcal{S}_2\mathcal{D}_2}$, $\widehat{\mathcal{S}_2\mathcal{D}_2\mathcal{D}_1}$, $\widehat{\mathcal{D}_2\mathcal{D}_1\mathcal{S}_1}$ and $\widehat{\mathcal{D}_1\mathcal{S}_1\mathcal{R}_n}$ are denoted by α_1, α_2, α_3, α_4 and α_{R_n}, respectively. The transmitting signal powers of the i-th source node and relay nodes are denoted by P_i, $i = 1, 2$ and P_R[2], respectively. The pathloss exponent between a pair of transceiver nodes is denoted by ν. The channel coefficients of $\mathcal{S}_i \to \mathcal{D}_i$, $\mathcal{S}_i \to \mathcal{R}_n$ and $\mathcal{R}_n \to \mathcal{D}_j$ links, are denoted by h_{ii}, h_{iR_n} and h_{R_nj}, respectively, which are assumed to be constant over the transmission of a data packet and vary independently in the next data packet with $\overline{|h_{11}|^2} = 1/d_1^\nu$, $\overline{|h_{22}|^2} = 1/d_3^\nu$, $\overline{|h_{iR_n}|^2} = 1/d_{iR_n}^\nu$ and $\overline{|h_{R_nj}|^2} = 1/d_{R_nj}^\nu$. Throughout this paper, $\bar{\omega}$ and $[\omega]_i$ denote the average and the i-th realization of a random variable ω, respectively.

III. Energy per Bit and Effective Delay in RMMNs

In this section, the efficiency metrics EB and ED of HARQ-IR protocols are derived for the RMMNs shown in Fig. 1.

A. EB and ED with PNC

Using the PNC scheme for HARQ-IR in RMMNs, in the first time slot, each relay performs joint decoding of two signals received from $\mathcal{S}_1$ and $\mathcal{S}_2$ in the MA phase [10], while $\mathcal{D}_1$ and $\mathcal{D}_2$ receive signals from $\mathcal{S}_1$ and $\mathcal{S}_2$, respectively, in the DR phase. Thus, the number of transmissions to $\mathcal{R}_n$, $n = 1, 2, \ldots, N$, in the MA phase can be determined through the MA channel capacity bound [6] as follows:

$$\begin{aligned}\tau^{(n)}_{\mathtt{PNC,MAC}} = \min\{k \Big| &\{\sum_{j=1}^{k} \log(1 + [\gamma_{1R_n}]_j) > r_1\} \\ &\cap \{\sum_{j=1}^{k} \log(1 + [\gamma_{2R_n}]_j) > r_2\} \\ &\cap \{\sum_{j=1}^{k} \log(1 + [\gamma_{1R_n}]_j + [\gamma_{2R_n}]_j) > r_1 + r_2\}\},\end{aligned} \tag{1}$$

where r_i, $i = 1, 2$, denotes the transmission rate at $\mathcal{S}_i$ and γ_{iR_n} denotes the signal-to-noise ratio (SNR) of $\mathcal{S}_i \to \mathcal{R}_n$ link. In fact, while the ultimate aim is to generate the network coded signal for two separated signals sent by $\mathcal{S}_1$ and $\mathcal{S}_2$, the relay nodes $\mathcal{R}^{(N)}$ may form the network coded signal from the received signal without explicitly decoding the original signals [10]. In this case, the condition at the right hand side of (1) becomes pessimistic and thus can be considered as an upper-bound.

[2]In this work, we assume that the relay nodes transmit signal with the same power.

In the DR phase, the number of transmissions required at $\mathcal{S}_i$ to transmit $\mathbf{s}_i$ to $\mathcal{D}_i$ can be computed by [6]

$$\tau_{\mathtt{PNC,DR}_i} = \min\{k \Big| \sum_{j=1}^{k} \log(1 + [\gamma_{ii}]_j) > r_i\}, \tag{2}$$

where γ_{ii}, $i = 1, 2$, denotes the SNR of $\mathcal{S}_i \to \mathcal{D}_i$ link and r_i denotes the transmission rate at $\mathcal{S}_i$. Since the data packet is retransmitted by $\mathcal{S}_i$ until $\mathcal{R}^{(N)}$ and $\mathcal{D}_i$ successfully decode, the number of transmissions at $\mathcal{S}_i$ and the total number of transmissions through $\mathcal{R}_n$ in the first time slot are respectively given by

$$\tau^{(n)}_{\mathtt{PNC,S}_i} = \max\{\tau^{(n)}_{\mathtt{PNC,MAC}}, \tau_{\mathtt{PNC,DR}_i}\}, \tag{3}$$

$$\tau^{(n)}_{\mathtt{PNC,1}} = \max\{\tau^{(n)}_{\mathtt{PNC,MAC}}, \tau_{\mathtt{PNC,DR}_1}, \tau_{\mathtt{PNC,DR}_2}\}. \tag{4}$$

After decoding successfully the data packets from both $\mathcal{S}_1$ and $\mathcal{S}_2$, each of $\mathcal{R}^{(N)}$ combines two decoded packets with the XOR operator [3], encodes the mixed packet, and then broadcasts the encoded packet to both $\mathcal{D}_1$ and $\mathcal{D}_2$ in the second time slot. The number of transmissions required at $\mathcal{R}_n$, $n = 1, 2, \ldots, N$, to transmit the mixed packet to $\mathcal{D}_i$, $i = 1, 2$, in the BC phase is similarly determined as

$$\tau^{(n)}_{\mathtt{PNC,BC}_i} = \min\{k \Big| \sum_{j=1}^{k} \log(1 + [\gamma_{R_ni}]_j) > r_{\bar{i}}\}, \tag{5}$$

where $\bar{i} = 1$ if $i = 2$ and $\bar{i} = 2$ if $i = 1$. Here, γ_{R_ni}, $i = 1, 2$, denotes the SNR of $\mathcal{R}_n \to \mathcal{D}_i$ link. In order to help both $\mathcal{D}_1$ and $\mathcal{D}_2$ detect the data packets from $\mathcal{S}_2$ and $\mathcal{S}_1$, respectively, $\mathcal{R}_n$ retransmits the packet until both $\mathcal{D}_1$ and $\mathcal{D}_2$ successfully detect it. Thus, the number of transmissions in the second time slot is computed by

$$\tau^{(n)}_{\mathtt{PNC,2}} = \max\{\tau^{(n)}_{\mathtt{PNC,BC}_1}, \tau^{(n)}_{\mathtt{PNC,BC}_2}\}. \tag{6}$$

Overall, the resulting ED and EB of the HARQ-IR PNC protocol for $\{\mathcal{S}_1, \mathcal{S}_2\} \to \mathcal{R}_n \to \{\mathcal{D}_1, \mathcal{D}_2\}$ links are respectively given by

$$\Delta^{(n)}_{\mathtt{PNC}} = \frac{\bar{\tau}^{(n)}_{\mathtt{PNC,1}} + \bar{\tau}^{(n)}_{\mathtt{PNC,2}}}{r_1 + r_2}, \tag{7}$$

$$\xi^{(n)}_{\mathtt{PNC}} = \frac{P_1\bar{\tau}^{(n)}_{\mathtt{PNC,S}_1} + P_2\bar{\tau}^{(n)}_{\mathtt{PNC,S}_2} + P_R\bar{\tau}^{(n)}_{\mathtt{PNC,2}}}{r_1 + r_2}. \tag{8}$$

B. EB and ED with ANC

With the ANC protocol, in the MA phase of the first time slot, $\mathcal{R}_n$, $n = 1, 2, \ldots, N$, receives the data packets from both $\mathcal{S}_1$ and $\mathcal{S}_2$, which can be written by

$$\mathbf{r}_n = \sqrt{P_1} h_{1R_n}\mathbf{s}_1 + \sqrt{P_2} h_{2R_n}\mathbf{s}_2 + \mathbf{n}_{R_n}, \tag{9}$$

where $\mathbf{n}_{R_n}$ denotes an independent CSCG noise vector of $\{\mathcal{S}_1, \mathcal{S}_2\} \to \mathcal{R}_n$ links with each entry having zero mean and unit variance. At the same time, $\mathcal{D}_i$, $i = 1, 2$, receives the data packet from $\mathcal{S}_i$ in the DR phase. Similarly, the number of transmissions $\tau_{\mathtt{ANC,DR}_i}$ is determined as $\tau_{\mathtt{PNC,DR}_i}$ in (2).

Prior to broadcasting the received signal to both $\mathcal{D}_1$ and $\mathcal{D}_2$, $\mathcal{R}_n$, $n = 1, 2, \ldots, N$, normalises its received signal $\mathbf{r}_n$ in (9) by a factor $\lambda_n = 1/\sqrt{E[|\mathbf{r}_n|^2]} = 1/\sqrt{\gamma_{1R_n} + \gamma_{2R_n} + 1}$ to have unit average energy. Thus, in the BC phase, the signals received at $\mathcal{D}_i$, $i = 1, 2$, from $\mathcal{R}_n$ can be written by

$$\mathbf{y}_{R_n i} = \sqrt{P_R} h_{R_n i} \lambda_n \mathbf{r}_n + \mathbf{n}_{R_n i}, \tag{10}$$

where $\mathbf{n}_{R_n i}$, $i = 1, 2$, $n = 1, 2, \ldots, N$, denotes an independent CSCG noise vector of $\mathcal{R}_n \to \mathcal{D}_i$ link with each entry having zero mean and unit variance. Then, $\mathcal{D}_i$ detects $\mathbf{s}_{\bar{i}}$ by cancelling $\mathbf{s}_i$ which is detected in the DR phase. The resulting SNR $\gamma_{\bar{i}}^{(n)}$ at $\mathcal{D}_i$ is expressed by

$$\gamma_{\bar{i}}^{(n)} = \frac{\gamma_{R_n i} \gamma_{\bar{i} R_n}}{\gamma_{R_n i} + \gamma_{\bar{i} R_n} + \gamma_{i R_n} + 1}. \tag{11}$$

Therefore, with the HARQ-IR protocol, the number of transmissions required at $\mathcal{S}_i$, $i = 1, 2$, to transmit $\mathbf{s}_i$ to $\mathcal{D}_{\bar{i}}$ through $\mathcal{R}_n$, $n = 1, 2, \ldots, N$, is determined by

$$\tau_{\mathtt{ANC}_i}^{(n)} = \min\{k \Big| \sum_{j=1}^{k} \log(1 + [\gamma_i^{(n)}]_j) > r_i\}. \tag{12}$$

The total number of transmissions at $\mathcal{S}_i$, $i = 1, 2$, is accordingly given by

$$\tau_{\mathtt{ANC},\mathtt{S}_i}^{(n)} = \max\{\tau_{\mathtt{ANC}_i}^{(n)}, \tau_{\mathtt{ANC},\mathtt{DR}_i}\} \tag{13}$$

It is noted that, with the ANC protocol, the retransmission of the lost packets at $\mathcal{D}_1$ and $\mathcal{D}_2$ is carried out by $\mathcal{S}_1$ and $\mathcal{S}_2$. $\mathcal{R}_n$ only amplifies and forwards to $\mathcal{D}_1$ and $\mathcal{D}_2$ the data received from $\mathcal{S}_1$ and $\mathcal{S}_2$. This means that the number of transmissions at $\mathcal{R}_n$ to assist $\mathcal{S}_1$ and $\mathcal{S}_2$ is also given by $\tau_{\mathtt{ANC}_1}^{(n)}$ and $\tau_{\mathtt{ANC}_2}^{(n)}$, respectively, and, $\mathcal{R}_n$ uses half power for each task. Therefore, the resulting ED and EB of the HARQ-IR ANC protocol are respectively obtained as

$$\Delta_{\mathtt{ANC}}^{(n)} = \frac{\bar{\tau}_{\mathtt{ANC},\mathtt{S}_1}^{(n)} + \bar{\tau}_{\mathtt{ANC},\mathtt{S}_2}^{(n)} + \bar{\tau}_{\mathtt{ANC}_1}^{(n)} + \bar{\tau}_{\mathtt{ANC}_2}^{(n)}}{r_1 + r_2}, \tag{14}$$

$$\xi_{\mathtt{ANC}}^{(n)} = \frac{P_1 \bar{\tau}_{\mathtt{ANC},\mathtt{S}_1}^{(n)} + P_2 \bar{\tau}_{\mathtt{ANC},\mathtt{S}_2}^{(n)} + \frac{P_R}{2} \bar{\tau}_{\mathtt{ANC},1}^{(n)} + \frac{P_R}{2} \bar{\tau}_{\mathtt{ANC},2}^{(n)}}{r_1 + r_2}. \tag{15}$$

IV. Relay Selection in RMMNs

In multi-relay networks, various RS schemes are considered to select the best relay to help the source forward data to the destination [7], [8]. However, the RS for HARQ-IR protocols in NC-based RMMNs for either optimal ED or EB has attracted little attention. In this section, based on the derived EB/ED in the previous section, we propose RS algorithms for given location and power constraints in RMMNs.

Let P denote the total power constraint of transmitting nodes $\mathcal{S}_1$, $\mathcal{S}_2$ and $\mathcal{R}^{(N)}$ (i.e. $P_n = P_1 + P_2 + NP_R$). Also, let us denote ρ_1, ρ_2 and $(1 - \rho_1 - \rho_2)/N$ as the fractions of power allocated to $\mathcal{S}_1$, $\mathcal{S}_2$ and $\mathcal{R}_n$, $n = 1, 2, \ldots, N$, respectively. Accordingly, $P_1 = \rho_1 P$, $P_2 = \rho_2 P$ and $P_{R_n} = (1 - \rho_1 - \rho_2) P_n / N$.

The problem relates to how to select the best relay node so as to either minimize the ED or EB of all the multicast transmissions from two source nodes to two destination nodes. As shown in Fig. 1, the location of $\mathcal{R}_n$, $n = 1, 2, \ldots, N$, can be determined through the distance between $\mathcal{S}_1$ and $\mathcal{R}_n$ (i.e. d_{1R_n}), and the angle $\widehat{\mathcal{D}_1 \mathcal{S}_1 \mathcal{R}_n}$ (i.e. α_{R_n}). Based on d_{1R_n} and α_{R_n}, we can easily evaluate the distance from $\mathcal{R}_n$ to $\mathcal{S}_2$, $\mathcal{D}_1$ and $\mathcal{D}_2$.

Let $\{d_{1R,\Delta_\mathtt{X}}^*, \alpha_{R,\Delta_\mathtt{X}}^*\}$ and $\{d_{1R,\xi_\mathtt{X}}^*\ \alpha_{R,\xi_\mathtt{X}}^*\}$, $\mathtt{X} \in \{\mathtt{PNC}, \mathtt{ANC}\}$, denote the positioning parameters of the best relay $\mathcal{R}^*$ using $\mathtt{X}$ protocol subject to minimizing $\{\Delta_\mathtt{X}^{(n)}\}$ and $\{\xi_\mathtt{X}^{(n)}\}$, respectively. The RS problem is expressed as

$$\mathcal{R}^* = \arg \min_{\{\mathcal{R}_n\}} \{\Delta_\mathtt{X}^{(n)}\}, \tag{16}$$

$$\mathcal{R}^* = \arg \min_{\{\mathcal{R}_n\}} \{\xi_\mathtt{X}^{(n)}\}, \tag{17}$$

where $\Delta_\mathtt{X}^{(n)}$ and $\xi_\mathtt{X}^{(n)}$ are generally given by (7), (14), (8) and (15), respectively.

It can be observed that the complexity of the above RS problem increases as the number of relay nodes increases due to the computation of $\{\Delta_\mathtt{X}^{(n)}\}$ and $\{\xi_\mathtt{X}^{(n)}\}$ for every relay node. Therefore, in this paper, we propose a new mapping-based RS scheme to reduce the search complexity. We first determine the optimal relay positions with respect to different power allocation at the source and relay nodes, different objectives (minimum ED/EB), and different relaying schemes (PNC/ANC). Then, for each scenario of power allocation, the best relay can be selected by choosing the relay which is located nearest to the determined optimal relay positions.

The optimal relay positions can be determined through

$$\{d_{1R,\Delta_\mathtt{X}}^*, \alpha_{R,\Delta_\mathtt{X}}^*\} = \arg \min_{\{d_{1R}, \alpha_R\}} \Delta_\mathtt{X}, \tag{18}$$

$$\{d_{1R,\xi_\mathtt{X}}^*, \alpha_{R,\xi_\mathtt{X}}^*\} = \arg \min_{\{d_{1R}, \alpha_R\}} \xi_\mathtt{X}. \tag{19}$$

For simplicity, let us investigate a specific scenario where $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \pi/2$. Accordingly, $d_1 = d_3$ and $d_2 = d_4$. With the total power constraint P and different power allocation at $\mathcal{S}_1$ and $\mathcal{S}_2$, there are three typical cases as follows:

A. Equal power at the source nodes

Due to the equal power allocation at $\mathcal{S}_1$ and $\mathcal{S}_2$, the optimal relay position should be located on the median line between the pair nodes $\{\mathcal{S}_1, \mathcal{D}_1\}$ and $\{\mathcal{S}_2, \mathcal{D}_2\}$. Let us denote $d_{0R} = \sqrt{d_{1R}^2 - d_2^2/4}$. The position of the optimal relay in (18) and (19) can be determined through

$$d_{0R,\Delta_\mathtt{X}}^* = \arg \min_{0 < d_{0R} < d_1} \Delta_\mathtt{X}^{(n)}, \tag{20}$$

$$d_{0R,\xi_\mathtt{X}}^* = \arg \min_{0 < d_{0R} < d_1} \xi_\mathtt{X}^{(n)}, \tag{21}$$

where $\mathtt{X} \in \{\mathtt{PNC}, \mathtt{ANC}\}$. Then, we can determine $\{d_{1R,\Delta_\mathtt{X}}^*, \alpha_{R,\Delta_\mathtt{X}}^*\}$ and $\{d_{1R,\xi_\mathtt{X}}^*\ \alpha_{1R,\xi_\mathtt{X}}^*\}$ as

$$d_{1R,\Delta_\mathtt{X}}^* = \sqrt{d_{0R,\Delta_\mathtt{X}}^{*2} + \frac{d_2^2}{4}}, \alpha_{R,\Delta_\mathtt{X}}^* = \tan^{-1}\left(\frac{d_2}{2d_{0R,\Delta_\mathtt{X}}^*}\right), \tag{22}$$

$$d_{1R,\xi_\mathtt{X}}^* = \sqrt{d_{0R,\xi_\mathtt{X}}^{*2} + \frac{d_2^2}{4}}, \alpha_{R,\xi_\mathtt{X}}^* = \tan^{-1}\left(\frac{d_2}{2d_{0R,\xi_\mathtt{X}}^*}\right). \tag{23}$$

TABLE I
POSITION OF RELAY NODES.

Relay	1	2	3	4	5	6	7
x	0.25	0.65	0.8	0.43	0.8	0.5	0.85
y	0.33	0.24	0.4	0.1	0.13	0.44	0.25

It can be observed that the search algorithms using (20), (21), (22) and (23) require a lower complexity processing than an exhaustive search of all available relay positions in the whole system.

B. More power at $\mathcal{S}_1$

In this scenario, the optimal relay should be located in the neighbourhood region of the pair node $\{\mathcal{S}_2, \mathcal{D}_2\}$. Thus, the search range for the optimal relay can be limited by two regions defined as follows:

$$\text{Region (I):} \begin{cases} \tan^{-1}\left(\frac{d_2}{2d_1}\right) < \alpha_{R_n} < \tan^{-1}\left(\frac{d_2}{d_1}\right), \\ \frac{d_2}{2\sin\alpha_{R_n}} < d_{1R_n} < \frac{d_1}{\cos\alpha_{R_n}}. \end{cases} \tag{24}$$

$$\text{Region (II):} \begin{cases} \tan^{-1}\left(\frac{d_2}{d_1}\right) < \alpha_{R_n} < \frac{\pi}{2}, \\ \frac{d_2}{2\sin\alpha_{R_n}} < d_{1R_n} < \frac{d_2}{\sin\alpha_{R_n}}. \end{cases} \tag{25}$$

With various relays in regions (I) and (II), we can then determine the location of the optimal relay $\{d^*_{1R,\Delta_\mathtt{X}}, \alpha^*_{R,\Delta_\mathtt{X}}\}$ and $\{d^*_{1R,\xi_\mathtt{X}}, \alpha^*_{R,\xi_\mathtt{X}}\}$, $\mathtt{X} \in \{\mathtt{PNC}, \mathtt{ANC}\}$, subject to minimizing either ED or EB as in (18) and (19). Regarding the search range in the context $P_1 > P_2$, it can be observed that the search regions (I) and (II) are narrower than the whole region, and thus the complexity of the search for the best relay is reduced.

C. More power at $\mathcal{S}_2$

Similarly, in this scenario, the optimal relay should be located near the two nodes $\mathcal{S}_1$ and $\mathcal{D}_1$. The search range for the optimal relay can thus be limited by two regions as:

$$\text{Region (III):} \begin{cases} 0 < \alpha_{R_n} < \tan^{-1}\left(\frac{d_2}{2d_1}\right), \\ 0 < d_{1R_n} < \frac{d_1}{\cos\alpha_{R_n}}. \end{cases} \tag{26}$$

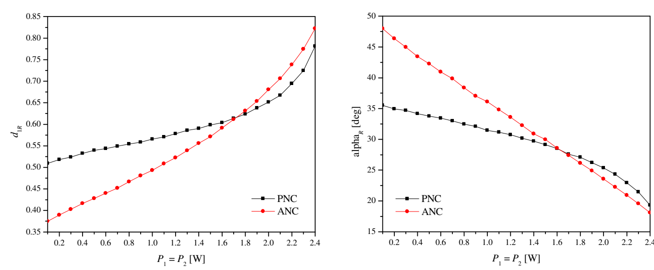
$$\text{Region (IV):} \begin{cases} \tan^{-1}\left(\frac{d_2}{2d_1}\right) < \alpha_{R_n} < \frac{\pi}{2}, \\ 0 < d_{1R_n} < \frac{d_2}{2\sin\alpha_{R_n}}. \end{cases} \tag{27}$$

Then, we can then determine the location of the optimal relay $\{d^*_{1R,\Delta_\mathtt{X}}, \alpha^*_{R,\Delta_\mathtt{X}}\}$ and $\{d^*_{1R,\xi_\mathtt{X}}, \alpha^*_{R,\xi_\mathtt{X}}\}$, $\mathtt{X} \in \{\mathtt{PNC}, \mathtt{ANC}\}$, in regions (III) and (IV) so as to minimize either ED or EB. Additionally, it can be observed that the search regions (III) and (IV) for the scenario $P_1 < P_2$ are also narrower than the whole region, and again a low-complexity search algorithm is achieved.

Given node locations, power allocation at source nodes, and relaying scheme at relay nodes, the algorithm corresponding to the proposed mapping-based RS scheme is summarized in Algorithm 1.

Algorithm 1 RS algorithm for HARQ-IR protocols

Step 1: Determine search range and optimal relay positions:
if $P_1 = P_2$ **then**
  Determine $\{d^*_{1R,\Delta_\mathtt{X}}, \alpha^*_{R,\Delta_\mathtt{X}}\}$ and $\{d^*_{1R,\xi_\mathtt{X}} \alpha^*_{R,\xi_\mathtt{X}}\}$ (see (22) and (23))
else if $P_1 > P_2$ **then**
  Determine $\{d^*_{1R,\Delta_\mathtt{X}}, \alpha^*_{R,\Delta_\mathtt{X}}\}$ and $\{d^*_{1R,\xi_\mathtt{X}} \alpha^*_{R,\xi_\mathtt{X}}\}$ in regions (I) and (II) (see (24) and (25))
else
  Determine $\{d^*_{1R,\Delta_\mathtt{X}}, \alpha^*_{R,\Delta_\mathtt{X}}\}$ and $\{d^*_{1R,\xi_\mathtt{X}} \alpha^*_{R,\xi_\mathtt{X}}\}$ in regions (III) and (IV) (see (26) and (27))
end if
Step 2: Find the best relay:
for $n = 1$ to N **do**
  Select $\mathcal{R}_n$ closest to $\{d^*_{1R,\Delta_\mathtt{X}}, \alpha^*_{R,\Delta_\mathtt{X}}\}$ for minimum ED
  Select $\mathcal{R}_n$ closest to $\{d^*_{1R,\xi_\mathtt{X}} \alpha^*_{R,\xi_\mathtt{X}}\}$ for minimum EB
end for



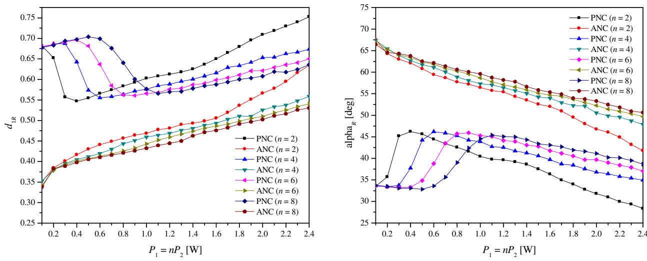
(a) (b)

Fig. 2. The optimal relay locations for minimum ED: (a) d_{1R}, (b) α_R.

V. SIMULATION RESULTS

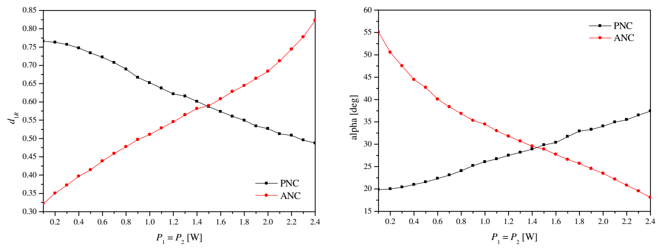
In this section, we present simulation results of the RS for either minimum ED or minimum EB in a RMMN using various HARQ-IR protocols. For simplicity, a symmetric network structure is considered with $d_1 = d_3 = 1$, $d_2 = d_4 = 1/2$ and $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \pi/2$. There are a total of 7 available relay nodes which are located in a scale of 1×1 unit of distance as shown in Table I. The data transmission from $\mathcal{S}_1$ and $\mathcal{S}_2$ to $\mathcal{D}_1$ and $\mathcal{D}_2$ is carried out at the same data rate (i.e. $r_1 = r_2 = R$) via HARQ-IR protocol with either DT or PNC or ANC schemes. All channels are assumed to experience quasi-static Rayleigh block fading. The pathloss exponent between a pair of transceiver nodes is assumed to be $\nu = 3$. Let us first investigate the search range and optimal relay locations for minimum ED and minimum EB. Based on these simulation results, we can easily choose the best relay nodes among available relays for HARQ-IR protocol with PNC and ANC schemes using Algorithm 1.

Figs. 2 and 3 sequentially plot the optimal relay locations for minimum ED as a function of power allocation when HARQ-IR protocols are employed with PNC and ANC for various scenarios of power allocations at the source nodes. Fig. 2 considers the scenario of equal power allocation (i.e. $P_1 = P_2$) while Fig. 3 investigates the scenario of unequal



(a) (b)

Fig. 3. The optimal relay locations for minimum ED : (a) d_{1R}, (b) α_R.



(a) (b)

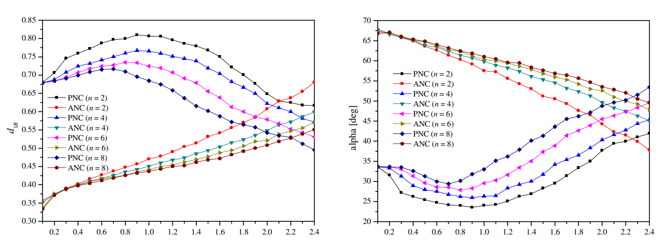
Fig. 4. The optimal relay locations for minimum EB: (a) d_{1R} and (b) α_R.

power allocation $P_1 = nP_2$, $n > 1$[3].We assume that $R = 5$ bps and $P_1 + P_2 + P_R = 5$ W. The optimal relay locations in Figs. 2 and 3 are determined through d_{1R} and α_R using the proposed algorithms in previous section for different power allocations.

Investigating the optimal relay locations for minimum EB in RMMNs, Figs. 4 and 5 sequentially plot the optimal relay locations as a function of power allocation with HARQ-IR PNC and ANC protocols for various scenarios of power allocations at the source nodes. The power allocations at the two source nodes are similarly assumed as in Figs. 2 and 3. We also assume that $R = 5$ bps and $P_1 + P_2 + P_R = 5$ W. Using the proposed algorithms in Section IV for different power allocations, the optimal relay locations for minimum EB are determined as in Figs. 4 and 5.

The above simulation results determine the optimal relay locations for minimum ED and minimum EB in RMMNs with respect to various NC-based HARQ-IR protocols and

[3]Note that, for the scenario $P_2 = nP_1$, the optimized relay locations can be similarly observed to be symmetric with those for the scenario $P_1 = nP_2$.



(a) (b)

Fig. 5. The optimal relay locations for minimum EB: (a) d_{1R}, (b) α_R.

TABLE II

THE BEST RELAY NODES FOR MINIMUM ED AND MINIMUM EB WITH VARIOUS HARQ-IR PROTOCOLS AND POWER ALLOCATIONS.

	Power allocation	PNC	ANC
Minimum ED	$P_1 = P_2 = 2.4$ W	$\mathcal{R}_2$	$\mathcal{R}_7$
	$P_1 = 2P_2 = 2.4$ W	$\mathcal{R}_5$	$\mathcal{R}_4$
	$P_1 = P_2/2 = 2.4$ W	$\mathcal{R}_6$	$\mathcal{R}_3$
Minimum EB	$P_1 = P_2 = 2.4$ W	$\mathcal{R}_1$	$\mathcal{R}_7$
	$P_1 = 2P_2 = 2.4$ W	$\mathcal{R}_5$	$\mathcal{R}_4$
	$P_1 = P_2/2 = 2.4$ W	$\mathcal{R}_6$	$\mathcal{R}_3$

various scenarios of power allocations at the source nodes. From Algorithm 1, we can easily select the best relay for different criteria as summarised in Table II. In this example, we assume $P_1 + P_2 + P_R = 5$ W and consider 3 scenarios: $P_1 = P_2 = 2.4$ W, $P_1 = 2P_2 = 2.4$ W and $P_1 = P_2/2 = 2.4$ W.

VI. CONCLUSIONS

In this paper, we have investigated the RS for data transmission in an HARQ-IR based RMMN consisting of two source nodes, multiple relay nodes and two destination nodes. The efficiency metrics, EB and ED, have been first derived for HARQ-IR protocols with PNC and ANC in RMMNs by taking into account the effects of both relay location and power allocation of source nodes. Algorithms for choosing the best relay have been developed for the HARQ-IR protocols with PNC and ANC to minimize either the ED or the EB in the RMMN. Finally, simulation results have been provided to determine the best relays for minimum ED and minimum EB in the RMMN.

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