Robust Optimization with Probabilistic Constraints for Power-Efficient and Secure SWIPT

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***Abstract*—In this paper, we propose beamforming schemes to simultaneously transmit data to multiple information receivers (IRs) while transfering power wirelessly to multiple energy-harvesting receivers (ERs). Taking into account the imperfection of the instantaneous channel state information, we introduce a probabilistic-constrained optimization problem to minimize the total transmit power while guaranteeing *data transmission reliability*, *secure data transmission*, and *power transfer reliability*. As the proposed optimization problem is non-convex and has an infinite number of constraints, we propose two robust reformulations of the original problem adopting safe-convex-approximation techniques. The derived robust formulations are in semidefinite programming forms, hence, they can be effectively solved by standard convex optimization packages. Simulation results confirm the superiority of the proposed approaches to a baseline scheme in guaranteeing transmission security.**

I. Introduction

Beamforming is a promising candidate to realize focused far-field electromagnetic radiation [1] for simultaneous wireless information and power transfer (SWIPT) in practice. Using advanced signal processing techniques for multiple antennas, transmit powers and phases across the transmit antennas can be designed such that information beams are steered towards multiple information receivers (IRs) while energy beams are directed at energy-harvesting receivers (ERs) [2]–[6]. The beamforming design problem is usually formulated as an optimization problem taking into account the system's quality of service (QoS) requirements[1]. In particular, channel state information (CSI) between the transmitter and the receivers is exploited to optimally control the phase and power of the beamformer. In practice, the system requirements are normally provided/decided by the system operator/designer while the CSI is obtained by some channel estimation technique. Due to the nature of wireless channels, errors in CSI estimation are unavoidable [5], [7]. However, using the estimated CSI directly as input for the beamforming design may result in a resource allocation mismatch. More importantly, such designs cannot guarantee any QoSs for users of the system. Therefore, robust beamforming designs taking into account the imperfection of the CSI are desirable for practical SWIPT systems.

[1]The system QoS requirements may include for example the minimum signal-to-interference-plus-noise ratios (SINRs) of the IRs, the minimum received power at the ERs, and the maximum tolerable leakage SINRs of the IRs to the ERs.

Robust beamforming designs for SWIPT have been mostly based on a worst-case approach [5], [7], [8]. Specifically, the imperfection of the CSI link between the transmitter and the receiver is captured by an error vector with random elements. Due to the randomness and continuity of the error vector, an infinite number of constraints are needed to guarantee the QoS which makes the problem intractable. To overcome this obstacle, the norms of the error vectors are assumed to be bounded by known values. Then, using the S-procedure [9], the QoS constraints of the SWIPT system are replaced by a finite number of constraints representing upper bounds on the CSI errors [5], [7], [8], [10]. This conservative design approach requires an exceeding amount of system resources to protect rarely occurring worst cases. Hence, less conservative approaches have recently been proposed by accepting the violation of the QoS constraints with certain probabilities [11], [12].

This paper focuses on power-efficient transmission strategies for SWIPT by minimizing the total transmit power. Taking into account the imperfection of the CSI, we first formulate an outage-based probabilistic optimization problem that minimizes the total transmit power subject to the following three sets of QoS constraints: i) the probability that the received SINR at each IR is above a required level is higher than a predefined target; ii) the probability that the IRs' leakage SINRs at ERs exceed a secure level is below a threshold; iii) the probability that the power received by an ER is above a required level is greater than a prescribed value. The aforementioned three types of constraints guarantee *data transmission reliability*, *secure data transmission*, and *power transfer reliability*, respectively. Since the error in the estimated CSI is modelled as a complex continuous random variable, the number of constraints in the proposed optimization problem is infinite. Furthermore, the probabilistic constraints are non-convex. To tackle these challenges, we adopt two mathematical tools, i.e., the S-procedure [9] and the Bernstein-type inequality [13], to formulate two safe approximations [14] of the proposed optimization problem. The derived safe approximations lead to tractable semidefinite programmes (SDPs) which are convex and serve as performance upper bounds for the original minimization problem.

This paper differs from the related works in [11], [12] in the following aspects. Communication security was not considered in [11]. While this paper studies a secure SWIPT wireless system, [12] considered a secure SWIPT cognitive system. Furthermore, the problem formulation proposed in this paper can guarantee that the IRs' leakage SINRs at the ERs remain below a secure level, e.g. below the decoding sensitivity should the ERs try to eavesdrop. In contrast, the approach in [12] cannot accomplish this. In fact, while the secrecy rate of the IR is kept above certain required level in [12], the ERs may still be able to eavesdrop IR's message if their decoding sensitivity levels are lower than the IR leakage SINR. Hence, the approach in [12] is less secure than the proposed approaches. Also, the optimization problem considered in this paper is more challenging than its counterparts in [11], [12] as secure information transmission for multiple IRs is ensured whereas the problem in [11] does not consider secrecy at all and the problem in [12] protects only a single IR.

Notation: The following notations are used in this paper. y or Y: a scalar; $\mathbf{y}$: a column vector; $\mathbf{Y}$: a matrix; $\|\cdot\|$: the standard Euclidean norm; $\|\cdot\|_F$: the Frobenius norm; $(\cdot)^H$: the complex conjugate transpose operator; $\mathrm{Tr}(\cdot)$: the trace operator; $\Pr(\cdot)$: the probability of an event; $\mathbf{Y} \succeq \mathbf{0}$: $\mathbf{Y}$ is positive semidefinite; $\mathbf{I}_M$: the $M \times M$ identity matrix; $\mathrm{Re}\{\cdot\}$: the real part of a complex number; $\mathrm{Eig}_{\max}(\mathbf{Y})$: the maximum eigenvalue of $\mathbf{Y}$; $s^+(\mathbf{Y})$: $\max\{\mathrm{Eig}_{\max}(\mathbf{Y}), 0\}$; $\mathrm{vec}(\mathbf{Y})$: stacking all the entries of $\mathbf{Y}$ into a column vector; $\mathbb{R}$: the set of all real scalars ; $\mathbb{C}^{M\times 1}$: the set of all $M \times 1$ vectors with complex elements; $\mathbb{H}^{M\times M}$: the set of all $M \times M$ Hermitian matrices; $y \sim \mathcal{CN}(0, \sigma^2)$: y is a zero-mean circularly symmetric complex Gaussian random variable with variance σ^2; $\mathbf{y} \sim \mathcal{CN}(\mathbf{0}, \mathbf{Y})$: $\mathbf{y}$ is a zero-mean circularly symmetric complex Gaussian random vector with covariance matrix $\mathbf{Y}$; and finally $\mathbf{Y}^{1/2}$: the square root of matrix $\mathbf{Y}$.

II. System Model

In this paper, we consider a scenario where a transmitter equipped with M antennas simultaneously transmits information and power to U IRs and N ERs, respectively, via radio frequency signals. All IRs and ERs are equipped with a single-antenna. Let $\mathbf{h}_i \in \mathbb{C}^{M\times 1}$ and $\mathbf{g}_t \in \mathbb{C}^{M\times 1}$ represent the actual channel coefficients of the ith IR and the tth ER, respectively. Let $\mathbf{w}_i \in \mathbb{C}^{M\times 1}$ and $s_i^{(I)} \sim \mathcal{CN}(0,1)$, respectively, denote the beamforming vector of and the data to be transmitted to the ith IR. Let $\mathbf{v}_t \in \mathbb{C}^{M\times 1}$ and $s_t^{(E)} \sim \mathcal{CN}(0,1)$, respectively, be the artificial-noise beamforming vector and the artificial noise for the tth ER. The overall signals received by the ith IR and the tth ER are, respectively, given by

$$y_i^{(I)} = \sum_{j=1}^{U} \mathbf{h}_i^H \mathbf{w}_j s_j^{(I)} + \sum_{t=1}^{N} \mathbf{h}_i^H \mathbf{v}_t s_t^{(E)} + n_i^{(I)} \text{ and } \quad (1)$$

$$y_t^{(E)} = \sum_{j=1}^{U} \mathbf{g}_t^H \mathbf{w}_j s_j^{(I)} + \sum_{p=1}^{N} \mathbf{g}_t^H \mathbf{v}_p s_p^{(E)} + n_t^{(E)}. \quad (2)$$

Here, $n_i^{(I)}$ and $n_t^{(E)}$ are the zero-mean circularly symmetric complex additive white Gaussian noises with variance σ^2, i.e., $n_i^{(I)}, n_t^{(E)} \sim \mathcal{CN}(0, \sigma^2)$, observed at the ith IR and the tth ER, respectively.

We assume that the CSI estimation is imperfect. This is modelled as $\mathbf{h}_i = \widetilde{\mathbf{h}}_i + \Delta\mathbf{h}_i$ and $\mathbf{g}_t = \widetilde{\mathbf{g}}_t + \Delta\mathbf{g}_t$, where $\widetilde{\mathbf{h}}_i \in \mathbb{C}^{M\times 1}$ and $\Delta\mathbf{h}_i \in \mathbb{C}^{M\times 1}$ are the estimated value of $\mathbf{h}_i$ and the corresponding error, respectively; $\widetilde{\mathbf{g}}_t \in \mathbb{C}^{M\times 1}$ and $\Delta\mathbf{g}_t \in \mathbb{C}^{M\times 1}$ are the estimated value of $\mathbf{g}_t$ and the corresponding error, respectively. We further assume that $\Delta\mathbf{h}_i \sim \mathcal{CN}(\mathbf{0}, \mathbf{H}_i)$ and $\Delta\mathbf{g}_t \sim \mathcal{CN}(\mathbf{0}, \mathbf{G}_t)$, where $\mathbf{H}_i \succeq \mathbf{0}$ and $\mathbf{G}_t \succeq \mathbf{0}$ are the error covariance matrices which are assumed to be known for beamformer design. Let $\Delta\mathbf{h}_i = \mathbf{H}_i^{1/2}\mathbf{e}_i$ and $\Delta\mathbf{g}_t = \mathbf{G}_t^{1/2}\mathbf{r}_t$, where $\mathbf{e}_i \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_M)$, $\mathbf{r}_t \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_M)$. Let $\{\mathbf{w}_i\} = \{\mathbf{w}_1, \cdots, \mathbf{w}_U\}$ be the set of candidate data beamforming vectors for all IRs and let $\{\mathbf{v}_t\} = \{\mathbf{v}_1, \cdots, \mathbf{v}_N\}$ be the set of candidate artificial-noise beamforming vectors. The SINR at the ith IR, denoted by $\Gamma_i(\{\mathbf{w}_i\}, \{\mathbf{v}_t\})$, and the *leakage SINR* of the ith IR observed at the tth ER, denoted by $\Gamma_i^{(t)}(\{\mathbf{w}_i\}, \{\mathbf{v}_t\})$, are given in (3) and (4) at the top of next page, respectively. The total power received by the tth ER, denoted by $\Phi_t(\{\mathbf{w}_i\}, \{\mathbf{v}_t\})$, is given as

$$\Phi_t(\{\mathbf{w}_i\}, \{\mathbf{v}_t\}) = \sum_{i=1}^{U} \mathbf{w}_i^H \left(\widetilde{\mathbf{g}}_t + \mathbf{G}_t^{1/2}\mathbf{r}_t\right)\left(\widetilde{\mathbf{g}}_t + \mathbf{G}_t^{1/2}\mathbf{r}_t\right)^H \mathbf{w}_i + \sum_{p=1}^{N} \mathbf{v}_p^H \left(\widetilde{\mathbf{g}}_t + \mathbf{G}_t^{1/2}\mathbf{r}_t\right)\left(\widetilde{\mathbf{g}}_t + \mathbf{G}_t^{1/2}\mathbf{r}_t\right)^H \mathbf{v}_p. \quad (5)$$

Hereafter, if otherwise stated, $\{i, j\} \in \{1, \cdots, U\}$, and $\{t, p\} \in \{1, \cdots, N\}$.

III. Proposed Robust Probabilistic-Constrained Optimization Problem

For secure information transmission, ERs are considered as potential eavesdroppers as they may be able to decode overheard messages intended for IRs. In order to reduce the information leakage, the quantities of the signals intended for the IRs but received at the ERs should be minimized. On the other hand, due to the low energy conversion efficiency at the ERs, a high received power level is required at each ER to compensate for the power-conversion loss. Therefore, artificial noise is utilized to satisfy these contradictory goals [5], [8].

The communication between the transmitter and the IRs and the power transfer to the ERs are considered as in outage if either one of the following cases occurs. (1) The SINR level at the ith IR falls bellow the required level γ_i, $\forall i$, which is referred to as *SINR outage*. (2) The leakage-SINR of the ith IR at the tth ER is above the secure level $\gamma_i^{(t)}$, $\forall i, \forall t$, which is referred to as *leakage-SINR outage*. (3) The received power at the tth ER is below the required level P_t, $\forall t$, which is referred to as *power-transfer outage*.

Aiming to design a power-efficient beamforming scheme, we find sets of data beamforming vectors $\{\mathbf{w}_i\}$ and artificial-noise beamforming vectors $\{\mathbf{v}_t\}$ that minimize the total transmit power subject to the probabilities of the SINR outage, leakage-SINR outage, and power-transfer outage being kept

$$\Gamma_i\left(\{\mathbf{w}_i\},\{\mathbf{v}_t\}\right)=\frac{\mathbf{w}_i^H\left(\widetilde{\mathbf{h}}_i+\mathbf{H}_i^{1/2}\mathbf{e}_i\right)\left(\widetilde{\mathbf{h}}_i+\mathbf{H}_i^{1/2}\mathbf{e}_i\right)^H\mathbf{w}_i}{\sum_{j=1,j\neq i}^{U}\mathbf{w}_j^H\left(\widetilde{\mathbf{h}}_i+\mathbf{H}_i^{1/2}\mathbf{e}_i\right)\left(\widetilde{\mathbf{h}}_i+\mathbf{H}_i^{1/2}\mathbf{e}_i\right)^H\mathbf{w}_j+\sum_{t=1}^{N}\mathbf{v}_t^H\left(\widetilde{\mathbf{h}}_i+\mathbf{H}_i^{1/2}\mathbf{e}_i\right)\left(\widetilde{\mathbf{h}}_i+\mathbf{H}_i^{1/2}\mathbf{e}_i\right)^H\mathbf{v}_t+\sigma^2} \quad (3)$$

$$\Gamma_i^{(t)}\left(\{\mathbf{w}_i\},\{\mathbf{v}_t\}\right)=\frac{\mathbf{w}_i^H\left(\widetilde{\mathbf{g}}_t+\mathbf{G}_t^{1/2}\mathbf{r}_t\right)\left(\widetilde{\mathbf{g}}_t+\mathbf{G}_t^{1/2}\mathbf{r}_t\right)^H\mathbf{w}_i}{\sum_{j=1,j\neq i}^{U}\mathbf{w}_j^H\left(\widetilde{\mathbf{g}}_t+\mathbf{G}_t^{1/2}\mathbf{r}_t\right)\left(\widetilde{\mathbf{g}}_t+\mathbf{G}_t^{1/2}\mathbf{r}_t\right)^H\mathbf{w}_j+\sum_{p=1}^{N}\mathbf{v}_p^H\left(\widetilde{\mathbf{g}}_t+\mathbf{G}_t^{1/2}\mathbf{r}_t\right)\left(\widetilde{\mathbf{g}}_t+\mathbf{G}_t^{1/2}\mathbf{r}_t\right)^H\mathbf{v}_p+\sigma^2} \quad (4)$$

below predefined maximum tolerable levels. Hence, we introduce the following optimization problem:

$$\begin{aligned}\min_{\{\mathbf{w}_i\},\{\mathbf{v}_t\}} \quad & \sum_{i=1}^{U}\mathbf{w}_i^H\mathbf{w}_i+\sum_{p=1}^{N}\mathbf{v}_p^H\mathbf{v}_p\\ \text{s. t.} \quad & \Pr\left(\Gamma_i\left(\{\mathbf{w}_i\},\{\mathbf{v}_t\}\right)\geq\gamma_i\right)\geq 1-\rho_i,\forall i,\\ & \Pr\left(\Gamma_i^{(t)}\left(\{\mathbf{w}_i\},\{\mathbf{v}_t\}\right)\leq\gamma_i^{(t)}\right)\geq 1-\rho_i^{(t)},\forall i,\forall t,\\ & \Pr\left(\Phi_t\left(\{\mathbf{w}_i\},\{\mathbf{v}_t\}\right)\geq P_t\right)\geq 1-\varrho_t,\forall t,\end{aligned} \quad (6)$$

where $\rho_i\in(0,1]$, $\rho_i^{(t)}\in(0,1]$, and $\varrho_t\in(0,1]$ are the maximum tolerable SINR outage, leakage-SINR outage, and power-transfer outage, respectively. The events in the first and second sets of probabilistic constraints in (6) are non-convex with respect to $\{\mathbf{w}_i\}$ and $\{\mathbf{v}_t\}$. In the sequel, we transform them into convex forms by introducing new variables. We also cast the events into quadratic forms of error vectors.

To this end, we define data beamforming matrix $\mathbf{W}_i=\mathbf{w}_i\mathbf{w}_i^H$ and artificial-noise beamforming matrix $\mathbf{V}_t=\mathbf{v}_t\mathbf{v}_t^H$ where $\mathbf{W}_i\succeq\mathbf{0}$, $\mathbf{V}_t\succeq\mathbf{0}$, $\mathbf{W}_i$ and $\mathbf{V}_t$ are rank-one matrices[2]. Then, using $\mathbf{x}^H\mathbf{y}\mathbf{y}^H\mathbf{x}=\mathbf{y}^H\mathbf{x}\mathbf{x}^H\mathbf{y}$, we rewrite $\Gamma_i\left(\{\mathbf{w}_i\},\{\mathbf{v}_t\}\right)\geq\gamma_i$ as:

$$\left(\widetilde{\mathbf{h}}_i+\mathbf{H}_i^{1/2}\mathbf{e}_i\right)^H\mathbf{A}_i\left(\widetilde{\mathbf{h}}_i+\mathbf{H}_i^{1/2}\mathbf{e}_i\right)\geq\sigma^2, \quad (7)$$

where $\mathbf{A}_i=\left(1+\frac{1}{\gamma_i}\right)\mathbf{W}_i-\mathbf{C}$ and $\mathbf{C}=\sum_{j=1}^{U}\mathbf{W}_j+\sum_{t=1}^{N}\mathbf{V}_t$. Further manipulations with a note of covariance matrix property, i.e., $\mathbf{H}_i^H=\mathbf{H}_i,\ \forall i$, lead to the following equivalent form of (7):

$$\begin{aligned}f_i(\mathbf{e}_i)\ \triangleq\ & \mathbf{e}_i^H\mathbf{H}_i^{1/2}\mathbf{A}_i\mathbf{H}_i^{1/2}\mathbf{e}_i+2\mathrm{Re}\{\mathbf{e}_i^H\mathbf{H}_i^{1/2}\mathbf{A}_i\widetilde{\mathbf{h}}_i\}\\ & +\widetilde{\mathbf{h}}_i^H\mathbf{A}_i\widetilde{\mathbf{h}}_i-\sigma^2\geq 0.\end{aligned} \quad (8)$$

Similarly, $\Gamma_i^{(t)}\left(\{\mathbf{w}_i\},\{\mathbf{v}_t\}\right)\leq\gamma_i^{(t)}$ can be recast as

$$\begin{aligned}k_i^{(t)}(\mathbf{r}_t)\ \triangleq\ & \mathbf{r}_t^H\mathbf{G}_t^{1/2}\mathbf{B}_i\mathbf{G}_t^{1/2}\mathbf{r}_t+2\mathrm{Re}\{\mathbf{r}_t^H\mathbf{G}_t^{1/2}\mathbf{B}_i\widetilde{\mathbf{g}}_t\}\\ & +\widetilde{\mathbf{g}}_t^H\mathbf{B}_i\widetilde{\mathbf{g}}_t+\sigma^2\geq 0,\end{aligned} \quad (9)$$

where $\mathbf{B}_i=\mathbf{C}-\left(1+\frac{1}{\gamma_i^{(t)}}\right)\mathbf{W}_i$. Furthermore, $\Phi_t\left(\{\mathbf{w}_i\},\{\mathbf{v}_t\}\right)\geq P_t$ is equivalent to:

$$\begin{aligned}d_t(\mathbf{r}_t)\ \triangleq\ & \mathbf{r}_t^H\mathbf{G}_t^{1/2}\mathbf{C}\mathbf{G}_t^{1/2}\mathbf{r}_t+2\mathrm{Re}\{\mathbf{r}_t^H\mathbf{G}_t^{1/2}\mathbf{C}\widetilde{\mathbf{g}}_t\}\\ & +\widetilde{\mathbf{g}}_t^H\mathbf{C}\widetilde{\mathbf{g}}_t-P_t\geq 0.\end{aligned} \quad (10)$$

[2]A matrix is rank-one if it has only one linearly independent column/row.

Using (8), (9), and (10) along with relaxing the rank-one constraints on $\mathbf{W}_i$ and $\mathbf{V}_t$, (6) is converted to the following optimization problem:

$$\begin{aligned}\min_{\{\mathbf{W}_i\},\{\mathbf{V}_t\}} \quad & \mathrm{Tr}\left(\sum_{i=1}^{U}\mathbf{W}_i+\sum_{p=1}^{N}\mathbf{V}_p\right)\\ \text{s. t.} \quad & \Pr\left(f_i(\mathbf{e}_i)\geq 0\right)\geq 1-\rho_i,\ \forall i,\\ & \Pr\left(k_i^{(t)}(\mathbf{r}_t)\geq 0\right)\geq 1-\rho_i^{(t)},\ \forall i,\forall t,\\ & \Pr\left(d_t(\mathbf{r}_t)\geq 0\right)\geq 1-\varrho_t,\forall t,\\ & \mathbf{W}_i\succeq\mathbf{0},\ \forall i,\ \mathbf{V}_t\succeq\mathbf{0},\ \forall t,\end{aligned} \quad (11)$$

where $\{\mathbf{W}_i\}=\{\mathbf{W}_1,\cdots,\mathbf{W}_U\}$ and $\{\mathbf{V}_t\}=\{\mathbf{V}_1,\cdots,\mathbf{V}_N\}$ are two sets of beamforming matrices. Since $\mathbf{e}_i$ and $\mathbf{r}_t$ are continuous random vectors, the number of constraints in optimization problem (11) is infinite. Furthermore, solving problem (11) is challenging due to the fact that the probabilistic constraints neither have simple closed-forms nor are they convex.

We tackle the intractable probabilistic constraints in (11) by replacing them by computationally tractable, i.e., convex, approximations [14]. These approximations result in optimization problem that are convex with respect to the original variables, i.e., $\{\mathbf{W}_i\}$ and $\{\mathbf{V}_t\}$, and possibly some additional variables. The process is regarded as a safe approximation if every feasible solution to the approximated problem is also feasible for the original problem (11). In other words, the optimal solution to the safe approximation problem is a feasible suboptimal solution to the original problem. Therefore, the problem based on the safe approximation serves as an upper bound for the original problem [15]. In the following sections, we introduce two approaches to derive robust problems using two different safe approximations of (11).

IV. S-Procedure Based Approach

Let us assume that $\mathbf{e}_i$ and $\mathbf{r}_t$ are confined to the complex spherical sets $\xi_i\triangleq\{\mathbf{e}_i\in\mathbb{C}^{M\times 1}\mid\|\mathbf{e}_i\|^2\leq R_i^2\}$ and $\psi_t\triangleq\{\mathbf{r}_t\in\mathbb{C}^{M\times 1}\mid\|\mathbf{r}_t\|^2\leq Q_t^2\}$ with M dimensions and radii R_i and Q_t, respectively. Since the error vector $\mathbf{e}_i\sim\mathcal{CN}(0,\mathbf{I}_M)$ is confined to the spherical set ξ_i, the probabilistic constraint

$$\Pr\left(f_i(\mathbf{e}_i)\geq 0\right)\geq 1-\rho_i \quad (12)$$

holds if [15]

$$f_i(\mathbf{e}_i)\geq 0 \text{ and } \Pr\left(\mathbf{e}_i\in\xi_i\right)\geq 1-\rho_i. \quad (13)$$

The second condition in (13) is always true if the radius of the spherical set ξ_i is selected such that

$$R_i = \sqrt{\frac{\mathfrak{T}_m\left(1-\rho_i\right)}{2}}, \tag{14}$$

where $\mathfrak{T}_m(\cdot)$ is the inverse cumulative distribution function of the Chi-square random variable with $m = 2M$ degrees of freedom. Therefore, using $\|\mathbf{e}_i\|^2 = \mathbf{e}_i^H \mathbf{I}_M \mathbf{e}_i$, one can conclude that (12) can be safely approximated by the following two constraints

$$f_i(\mathbf{e}_i) \geq 0 \text{ and } \mathbf{e}_i^H \mathbf{I}_M \mathbf{e}_i - \frac{\mathfrak{T}_m\left(1-\rho_i\right)}{2} \leq 0. \tag{15}$$

Applying similar steps for $\Pr\left(k_i^{(t)}(\mathbf{r}_t) \geq 0\right) \geq \rho_i^{(t)}$ and $\Pr\left(d_t(\mathbf{r}_t) \geq 0\right) \geq \varrho_t$, we introduce the safe approximation to problem (11) as

$$\begin{aligned}
\min_{\{\mathbf{W}_i\},\{\mathbf{V}_t\}} \quad & \operatorname{Tr}\left(\sum_{i=1}^{U} \mathbf{W}_i + \sum_{p=1}^{N} \mathbf{V}_p\right) \\
\text{s. t.} \quad & f_i(\mathbf{e}_i) \geq 0, \ \forall i, \\
& \mathbf{e}_i^H \mathbf{I}_M \mathbf{e}_i - \frac{\mathfrak{T}_m\left(1-\rho_i\right)}{2} \leq 0, \ \forall i, \\
& k_i^{(t)}(\mathbf{r}_t) \geq 0, \ \forall i, \forall t, \\
& \mathbf{r}_t^H \mathbf{I}_M \mathbf{r}_t - \Omega \leq 0, \ \forall t, \\
& d_t(\mathbf{r}_t) \geq 0, \forall t, \\
& \mathbf{r}_t^H \mathbf{I}_M \mathbf{r}_t - \Omega \leq 0, \ \forall t, \\
& \mathbf{W}_i \succeq \mathbf{0}, \ \forall i, \ \mathbf{V}_t \succeq \mathbf{0}, \ \forall t,
\end{aligned} \tag{16}$$

where $\Omega = \min\left(\frac{\mathfrak{T}_m\left(1-\rho_i^{(t)}\right)}{2}, \frac{\mathfrak{T}_m\left(1-\varrho_t\right)}{2}\right)$.

Remark 1: The values of the radii R_i and Q_t of the spherical sets ξ_i and ψ_t are not required in (16) as they have been implicit incorporated into the outages, e.g. $R_i = \sqrt{\frac{\mathfrak{T}_m(1-\rho_i)}{2}}$.

The number of constraints in (16) is still infinite[3] due to the randomness of the error vectors $\mathbf{e}_i$ and $\mathbf{r}_t$. To proceed, we first introduce the following lemma.

Lemma 1 (S-Procedure [9]): Let

$$m_n(\mathbf{x}) = \mathbf{x}^H \mathbf{Y}_n \mathbf{x} + 2\text{Re}\{\mathbf{x}^H \mathbf{y}_n\} + c_n, \ n \in \{1, 2\}, \tag{17}$$

where $\mathbf{Y}_n \in \mathbb{H}^{M\times M}$, $\mathbf{y}_n \in \mathbb{C}^{M\times 1}$, and $c_n \in \mathbb{R}$. If there exists an $\check{\mathbf{x}}$ such that $m_n(\check{\mathbf{x}}) < 0$, then $\forall \mathbf{x} \in \mathbb{C}^{M\times 1}$, the following statements are equivalent:

1) $m_1(\mathbf{x}) \geq 0$ and $m_2(\mathbf{x}) \leq 0$ are satisfied $\forall \mathbf{x} \in \mathbb{C}^{M\times 1}$.
2) There exists a $\beta \geq 0$ such that

$$\begin{bmatrix} \mathbf{Y}_1 + \beta \mathbf{Y}_2 & \mathbf{b}_1 + \beta \mathbf{b}_2 \\ \mathbf{b}_1^H + \beta \mathbf{b}_2^H & c_1 + \beta c_2 \end{bmatrix} \succeq \mathbf{0}. \tag{18}$$

Adopting Lemma 1, one can transform optimization problem (16) into the standard convex SDP form in (19) given at the top of next page where α_i, λ_i^t, and β_t are auxiliary variables.

[3]Problem (16) is an semi-infinite optimization problem, i.e., an optimization problem with a finite number of variables and an infinite number of constraints.

To arrive at (19), we have relaxed the rank-one conditions on the beamforming matrices $\mathbf{W}_i$ and $\mathbf{V}_t$. From our simulation results, we have observed that if problem (19) is feasible then it yields rank-one optimal solutions[4]. As a result, beamforming vectors $\mathbf{w}_i^\star$ and $\mathbf{v}_t^\star$ are obtained as the products of the eigenvectors and their corresponding eigenvalues of the optimal rank-one matrices $\mathbf{W}_i^\star$ and $\mathbf{V}_t^\star$, respectively. Since (19) is a safe approximation of (6), $\mathbf{w}_i^\star$ and $\mathbf{v}_t^\star$ are suboptimal solutions to the original problem (6).

V. Bernstein-type-inequality based Approach

In this section, the bounded-norm conditions on the error vectors are relaxed. We adopt a different approach to find a robust formulation, i.e., safe convex approximation to the original problem (11). To begin, let us recall the following lemma.

Lemma 2 (Bernstein-type inequality [13]): Consider the following random expression $f(\mathbf{x}) = \mathbf{x}^H \mathbf{Y} \mathbf{x} + 2\text{Re}\{\mathbf{x}^H \mathbf{u}\}$, where $\mathbf{x} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_M)$, $\mathbf{Y} \in \mathbb{H}^{M\times M}$, and $\mathbf{u} \in \mathbb{C}^{M\times 1}$. For all $\delta > 0$, the following statement always holds:

$$\Pr\left(f(\mathbf{x}) \geq \operatorname{Tr}(\mathbf{Y}) - \sqrt{2\delta}\sqrt{\|\mathbf{Y}\|_F^2 + 2\|\mathbf{u}\|^2} - \delta s^+(\mathbf{Y})\right) \geq 1 - e^{-\delta}. \tag{20}$$

With $\delta_i = -\ln \rho_i$ and Lemma 2, the constraint $\Pr\left(f_i(\mathbf{e}_i) \geq 0\right) \geq 1 - \rho_i$ in (11) can be rewritten as (21) at the top of the next page. Then, by introducing two auxiliary variables θ_i and ϑ_i, (21) is further cast as:

$$\operatorname{Tr}\left(\mathbf{H}_i^{1/2} \mathbf{A}_i \mathbf{H}_i^{1/2}\right) - \sqrt{2\delta_i}\theta_i - \delta_i \vartheta_i \geq \sigma^2 - \widetilde{\mathbf{h}}_i^H \mathbf{A}_i \widetilde{\mathbf{h}}_i, \tag{22}$$

$$\sqrt{\|\mathbf{H}_i^{1/2} \mathbf{A}_i \mathbf{H}_i^{1/2}\|_F^2 + 2\|\mathbf{H}_i^{1/2} \mathbf{A}_i \widetilde{\mathbf{h}}_i\|^2} \leq \theta_i, \tag{23}$$

$$\vartheta_i \mathbf{I}_M + \mathbf{H}_i^{1/2} \mathbf{A}_i \mathbf{H}_i^{1/2} \succeq \mathbf{0}, \tag{24}$$

$$\vartheta_i \geq 0. \tag{25}$$

Note that (23) can be equivalently written as a second-order-cone (SOC) constraint

$$\left\| \begin{bmatrix} \operatorname{vec}\left(\mathbf{H}_i^{1/2} \mathbf{A}_i \mathbf{H}_i^{1/2}\right) \\ \sqrt{2}\mathbf{H}_i^{1/2} \mathbf{A}_i \widetilde{\mathbf{h}}_i \end{bmatrix} \right\| \leq x_i. \tag{26}$$

Similarly, setting $\delta_i^{(t)} = -\ln \rho_i^{(t)}$, and using two auxiliary variables $\theta_i^{(t)}$ and $\vartheta_i^{(t)}$, the constraint $\Pr\left(k_i^{(t)}(\mathbf{r}_t) \geq 0\right) \geq 1 - \rho_i^{(t)}$ in (11) is equivalent to the following constraints:

$$\operatorname{Tr}\left(\mathbf{G}_t^{1/2} \mathbf{B}_i \mathbf{G}_t^{1/2}\right) - \sqrt{2\delta_i^{(t)}}\theta_i^{(t)} - \delta_i^{(t)} \vartheta_i^{(t)} \geq -\sigma^2 - \widetilde{\mathbf{g}}_t^H \mathbf{B}_i \widetilde{\mathbf{g}}_t, \tag{27}$$

$$\left\| \begin{bmatrix} \operatorname{vec}\left(\mathbf{G}_t^{1/2} \mathbf{B}_i \mathbf{G}_t^{1/2}\right) \\ \sqrt{2}\mathbf{G}_t^{1/2} \mathbf{B}_i \widetilde{\mathbf{g}}_t \end{bmatrix} \right\| \leq \theta_i^{(t)}, \tag{28}$$

$$\vartheta_i^{(t)} \mathbf{I}_M + \mathbf{G}_t^{1/2} \mathbf{B}_i \mathbf{G}_t^{1/2} \succeq \mathbf{0}, \tag{29}$$

$$\vartheta_i^{(t)} \geq 0. \tag{30}$$

[4]A proof for the rank-one optimal solution to problem (19) is excluded here due to the constrained space. The proof will be provided in a full report of this work.

$$\begin{aligned}
\min_{\{\mathbf{W}_i\},\{\mathbf{V}_t\},\alpha_i,\lambda_i^t,\beta_t} \quad & \mathrm{Tr}\left(\sum_{i=1}^{U}\mathbf{W}_i+\sum_{p=1}^{N}\mathbf{V}_p\right)\\
\text{s. t.} \quad & \begin{bmatrix}\mathbf{H}_i^{1/2}\mathbf{A}_i\mathbf{H}_i^{1/2}+\alpha_i\mathbf{I}_M & \mathbf{H}_i^{1/2}\mathbf{A}_i\tilde{\mathbf{h}}_i\\ \tilde{\mathbf{h}}_i^H\mathbf{A}_i\mathbf{H}_i^{1/2} & \tilde{\mathbf{h}}_i^H\mathbf{A}_i\mathbf{h}_i-\sigma^2-\alpha_i\frac{\mathfrak{I}_m(1-\rho_i)}{2}\end{bmatrix}\succeq\mathbf{0},\ \alpha_i\geq 0,\ \forall i,\\
& \begin{bmatrix}\mathbf{G}_t^{1/2}\mathbf{B}_i\mathbf{G}_t^{1/2}+\lambda_i^{(t)}\mathbf{I}_M & \mathbf{G}_t^{1/2}\mathbf{B}_i\tilde{\mathbf{g}}_t\\ \tilde{\mathbf{g}}_t^H\mathbf{B}_i\mathbf{G}_t^{1/2} & \tilde{\mathbf{g}}_t^H\mathbf{B}_i\tilde{\mathbf{g}}_t+\sigma^2-\lambda_i^{(t)}\Omega\end{bmatrix}\succeq\mathbf{0},\ \lambda_i^{(t)}\geq 0,\ \forall i,\forall t,\\
& \begin{bmatrix}\mathbf{G}_t^{1/2}\mathbf{C}\mathbf{G}_t^{1/2}+\beta_t\mathbf{I}_M & \mathbf{G}_t^{1/2}\mathbf{C}\tilde{\mathbf{g}}_t\\ \tilde{\mathbf{g}}_t^H\mathbf{C}\mathbf{G}_t^{1/2} & \tilde{\mathbf{g}}_t^H\mathbf{C}\mathbf{g}_t-P_t-\beta_t\Omega\end{bmatrix}\succeq\mathbf{0},\ \beta_t\geq 0,\ \forall t,\\
& \mathbf{W}_i\succeq\mathbf{0},\ \forall i,\ \mathbf{V}_t\succeq\mathbf{0},\ \forall t.
\end{aligned} \tag{19}$$

$$\mathrm{Tr}\left(\mathbf{H}_i^{1/2}\mathbf{A}_i\mathbf{H}_i^{1/2}\right)-\sqrt{2\delta_i}\sqrt{\|\mathbf{H}_i^{1/2}\mathbf{A}_i\mathbf{H}_i^{1/2}\|_F^2+2\|\mathbf{H}_i^{1/2}\mathbf{A}_i\widetilde{\mathbf{h}}_i\|^2}-\delta_i s^+\left(\mathbf{H}_i^{1/2}\mathbf{A}_i\mathbf{H}_i^{1/2}\right)\geq\sigma^2-\widetilde{\mathbf{h}}_i^H\mathbf{A}_i\widetilde{\mathbf{h}}_i. \tag{21}$$

Using the same approach, the constraint $\Pr\left(d_t(\mathbf{r}_t)\geq 0\right)\geq 1-\varrho_t$ in (11) is recast as:

$$\mathrm{Tr}\left(\mathbf{G}_t^{1/2}\mathbf{C}\mathbf{G}_t^{1/2}\right)-\sqrt{2\mu_t}a_t-\mu_t b_t\geq P_t-\widetilde{\mathbf{g}}_t^H\mathbf{C}\widetilde{\mathbf{g}}_t, \tag{31}$$

$$\left\|\begin{bmatrix}\mathrm{vec}\left(\mathbf{G}_t^{1/2}\mathbf{C}\mathbf{G}_t^{1/2}\right)\\ \sqrt{2}\mathbf{G}_t^{1/2}\mathbf{C}\widetilde{\mathbf{g}}_t\end{bmatrix}\right\|\leq a_t, \tag{32}$$

$$b_t\mathbf{I}_M+\mathbf{G}_t^{1/2}\mathbf{C}\mathbf{G}_t^{1/2}\succeq\mathbf{0}, \tag{33}$$

$$b_t\geq 0, \tag{34}$$

where $\mu_t=-\ln\varrho_t$; a_t and b_t are auxiliary variables. Therefore, the safe approximation to problem (11) can be written as

$$\begin{aligned}
\min_{\{\mathbf{W}_i\},\{\mathbf{V}_t\},\theta_i,\vartheta_i,\theta_i^{(t)},\vartheta_i^{(t)},a_t,b_t} \quad & \mathrm{Tr}\left(\sum_{i=1}^{U}\mathbf{W}_i+\sum_{p=1}^{N}\mathbf{V}_p\right)\\
\text{s. t.} \quad & (22),(26),(24),(25)\ \forall i,\\
& (27),(28),(29),(30),\ \forall i,\forall t,\\
& (31),(32),(33),(34),\forall t,\\
& \mathbf{W}_i\succeq\mathbf{0},\ \forall i,\ \mathbf{V}_t\succeq\mathbf{0},\ \forall t.
\end{aligned} \tag{35}$$

The SOC constraints in (26), (28), and (32) are simple cases of an SDP since any SOC constraint can be recast in SDP form using the Schur complement [9]. Hence, optimization problem (35) is convex.

We note that we have also relaxed the rank-one conditions on the beamforming matrices $\mathbf{W}_i$ and $\mathbf{V}_t$ to derive the safe approximation (35) of the original problem (6). However, from our simulations, the optimal solutions to (35) are always rank-one if the problem is feasible[5]. Hence, we can obtain beamforming vectors $\mathbf{w}_i^\star$ and $\mathbf{v}_t^\star$ using the same approach as in Section IV as suboptimal solutions to the original problem.

[5]Again, a proof for the rank-one optimal solution to problem (35) is excluded here due to the constrained space. The proof will be provided in a full report of this work.

VI. Simulation Results

We evaluate the performance of the two proposed approaches and compare them against the probabilistic-constraint-based scheme introduced in [11] which is considered as the baseline scheme. For the baseline scheme, there is no guarantee for information security, i.e., no leakage-SINR-outage constraint. Hereafter, we refer to the S-Procedure based approach and the Bernstein-type-inequality based approach as Approach-I and Approach-II, respectively. Since the optimization problems in (19), i.e., Approach-I, and (35), i.e., Approach-II, are convex [9], numerical convex program solvers such as the SeDuMi provided by the CVX optimization package [16] can be adopted to obtain the sets of optimal beamforming matrices $\mathbf{W}_i^\star$ and $\mathbf{V}_t^\star$.

A. Simulation Setup

Consider a transmitter supporting two IRs and two ERs, i.e., $U=N=2$. The estimated channel vectors $\widetilde{\mathbf{h}}_i$ and $\widetilde{\mathbf{g}}_t$ are respectively modelled as:

$$\widetilde{\mathbf{h}}_i=H_i(l_i^{(I)})\mathbf{h}_{i,w},\ \text{and}\ \widetilde{\mathbf{g}}_t=G_t(l_t^{(E)})\mathbf{g}_{t,w},$$

where $\mathbf{h}_{i,w}\sim\mathcal{CN}(\mathbf{0},\mathbf{I}_M)$; $\mathbf{g}_{t,w}\sim\mathcal{CN}(\mathbf{0},\mathbf{I}_M)$;

$$H_i(l_i^{(I)})=\frac{C}{4\pi f_c}\left(\frac{1}{l_i^{(I)}}\right)^{\frac{\kappa}{2}};\ G_t(l_t^{(E)})=\frac{C}{4\pi f_c}\left(\frac{1}{l_t^{(E)}}\right)^{\frac{\kappa}{2}};$$

$l_i^{(I)}=100$ m, $\forall i$, and $l_t^{(E)}=9$ m, $\forall t$, are distances from the transmitter to an IR and an ER, respectively; $C=3\times 10^8$ ms^{-1} is the speed of light; $f_c=900$ MHz is the carrier frequency; and $\kappa=2.7$ is the pathloss exponent. The noise power at each IR and ER is assumed to be -70 dBm. The error covariance matrices are given as $\mathbf{H}_i=\varepsilon\left(H_i(l_i^{(I)})\right)^2\mathbf{I}_M$ and $\mathbf{G}_t=\varepsilon\left(G_t(l_t^{(E)})\right)^2\mathbf{I}_M$ where $\varepsilon=0.001$. Monte-Carlo simulations have been carried out over 500 channel realizations.

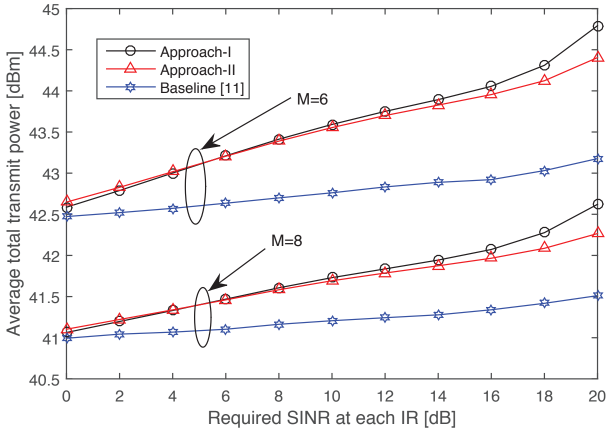

Fig. 1: Average total transmit power versus required IRs' SINR with different numbers of antennas. The secure level is set to $\gamma_i^{(t)} = -5$ dB $\forall i, \forall t$. The SINR outage, leakage-SINR outage, and power-transfer outage are set equal to 10 %, i.e., $\rho_i = \rho_i^{(t)} = \varrho_t = 0.1,\ \forall i, \forall t$. The required power level at the ERs are $P_t = -10$ dBm, $\forall t$.

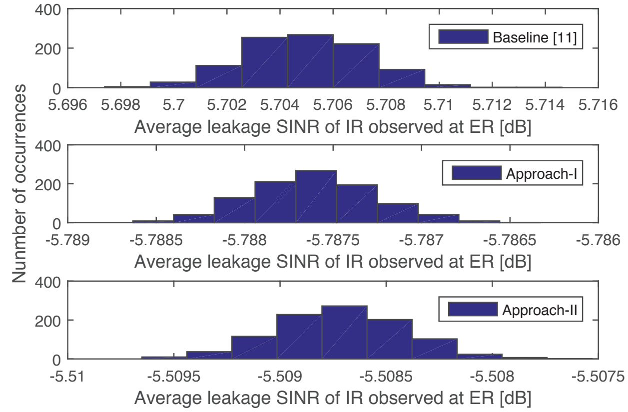

Fig. 2: The histograms of the average leakage SINRs of the IR signals observed at the ERs when the required IR SINR target is $\gamma_i = 18$ dB, $\forall i$. The number of antennas is $M = 6$. The secure level is $\gamma_i^{(t)} = -5$ dB, $\forall i, \forall t$. The SINR outage, leakage-SINR outage, and power-transfer outage are set equal to 10 %, i.e., $\rho_i = \rho_i^{(t)} = \varrho_t = 0.1,\ \forall i, \forall t$. The required power level at the ERs are $P_t = -10$ dBm, $\forall t$.

B. Performance Evaluation

Fig. 1 shows the average total transmit power versus the required SINR at each IR for different numbers of antennas for the proposed approaches and the baseline scheme in [11]. This figure indicates that the performances of the two proposed approaches are almost identical for IR SINRs from 0 dB to 12 dB. Approach-I consumes more power than Approach-II for relatively high target SINRs. The performance gap gradually increases from 0.05 dB to around 0.4 dB as the IR's SINR increases from 12 dB to 20 dB for both 6 and 8 antennas. This is due to the fact that a bounded-norm model has been imposed on the uncertainty set of the CSI for the derivation of Approach-I but not for Approach-II. This leads to a tighter approximation to the original problem for the latter compared to the former. The same performance trend of the two types of approximation have also been reported in [15] for conventional information transmission in MISO downlink scenarios.

Fig. 1 shows that the baseline scheme consumes the lowest power, e.g. around 0.8 dB and 0.5 dB less than the proposed approach with 6 and 8 antennas, respectively, at IR target SINRs of 10 dB. The price paid for this low power consumption is that there is no control on the leakage SINR of the IR signals observed at the ERs. This will be shown and discussed more in detail in Fig. 2. At relatively low IR target SINRs, i.e., less than 6 dB, the performances of the proposed approaches are close to that of the baseline. However, as the IR target SINRs increase, the performance gap between the proposed approaches and the baseline widens, i.e., up to 1.25 dB at a IR target SINR of 20 dB. The reason for this is that the leakage SINR of the IRs increases with the IR's SINR requirement. Therefore, a higher power consumption is required by the proposed approaches for pumping more artificial noise into the communication channels to confuse the ERs, hence, guaranteeing the required level of security for all IRs.

Fig. 2 shows the histogram of the average leakage SINR of the IRs measured at the ERs. To obtain the result in this figure, for each feasible channel realization[6], we first obtained beamforming vectors for all approaches. We then generated 1000 random error vectors associated with that channel realization to test the performance of each approach. The resulting leakage SINRs were averaged over the number of ERs and the number of feasible CSIs. In this experiment, the required secure level $\gamma_i^{(t)}$ is set to be -5 dB for all i and t. The results indicate that the information delivered to the IRs by the baseline approach are at a very high risk of being decoded by the ERs as the leakage SINR is about 11 dB higher than the required secure level for all occurrences. On the other hand, the proposed approaches successfully guarantee secure information transmission to the IRs as they push the leakage SINR well below the required secure level for all cases. The conservatism of Approach-I can be observed here as it pushes the leakage SINR around 0.7 dB below the requirement, i.e., 0.2 dB lower than Approach-II.

Fig. 3 shows the average total transmit power versus required power level P_t at the ERs for different numbers of antennas, i.e., $M = 6$ and $M = 8$, and different required IR SINR targets, i.e., $\gamma_i = 10$ dB in Fig. 3 (a) and $\gamma_i = 18$ dB in Fig. 3 (b), $\forall i$. The average transmit power appears to be a monotonically increasing function of the required power at the ERs. For a required receive power range of -10 dBm to 0 dBm, the performances of the two approaches are similar. The performance gap widens to around 0.8 dB as the ER's demand increases to 10 dBm. This again confirms the tightness approximation of Approach-I over Approach-II for relatively high ER power demands.

Fig. 4 illustrates the average total transmit power versus the outage level for different numbers of antennas, i.e., $M = 6$ in

[6]For feasible channel condition, all considered schemes return feasible solutions.

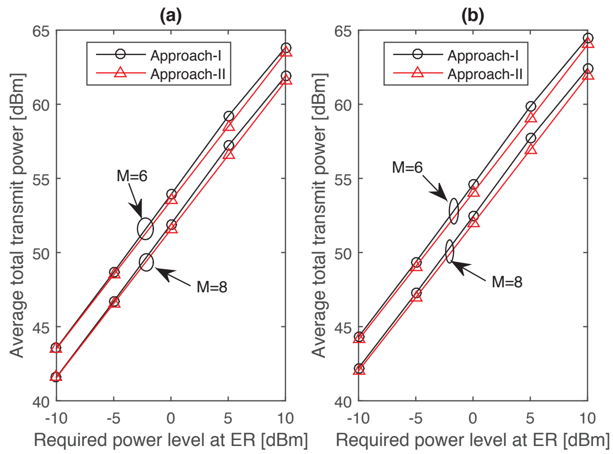

Fig. 3: Average total transmit power versus required power at each ER for different required IR SINR targets: (a) $\gamma_i = 10$ dB, $\forall i$; (b) $\gamma_i = 18$ dB, $\forall i$. The secure level $\gamma_i^{(t)} = -5$ dB $\forall i, \forall t$. The SINR outage, leakage-SINR outage, and power-transfer outage are set equal to 10 %, i.e., $\rho_i = \rho_i^{(t)} = \varrho_t = 0.1, \ \forall i, \forall t$.

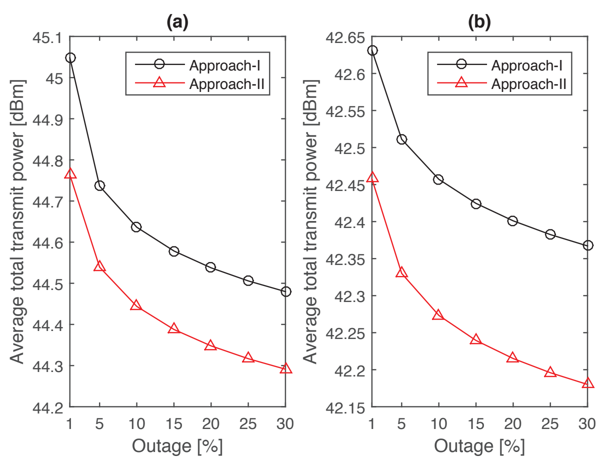

Fig. 4: Average total transmit power versus outage level for different numbers of transmit antennas: (a) $M = 6$; (b) $M = 8$. Here, we set equal outage levels for all types of constraints, i.e., $\rho_i = \rho_i^{(t)} = \varrho_t, \ \forall i, t$. The required IR SINR target is $\gamma_i = 18$ dB, $\forall i$. The secure level is $\gamma_i^{(t)} = -5$ dB, $\forall i, \forall t$. The required power level at the ERs are $P_t = -10$ dBm, $\forall t$.

Fig. 4 (a) and $M = 8$ in Fig. 4 (b). It can be seen from the figure that a small drop in the transmit power level significantly degrades the QoS. For instance, with $M = 6$, a drop of 0.57 dB in the transmit power of Approach-I and a drop of 0.47 dB in that of Approach-II causes a large increase in the outage from 1 % to 30 %. We have noted from our simulations that the stricter the requirements in terms of the outage are, the more likely it is that the problem becomes infeasible. This is because of the fact that the feasibility regions of our optimization problems are smaller for lower required outage probabilities.

Finally, from Figs. 1, 3, and 4, one can conclude that increasing the number of antennas reduces the power consumption of the considered approaches. This is a result of the improved beamformer resolution due to the extra spatial degrees of freedom introduced by additional antennas.

VII. Conclusions

We have proposed a probabilistic-constrained optimization problem for a SWIPT system to tackle the imperfection of the instantaneous CSI. To handle the non-convex QoS constraints, we have derived two robust formulations for the proposed problem adopting safe approximation techniques. The derived robust formulations are convex and the obtained optimal solutions can be considered as upper bounds for the original power minimization problem. Simulation results confirmed the superiority of the proposed approaches to a baseline scheme in guaranteeing secure data transmissions.

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