Economic Inquiry

THEORIES OF RISK: TESTING INVESTOR BEHAVIOR ON THE TAIWAN STOCK AND STOCK INDEX FUTURES MARKETS

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This article considers four utility functions—concave, convex, S-shaped, and reverse S-shaped—to analyze the behavior of different types of investors on the Taiwan stock index and its corresponding index futures. Using stochastic dominance (SD) rules, we show that the existence of all four investor types is plausible. Risk averters prefer spot to futures, whereas risk seekers prefer futures to spot. Investors with S-shaped utility functions prefer spot (futures) to futures (spot) when markets move upward (downward). Investors with reverse S-shaped utility functions prefer futures (spot) to spot (futures) when markets move upward (downward). We show that both spot and futures markets can exist when only risk averters are present, but futures can dominate spot only if there is some risk-seeking behavior. These results are robust with respect to subperiods, spot returns including dividends, and diversification. (*JEL* C14, G12, G15)

I. INTRODUCTION

Expected utility maximization and investor behavior toward risk lie at the heart of economic decision making in general and modern investment theory and practice in particular. Within this comprehensive framework, the intuitive attractiveness of mean-variance (MV) optimization, based on a single measure of risk, is the special case that is most widely accepted throughout the financial profession. However, the conditions for MV to be analytically consistent with expected utility maximization, such as quadratic utility functions or normally distributed returns, seldom hold in practice. Stochastic dominance (SD) is an alternative, more general approach to expected utility maximization that does not share this handicap. It requires neither a specific utility function nor a specific return distribution and is expressed in terms of probability distributions rather than the usual MV parameters of standard deviation and return. In this article, we use both the MV criterion (Markowitz 1952) and SD procedures (Hanoch and Levy 1969) to examine the preferences for different types of investors on the Taiwan stock index and its corresponding index futures. Our findings have implications for risk preference theory and behavioral economics.

Compared with traditional methods of portfolio evaluation, such as the MV criterion developed by Markowitz (1952) and the capital

*The authors would like to thank the Editor, Professor Wesley W. Wilson and two anonymous referees for their insightful comments and helpful suggestions on earlier drafts of the paper. Ephraim Clark would like to thank financial support from Middlesex University. The corresponding author of this paper, Zhuo Qiao, gratefully acknowledges financial support from University of Macau (research grant no. MYRG014 (Y1-L1)-FBA11-QZ). Wing-Keung Wong would like to thank financial support from Hong Kong Baptist University (FRG2/14-15/040 and FRG2/14-15/106) and Research Grants Council of Hong Kong (Project Nos. 12502814 and 12500915). He would like to thank Professors Robert B. Miller and Howard E. Thompson for their continuous guidance and encouragement.

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ABBREVIATIONS

ACDF: Ascending Cumulative Distribution Function
ASD: Ascending Stochastic Dominance
CAPM: Capital Asset Pricing Model
CDF: Cumulative Distribution Function
DARA: Decreasing Absolute Risk Aversion
DCDF: Descending Cumulative Distribution Function
DD: Davidson and Duclos
DSD: Descending Stochastic Dominance
MV: Mean-Variance
PDF: Probability Density Function
SD: Stochastic Dominance
TAIEX: Taiwan Stock Exchange Capitalization Weighted Stock Index
TAIFEX: Taiwan Futures Exchange
TX: Taiwan Stock Exchange Index Futures
UMPU: Uniformly Most Powerful Unbiased

Economic Inquiry
(ISSN 0095-2583)
Vol. 54, No. 2, April 2016, 907–924
doi:10.1111/ecin.12288
Online Early publication October 28, 2015

asset pricing model (CAPM) statistics developed by Jensen (1969), Sharpe (1964), and Treynor (1965), the SD approach provides a very general framework to assess portfolio choice without the need for asset-pricing benchmarks. Whereas the MV criterion and CAPM statistics rely on the assumptions of normal distributions and quadratic utility functions, SD theory makes no such assumptions. It can accommodate any return distribution, both normal and non-normal, and a wide range of underlying utility functions including the standard linear utility functions satisfying von-Neumann–Morgenstern axioms as well as a variety of nonlinear utility functions based on substantially weaker axioms (Fishburn 1989). In addition, SD criteria work well for a wide range of nonexpected utility theories of choice under uncertainty (Wong and Ma 2008). Importantly for the focus of this article, SD theory can be applied to risk seekers as well as risk averters (see Li and Wong 1999 and Wong and Li 1999 for more discussion).[1]

To employ the SD tests in this article, we use data from the Taiwan stock index and its corresponding index futures. First, we apply a test of SD for risk averters developed by Davidson and Duclos (DD, 2000) that allows for dependent observations and has simple asymptotic properties. We then modify the test so that it can be applied to risk seekers. Finally, we apply both tests for risk averters and risk seekers in the positive and negative domains of the return distributions. This enables us to reveal risk aversion and risk-seeking preferences in both the positive and negative domains, which, in turn, enables us to analyze the preferences of investors suggested by two competing hypotheses of choice under risk as proposed in prospect theory. The first is the hypothesis of Kahneman and Tversky (1979) that people integrate the outcomes of sequential gambles, which leads to an S-shaped utility function where investors are risk seeking in losses and risk averting in gains. The second hypothesis, stemming from the experimental work of Thaler and Johnson (1990) (we call it the Thaler–Johnson hypothesis), is that sequential outcomes are segregated, which can lead to a reverse S-shaped utility function where people are risk averse in losses and risk seeking in gains.[2]

When we employ the MV criterion, our findings are limited. They provide no evidence of a preference for futures or spot markets for risk averters, but they do provide evidence that risk seekers prefer futures markets to spot markets. The SD procedures provide evidence of more complex behavior. By partitioning returns of the Taiwan stock index and its corresponding index futures into negative and positive return regions, we find that risk averters prefer spot to futures, whereas risk seekers prefer futures to spot. Our findings show that investors with S-shaped utility functions prefer spot (futures) to futures (spot) when markets move upward (downward). Finally, our results also imply that investors with reverse S-shaped utility functions prefer futures (spot) to spot (futures) when markets move upward (downward). These results are robust with respect to subperiods, spot returns including dividends and diversification.

The rest of this article is organized as follows. Section II reviews theories of decision making under risk that incorporates risk aversion as well as risk seeking in gains and in losses. Section III describes the dataset and presents the descriptive statistics. Section IV discusses the theory of the MV criterion and SD theory for different types of investors. Section V presents our empirical findings on the preferences of the Taiwan stock index and its corresponding index futures for different types of investors. In Section VI, we discuss our results and their implications for market efficiency and the existence of heterogeneous investor behavior, and provide our concluding remarks.

II. THEORIES OF RISK PREFERENCES

Expected utility theory is the predominant approach to analyzing individual risk preferences. Risk aversion reflected in strictly concave utility functions is the standard assumption in economics and finance. However, global risk aversion has been criticized for not describing how investors actually behave. For example, examining the relative attractiveness of various forms of investments, Friedman and Savage (1948) claim that the strictly concave functions may not be able to explain why investors buy insurance or lottery tickets. Several alternative

1. Levy and Wiener (1998) further develop the theory for the reverse S-shaped utility functions for investors. Levy and Levy (2002) are the first to extend the work of Markowitz (1952) and others by developing new SD criteria to determine the dominance of one investment alternative over another for all S-shaped or reverse S-shaped utility functions. In addition, Wong and Chan (2008) extend the theory to third-order stochastic dominance.

2. See Barberis, Huang, and Santos (2001, 18–19) for a discussion of these two competing hypotheses.

theories have been proposed to provide more realistic descriptions of individual risk preferences. For example, Hartley and Farrell (2002) and others propose using global convex utility functions, the functions for risk seekers, to indicate risk-seeking behavior. Markowitz (1952) addresses Friedman and Savage's concern and proposes a utility function that has convex and concave regions in both the positive and the negative domains.

To support Markowitz's proposed utility function, Williams (1966) reports data whereby a translation of outcomes produces a dramatic shift from risk aversion to risk seeking, while Fishburn and Kochenberger (1979) document the prevalence of risk seeking in choices between negative prospects. Kahneman and Tversky's (1979) analysis of decision making under uncertainty, called prospect theory, has shown the importance of "The location of the reference point, and the manner in which choice problems are coded and edited … " (1979, 288). Under the hypothesis that investors integrate the outcomes of sequential gambles, Kahneman and Tversky (1979) and Tversky and Kahneman (1992) find investor behavior that is consistent with a (value) utility function that is concave for gains and convex for losses, yielding an S-shaped function. Thereafter, there is a stream of papers that build economic or financial models based on prospect theory and many empirical and experimental attempts to test it, for example, the equity premium puzzle by Benartzi and Thaler (1995) and the buying strategies of hog farmers by Pennings and Smidts (2003).

Although the hypothesis of integrated outcomes of sequential gambles has received some experimental support, many studies have found evidence against it. For example, based on segregated outcomes of sequential gambles, Thaler and Johnson (1990) show that subjects are more willing to take risk if they made money on prior gambles than if they lost. Their findings support the existence of a reverse S-shaped utility function where investors are risk seeking in the gain and risk averse in the loss. Barberis, Huang, and Santos (2001) find that after prior gains, investors become less loss averse: the prior gains will cushion any subsequent loss, making it more bearable. Conversely, after a prior loss, investors become more loss averse: after being burned by the initial loss, they are more sensitive to additional setbacks. Post and Levy (2005) conclude that investors are risk averse in bear markets and risk seeking in bull markets, and hence, investor preferences are best represented by the reverse S-shaped utility function. In addition, Fong, Lean, and Wong (2008) and Post, Van Vliet, and Levy (2008) also find evidence to support the reverse S-shaped utility function. Readers can also refer to Broll et al. (2010) and Egozcue et al. (2011) for more properties on the theory of reverse S-shaped utility functions.

The upshot of all this is that investors' risk preferences may depend on whether returns are in the positive or negative domain of an empirical return distribution. Risk-averting behavior in the positive domain and risk-seeking behavior in the negative domain imply the existence of S-shaped utility functions. Alternatively, risk-seeking behavior in the positive domain and risk-averting behavior in the negative domain imply the existence of reverse S-shaped utility functions. Shefrin and Statman (1993) exploit behavioral finance concepts and suggest that investors aspire to riches and seek to avoid losses or poverty. They note that investors may have different risk-return preferences for the same class of securities because they view different parts of their portfolios differently.

In this study, we consider all four utility functions: concave, convex, S-shaped, and reverse S-shaped to analyze the behavior of different types of investors on the Taiwan stock index and its corresponding index futures. Our findings provide insights into investor behavior with respect to risk aversion, risk seeking, prospect theory, and the Thaler–Johnson hypothesis.

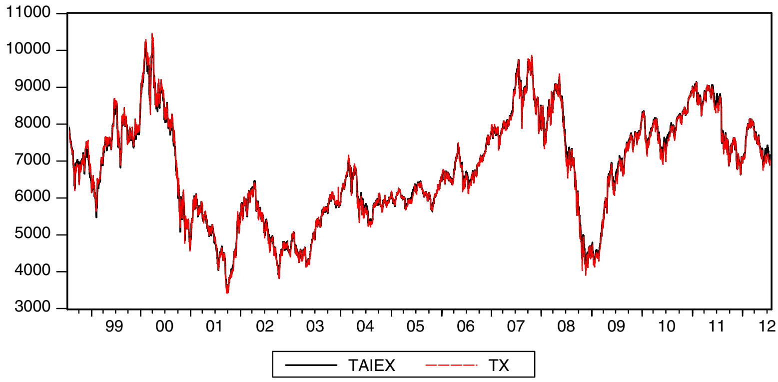
III. DATA

We compare the performance of the futures and spot markets by examining the daily closing prices of the Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX) and its index futures (TX).[3] Our sample is taken from *DataStream International* and starts on July 21,

3. The futures contract size equals NTD200 times the index point. The delivery months related to the contracts that constitute the TX are the spot month, the next calendar month, and the next three quarterly months. The last trading day for each of these contracts is the third Wednesday of the delivery month of each contract. The daily settlement price is the volume-weighted average price of these individual contracts, which is calculated using prices and volumes recorded within the last 1 minute of trading or as otherwise determined by the Taiwan Futures Exchange according to the Trading Rules. The final settlement day for the TX is the same day as the last trading day. The final settlement price is the average price of the underlying index disclosed within the last 30 minutes prior to the close of trading on the final settlement day. For more information, please refer to http://www.taifex.com.tw/eng/eng_home.htm. Our dataset excludes dates when the exchanges are closed for holidays.

FIGURE 1

Time Series of Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX) and Its Index Futures (TX)



Notes: This figure plots the time series of the Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX) and its index futures (TX). Our sample starts on July 21, 1998, when the TX was launched by the Taiwan Futures Exchange (TAIFEX), through July 20, 2012.

1998, when the TX was launched by the Taiwan Futures Exchange (TAIFEX), through July 20, 2012. In the testing for robustness, the sample was also divided into two subsamples that have roughly the same number of observations. The first subsample covers the period from July 21, 1998 to July 20, 2005, and the second covers the period from July 21, 2005 to July 20, 2012. The plots of the TAIEX and its index futures TX in Figure 1 show clearly that the two time series move closely together.

Comparing futures returns with spot returns is complicated by the fact that the futures market requires only a relatively small outlay of funds in the form of margin, while the spot market requires an outlay of the full amount of the investment. In order to account for this, we introduce a collateralized version of the futures portfolio by adding the futures position to an investment in a risk-free asset with the "same" initial capital. This makes it possible to compare the return of the futures-deposit portfolio, R_t^P, with the return on the spot portfolio, R_t^S, at time t because the amounts invested in the futures portfolio and spot portfolio are equal.

Let W_{t-1}^S be the amount of wealth invested in the spot market at time $t-1$. Consider a futures-deposit portfolio (W_{t-1}^{FD}) composed of a risk-free deposit denoted W_{t-1}^D of an amount equal to W_{t-1}^S and a long futures position denoted W_{t-1}^F of an amount equal to W_{t-1}^S. Because the long futures position requires no outlay, the total amount invested in this portfolio is $W_{t-1}^D = W_{t-1}^S$, such that $W_{t-1}^{FD} = W_{t-1}^F = W_{t-1}^D = W_{t-1}^S$. Let r_{t-1}^f be the risk-free rate at time $t-1$ measured as the Taiwan bank deposit rate. Then, the return on investing (W_{t-1}^D) in the bank deposit from time $t-1$ to time t is

$$R_t^D = \left(W_{t-1}^D \times \left(1 + r_{t-1}^f\right) - W_{t-1}^D\right) / W_{t-1}^D = r_{t-1}^f.$$

The return of investing "W_{t-1}^F" in the futures market from time $t-1$ to time t is

$$R_t^F = \left(W_t^F - W_{t-1}^F\right) / W_{t-1}^F.$$

Thus, the return on the futures-deposit portfolio obtained by investing "W_{t-1}^F" in futures and W_{t-1}^D in the bank deposit (this is equivalent to investing $W_{t-1}^{FD} = W_{t-1}^D$ in the futures-deposit portfolio because "W_{t-1}^F" generates no cash flow) from time $t-1$ to time t is

$$R_t^{FD} = R_t^F + R_t^D = R_t^F + r_{t-1}^f.$$

Similarly, if S_t is the spot price at time t, the return of investing W_{t-1}^S in spot from time $t-1$ to time t is

$$R_t^S = \left(W_t^S - W_{t-1}^S\right) / W_{t-1}^S.$$

Thus, we can compare the returns of the futures-deposit portfolio ($R_t^{FD} = R_t^F + r_{t-1}^D$) with the returns on the spot portfolio (R_t^S) because the same amount has been invested in each portfolio.

IV. METHODOLOGY

In this section, we discuss the MV criterion and the SD procedures used to examine the preferences of different types of investors in the Taiwan stock index and its corresponding index futures. We start with the MV criterion.

A. *MV Criterion for Risk Averters and Risk Seekers*

For any two returns Y_i and Y_j with means μ_i and μ_j and standard deviations σ_i and σ_j, respectively, it is well known that Y_j is said to dominate Y_i by the MV rule for risk averters, denoted by $Y_j\ MV_A\ Y_i$, if $\mu_j \geq \mu_i$ and $\sigma_j \leq \sigma_i$ (Markowitz 1952), and the inequality holds in at least one of the two.[4] In addition, Wong (2007) defines an MV rule for risk seekers in which Y_j is said to dominate Y_i if $\mu_j \geq \mu_i$, $\sigma_j \geq \sigma_i$ and the inequality holds in at least one of the two. He has proved that if both Y_j and Y_i belong to the same location-scale family or the same linear combination of location-scale families, $Y_j\ MV_A\ (MV_D)\ Y_i$ implies $E[u(Y_j)] \geq E[u(Y_i)]$ for any risk-averse (risk-seeking) investor.

B. *SD Theory for Different Types of Investors*

Let F and G be the cumulative distribution functions (CDFs) and let f and g be the corresponding probability density functions (PDFs) of X and Y, for the returns of futures and spot,[5] respectively, with common support $[a, b]$ where $a < b$. Define:

$$(1)\quad \mu_F = \mu_X = E(X) = \int_a^b x\mathrm{d}F(x),$$

$$\mu_G = \mu_Y = E(Y) = \int_a^b x\mathrm{d}G(x),$$

$$H_0^A = H_0^D = h,\ H_j^A(x) = \int_a^x H_{j-1}^A(t)\ \mathrm{d}t,$$

$$\text{and } H_j^D(x) = \int_x^b H_{j-1}^D(t)\ \mathrm{d}t$$

for $h = f, g$, $H = F, G$, and $j = 1, 2, 3$.

4. We note that Bai et al. (2012) introduce the MV ratio test to analyze the performance of asset returns. They have proved that their test is the uniformly most powerful unbiased (UMPU) test for small samples. Readers may apply their test to explore the MV relationship between spot and futures further. Because our sample is not small, we do not apply their test in our analysis.

5. In this article when we say "invest in futures" or "invest in index futures," it refers to investing in the futures-deposit portfolio as discussed in Section III.

Quirk and Saposnik (1962) and others use H_j^A to develop the SD theory for risk averters, whereas Li and Wong (1999) and others use H_j^D to develop the SD theory for risk seekers. When H_j^A is integrated from H_{j-1}^A in ascending order from the leftmost point of downside risk, the SD for risk averters is denoted as ascending stochastic dominance (ASD). The integral of H_j^A is the jth order ascending cumulative distribution function (ACDF) or simply the jth order ASD integral. Similarly, when H_j^D is integrated from H_{j-1}^D in descending order from the rightmost point of upside profit, the SD for risk seekers is referred to as descending stochastic dominance (DSD). Also, the integral of H_j^D is the jth order descending cumulative distribution function (DCDF) or simply the jth order DSD integral for $j = 1$, 2, and 3 and for $H = F$ and G. These definitions can be used to examine both risk-averting and risk-seeking preferences. Hence, FASD (FDSD), SASD (SDSD), and TASD (TDSD) refer to first-, second-, and third-order SD for risk averters (risk seekers). Their definitions (Wong and Li 1999) are as follows:

DEFINITION 1. X *dominates* Y *by FASD (SASD, TASD), denoted by* $X \succ_1 Y$ *or* $F \succ_1 G$ *(*$X \succ_2 Y$ *or* $F \succ_2 G$, $X \succ_3 Y$ *or* $F \succ_3 G$*) if and only if* $F_1^A(x) \leq G_1^A(x)$ *(*$F_2^A(x) \leq G_2^A(x)$, $F_3^A(x) \leq G_3^A(x)$*) for all possible returns* x, *and the strict inequality holds in a nonempty interval.*

DEFINITION 2. X *dominates* Y *by FDSD (SDSD, TDSD), denoted by* $X \succ^1 Y$ *or* $F \succ^1 G$ *(*$X \succ^2 Y$ *or* $F \succ^2 G$, $X \succ^3 Y$ *or* $F \succ^3 G$*) if and only if* $F_1^D(x) \geq G_1^D(x)$ *(*$F_2^D(x) \geq G_2^D(x)$, $F_3^D(x) \geq G_3^D(x)$*) for all possible returns* x, *and the strict inequality holds in a nonempty interval.*

For $n = 1$, 2, 3, ASD corresponds to three broadly defined utility functions, U_n^A, for risk averters; DSD corresponds to three broadly defined utility functions, U_n^D, for risk seekers. The utility functions U_n^S for investors with S-shaped and U_n^R for investors with reversed S-shaped could be defined as follows (Wong and Chan 2008):

DEFINITION 3. *Let* u *be a utility function. For* n = *1, 2, 3,*

(a) U_n^A *is the set of utility functions such that* $U_n^A = \left\{u : (-1)^{i+1} u^{(i)} \geq 0, i = 1, \cdots, n\right\}$*;*

(b) U_n^D *is the set of utility functions such that* $U_n^D = \{u : u^{(i)} \geq 0, i = 1, \cdots, n\}$;
(c) $U_n^S = \{u : u^+ \in U_n^A$ *and* $u^- \in U_n^D$, $i = 1, \cdots, n\}$;
(d) $U_n^R = \{u : u^+ \in U_n^D$ *and* $u^- \in U_n^A$, $i = 1, \cdots, n\}$,

where $\mathrm{u}^{(\mathrm{i})}$ *is the* i*th derivative of the utility function* u.

Quirk and Saposnik (1962) and others relate ASD to utility maximization for risk averters for $n = 1$, 2, and 3, because $F \succ_n G$ if and only if $E[u(X)] \geq E[u(Y)]$ for any u in U_n^A. Thus, risk-averse investors exhibit FASD (SASD, TASD) if their utility functions u belong to U_1^A (U_2^A, U_3^A). On the other hand, Li and Wong (1999) and others relate DSD to utility maximization for risk seekers. For $n = 1$, 2, and 3, we have $F \succ^n G$ if and only if $E[u(X)] \geq E[u(Y)]$ for any u in U_n^D. Thus, risk-seeking investors exhibit FDSD (SDSD, TDSD) if their utility functions u belong to U_1^D (U_2^D, U_3^D). The existence of ASD (DSD) implies that the expected utility of the risk-averse (risk-seeking) investor is always higher when holding the dominant asset than when holding the dominated asset and, consequently, the dominated asset would never be chosen.

We note that a hierarchical relation exists in ASD and DSD (Levy 2006; Sriboonchita et al. 2009). FASD implies SASD, which, in turn, implies TASD. However, the converse is not true: the existence of SASD does not imply the existence of FASD. Likewise, the existence of TASD does not imply the existence of SASD or FASD. A similar hierarchical relation also exists in DSD. Thus, only the lowest dominance order of ASD and DSD is reported.

The SD test for risk averters developed by Davidson and Duclos (2000) is one of the most powerful tests of SD significance and yet one of the least conservative in size.[6] Let $\{(f_i, g_i)\}$ $(i = 1, \ldots, n)$ be pairs of observations drawn from the index futures and stock returns with CDFs F and G, respectively. For a grid of preselected points, $x_1, x_2, \ldots, x_k$, the jth order DD test statistic for risk averters, $T_j^A(x)$ ($j = 1$, 2, and 3, denoted by ASD test), is:

$$(2) \qquad T_j^A(x) = \left(\widehat{F}_j^A(x) - \widehat{G}_j^A(x)\right) / \sqrt{\widehat{V}_j^A(x)}$$

where

$$\widehat{V}_j^A(x) = \widehat{V}_{F_j}^A(x) + \widehat{V}_{G_j}^A(x) - 2\widehat{V}_{FG_j}^A(x),$$

$$\widehat{H}_j^A(x) = \frac{1}{N(j-1)!}\sum_{i=1}^{N}(x - z_i)_+^{j-1},$$

$$\widehat{V}_{H_j}^A(x) = \frac{1}{N}\left[\frac{1}{N((j-1)!)^2}\sum_{i=1}^{N}\left(x - z_i\right)_+^{2(j-1)} - \widehat{H}_j^A(x)^2\right],$$

$$H = F, G; \; z = f, g;$$

$$\widehat{V}_{FG_j}^A(x) = \frac{1}{N}\left[\frac{1}{N((j-1)!)^2}\sum_{i=1}^{N}\left(x - f_i\right)_+^{j-1} \left(x - s_i\right)_+^{j-1} - \widehat{F}_j^A(x)\,\widehat{G}_j^A(x)\right];$$

$$(x)_+ = \max\{x, 0\}.$$

It is empirically impossible to test the null hypothesis for the full support of the distributions. Thus, Bishop, Formly, and Thistle (1992) propose to test the null hypothesis for a predesigned finite number of values x. Specifically, for all $i = 1, 2, \ldots, k$, the following hypotheses are tested:

$$H_0 : F_j^A(x_i) = G_j^A(x_i) \text{ for all } x_i,$$

$$H_A : F_j^A(x_i) \neq G_j^A(x_i) \text{ for some } x_i;$$

$$H_{A1} : F_j^A(x_i) \leq G_j^A(x_i) \text{ for all } x_i,$$

$$F_j^A(x_i) < G_j^A(x_i) \text{ for some } x_i;$$

$$H_{A2} : F_j^A(x_i) \geq G_j^A(x_i) \text{ for all } x_i,$$

$$F_j^A(x_i) > G_j^A(x_i) \text{ for some } x_i.$$

Accepting either H_0 or H_A implies no SD between the returns of index futures and stock, no arbitrage opportunity and no preference for either of them. However, if H_{A1} (H_{A2}) of order one is accepted, index futures (stock) stochastically dominate stock (index futures) at the first-order ASD. In this situation and under certain regularity conditions,[7] an arbitrage opportunity exists and any nonsatiated investor (who prefers more to less) will be better off by switching from the

6. Readers may refer to Lean, Wong, and Zhang (2008) and the references they cite for more information.

7. Refer to Jarrow (1986) for the conditions.

dominated asset to the dominant asset.[8] On the other hand, if H_{A1} (H_{A2}) is accepted for order two or three, index futures (stock) stochastically dominate stock (index futures) at the second or the third order. In this situation, no arbitrage opportunities are available, but switching from the dominated asset to the dominant asset will increase risk averters' expected utilities, but not their wealth.

To test SD for risk seekers, we modify the ASD test to be the SD test for descending SD, denoted by the DSD test such that:

$$(3) \qquad T_j^D(x) = \left(\widehat{F}_j^D(x) - \widehat{G}_j^D(x)\right) / \sqrt{\widehat{V}_j^D(x)}$$

where

$$\widehat{V}_j^D(x) = \widehat{V}_{F_j}^D(x) + \widehat{V}_{G_j}^D(x) - 2\widehat{V}_{FG_j}^D(x),$$

$$\widehat{H}_j^D(x) = \frac{1}{N(j-1)!}\sum_{i=1}^{N}\left(z_i - x\right)_+^{j-1}$$

$$\widehat{V}_{H_j}^D(x) = \frac{1}{N}\left[\frac{1}{N((j-1)!)^2}\sum_{i=1}^{N}\left(z_i - x\right)_+^{2(j-1)} - \widehat{H}_j^D(x)^2\right],$$

$$H = F, G;\ z = f, g;$$

$$\widehat{V}_{FG_j}^D(x) = \frac{1}{N}\left[\frac{1}{N((j-1)!)^2}\sum_{i=1}^{N}\left(f_i - x\right)_+^{j-1}\left(s_i - x\right)_+^{j-1} - \widehat{F}_j^D(x)\,\widehat{G}_j^D(x)\right];$$

where the integrals $F_j^D(x)$ and $G_j^D(x)$ are defined in Equation (1) for $j = 1, 2, 3$. For $i = 1, 2, \ldots, k$, the following hypotheses are tested for risk seekers:

$$H_0 : F_j^D\left(x_i\right) = G_j^D\left(x_i\right) \text{ for all } x_i;$$

$$H_D : F_j^D\left(x_i\right) \neq G_j^D\left(x_i\right) \text{ for some } x_i;$$

$$H_{D1} : F_j^D\left(x_i\right) \geq G_j^D\left(x_i\right) \text{ for all } x_i,\quad F_j^D\left(x_i\right) > G_j^D\left(x_i\right) \text{ for some } x_i;$$

$$H_{D2} : F_j^D\left(x_i\right) \leq G_j^D\left(x_i\right) \text{ for all } x_i,\quad F_j^D\left(x_i\right) < G_j^D\left(x_i\right) \text{ for some } x_i.$$

8. Readers may refer to Chan et al. (2012), Lean, McAleer, and Wong (2010), and Wong, Phoon, and Lean (2008), and the references therein for a discussion of arbitrage opportunity.

Similar to the test for risk averters, accepting either H_0 or H_D implies no SD between the returns of index futures and stock, no arbitrage opportunity and no preference for either of them. If H_{D1} (H_{D2}) of order one is accepted, index futures (stock) stochastically dominate stock (index futures) at the first-order DSD. In this situation, an arbitrage opportunity exists and any nonsatiated investor will be better off by switching from the dominated asset to the dominant asset. On the other hand, if H_{D1} or H_{D2} is accepted for order two or three, index futures (stock) stochastically dominate stock (index futures) at the second or the third order. In this situation, although no arbitrage opportunity exists, switching from the dominated asset to the dominant asset will increase risk seekers' expected utilities.

The ASD and DSD tests compare the distributions at a finite number of grid points. The null hypothesis is rejected when some t-statistic values across these grid points are significant. We follow Fong, Wong, and Lean (2005), Gasbarro, Wong, and Zumwalt (2007), and others to make ten major partitions with ten minor partitions within any two consecutive major partitions in each comparison. In addition, we follow Bai et al. (2011) to adopt a bootstrap method to decide the simulated critical values of the ASD and DSD tests.

From Definition 3, one can see that investors with S-shaped utility functions possess the same utility functions as risk averters in the positive domain and the same utility functions as risk seekers in the negative domain, whereas investors with reverse S-shaped utility functions possess the same utility functions as risk seekers in the positive domain and the same utility functions as risk averters in the negative domain. Thus, in this article, we suggest examining T_j^A over the positive domain and T_j^D over the negative domain to identify the risk preferences of investors with jth order S-shaped utility functions. Finally, we examine T_j^D over the positive domain and T_j^A over the negative domain to identify investors with jth order reverse S-shaped utility functions. These investors exhibit jth order risk seeking over the positive domain and risk aversion over the negative domain. Thus, combining the ASD and DSD tests for risk aversion and risk seeking on both the positive and the negative domains allows an identification of the preference of investors with S-shaped and reverse S-shaped utility functions.

TABLE 1
Descriptive Statistics for Daily Returns of the Spot and Index Futures

	Full Sample		Subsample 1		Subsample 2	
Variable	**Spot**	**Futures**	**Spot**	**Futures**	**Spot**	**Futures**
Mean	0.0001	0.0002	0.0000	0.0001	0.0001	0.0002
Median	0.0002	0.0004	−0.0007	−0.0005	0.0008	0.0011
Maximum	0.0889	0.1116	0.0889	0.1116	0.0674	0.0700
Minimum	−0.0946	−0.1048	−0.0946	−0.1048	−0.0651	−0.0699
SD	0.0158	0.0181	0.0174	0.0198	0.0141	0.0163
Skewness	−0.0208	0.0228	0.1143	0.1501	−0.2678	−0.2030
Kurtosis	5.3663	6.1125	4.9859	5.7118	5.5102	6.2858
Jarque−Bera	798.5971***	1381.587***	284.2101***	529.4402***	470.7648***	783.2585***
t test	0.2625		0.1906		0.1815	
F test	1.3114***		1.2950***		1.3364***	

Notes: These are descriptive statistics of the daily returns of the spot and futures. Our sample starts on July 21, 1998 and runs through July 20, 2012. Subsample 1 covers July 21, 1998 to July 20, 2005 and subsample 2 covers July 21, 2005 to July 20, 2012.
***, **, and * indicate significance at the 1%, 5% and 10% levels, respectively.

V. EMPRICAL RESULTS

A. MV Analysis

In Table 1, we display the descriptive statistics for the daily returns of spot and index futures. From the table, we notice that both the means and the standard deviations of futures returns for the full sample and two subsample periods are higher than those of spot. Recall that for any two returns Y_i and Y_j with means μ_i and μ_j and standard deviations σ_i and σ_j, respectively, Y_j dominates Y_i by the MV rule for risk averters (risk seekers) if $\mu_j \geq \mu_i$, $\sigma_j \leq \sigma_i$ ($\sigma_j \geq \sigma_i$), and the inequality holds in at least one of the two. In this sense, our findings on the means and the standard deviations of futures and spot returns do not imply any preference for spot or futures for risk averters. Nevertheless, although the mean return of futures is larger (but insignificant) than that of spot, the *F* test shown in Table 1 reveals that the standard deviation of futures returns is significantly larger than that of spot returns at the 1% significance level in the full sample and the two subsamples. Thus, according to the MV rule for risk seekers, our findings on the means and the standard deviations of futures and spot returns imply that risk seekers prefer futures to spot.

B. SD Analysis for Risk Averters

Given that our data are not normally distributed, as indicated by the highly significant Jarque−Bera statistics in Table 1, the inference from the MV analysis may not be meaningful. To circumvent this limitation, we continue our study using the SD rules to examine the preference for different types of investors on spot and futures. We first apply the ASD test to study the preference of risk averters for spot and futures.

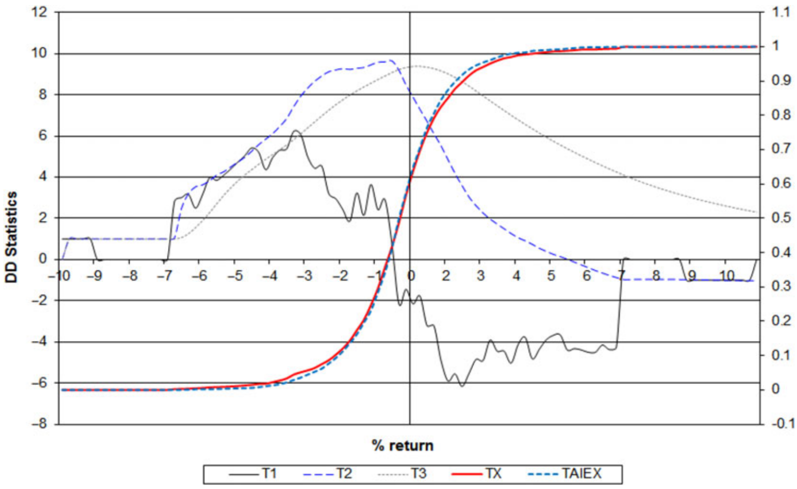
In Figure 2, we plot the empirical CDFs of TAIEX and TX returns. We also plot the first-order, second-order, and third-order ASD statistics (i.e., T_1^A, T_2^A, and T_3^A) for the risk averters as defined in Equation (2) for the entire sample period. From the figure, we find that their ACDFs cross with each other and T_1^A changes its sign from positive in the negative return domain to negative in the positive return domain, implying that there is no FASD between the two returns and that spot dominates futures on the downside, while futures dominate spot in the upside profit range.[9]

To verify this formally, we apply the ASD test, T_j^A, for risk averters to the two series and display the results in Table 2. To minimize the Type I error and to avoid almost-SD (Guo et al. 2014; Leshno and Levy 2002), we use a 5% cut-off point for the proportion of the test statistic in our statistical inference.[10] Using the 5% cut-off

9. There are two methods to check whether there is FASD between futures and spot. The first method is to check ACDFs of futures and spot using Definition 1 of Section IV. If spot (futures) dominates futures (spot) in the sense of FASD, we should observe ACDF curve of spot returns lies below (above) ACDF curve of futures returns. If these two ACDF curves cross each other, then there is no FASD between futures and spot. The second way is to look at the first-order DD test statistics for risk averters over 100 grid points. In Figure 2, we plot these 100 DD test statistics (i.e., T1). Then we could check the percentage of significant T1, which is reported in Table 2. To check whether there is SASD and TASD, we can only look at the second-order and third-order DD test statistics for risk averters (i.e., T2 and T3 in Figure 2). Table 2 also reports the percentages of significant T2 and T3, respectively.

10. We note that Leshno and Levy (2002) use an example of 1% to state the problem of almost-SD. In this article, we follow Fong, Wong, and Lean (2005), Gasbarro, Wong, and Zumwalt (2007), and others to choose a more conservative 5% cut-off point to avoid the problem of almost-SD. The conclusion drawn in our paper holds if one uses any less conservative cut-off point, say 1%.

FIGURE 2
ACDF of Returns and ASD Statistics for Risk Averters – Full Sample



Notes: TX and TAIEX are the CDFs of futures and spot returns, respectively. T_j is the *j*th order ASD test statistic for risk averters, T_j^A ($j = 1$, 2, and 3) defined in Equation (2) with F_j^A and G_j^A denoting the *j*th order ACDFs of the results of futures and spot, respectively.

point, if futures dominate spot, we should find at least 5% of T_j^A to be significantly negative and no portion of T_j^A to be significantly positive. The reverse holds if spot dominates futures. From the table, we find that, for the full sample, 28% (22%) of T_1^A is significantly negative (positive). Thus, the results invalidate the hypothesis that futures stochastically dominate spot at the first order and vice versa.

The absence of FASD leads us to focus the analysis on higher orders to derive utility interpretations with respect to investors' risk aversion and decreasing absolute risk aversion (DARA), respectively. Table 2 shows that 42% (78%) of the T_2^A (T_3^A) is significantly positive and no T_2^A (T_3^A) is significantly negative at the 5% level. Hence, our finding implies that risk averters significantly prefer spot to futures in the sense of both SASD and TASD.

C. SD Analysis for Risk Seekers

We turn to analyzing risk seekers' preferences. Figure 3 shows the empirical first-order DCDFs of returns for TAIEX and TX, and their corresponding DSD statistics for risk seekers, T_j^D, for the entire sample period. The DCDFs of the returns for TAIEX and TX cross and T_1^D changes sign from positive in the positive return domain to negative in the negative return domain. The inference here is that there is no FDSD between the two returns and futures are preferred to spot for upside returns, while spot is preferred to futures for downside returns.[11]

To test this formally, we apply the DSD test, T_j^D, for the risk seekers and display the results in Table 3. It shows that, for the full sample, 28% (22%) of T_1^D is significantly positive (negative), from which we can infer no dominance in FDSD. Because there is no FDSD, we examine the T_j^D for the second and third orders. Both T_2^D and T_3^D depicted in Figure 3 are positive for the entire range and Table 3 shows that 51% (75%) of T_2^D (T_3^D) is significantly positive and no T_2^D (T_3^D) is significantly negative at the 5% level. This

11. Similar to what we introduced in Footnote 9, there are also two methods to check whether there is FDSD between futures and spot. The first method is to check DCDFs of futures and spot using Definition 2 of Section IV. If spot (futures) dominates futures (spot) in the sense of FDSD, we should observe DCDF curve of spot returns lies above (below) DCDF curve of futures returns. If these two DCDF curves cross each other, then there is no FDSD between futures and spot. The second way is to look at the first-order DD test statistics for risk seekers over 100 grid points. In Figure 3, we plot these 100 DD test statistics (i.e., T1). Then we could check the percentage of significant T1, which is reported in Table 3. To check whether there is SDSD and TDSD, we can only look at the second-order and third-order DD test statistics for risk seekers (i.e., T2 and T3 in Figure 3). Table 3 also reports the percentages of significant T2 and T3, respectively.

TABLE 2
Results of ASD Test for Risk Averters

	FASD		SASD		TASD	
	$\%T_1^A > 0$	$\%T_1^A < 0$	$\%T_2^A > 0$	$\%T_2^A < 0$	$\%T_3^A > 0$	$\%T_3^A < 0$
Full sample						
Total	22	28	42	0	78	0
Positive domain	0	28	12	0	52	0
Negative domain	22	0	30	0	26	0
$\left\|\max\left(T_j^A\right)\right\|$	6.229	6.163	9.649	1.034	9.382	0
Subsample 1						
Total	12	17	40	0	62	0
Positive domain	0	17	10	0	38	0
Negative domain	12	0	30	0	24	0
$\left\|\max\left(T_j^A\right)\right\|$	4.955	3.816	6.568	0.716	6.284	0
Subsample 2						
Total	22	23	56	0	90	0
Positive domain	0	23	14	0	50	0
Negative domain	22	0	42	0	40	0
$\left\|\max\left(T_j^A\right)\right\|$	3.917	5.390	8.657	0.776	7.492	0

Notes: This table summarizes the ASD test T_j^A results for risk averters in which T_j^A ($j = 1$, 2, and 3) is defined in Equation (2) with F_j^A and G_j^A denoting the *j*th order ACDFs of the results of futures and spot, respectively. The spot returns are calculated using TAIEX index. The ASD test statistics are computed over a grid of 100 on the range of the empirical distributions of stock and futures returns. The table reports the percentage of ASD statistics that are significantly negative or positive at the 5% significance level, based on the critical value generated from a bootstrap method proposed by Bai et al. (2011). The full sample covers July 21, 1998 to July 20, 2012. Subsample 1 covers July 21, 1998 to July 20, 2005 and subsample 2 covers July 21, 2005 to July 20, 2012.

implies that futures stochastically dominate spot in the sense of both SDSD and TDSD and risk seekers prefer futures to spot to maximize their expected utilities.

D. SD Analysis of Investors with S-Shaped and Reverse S-Shaped Utility Functions

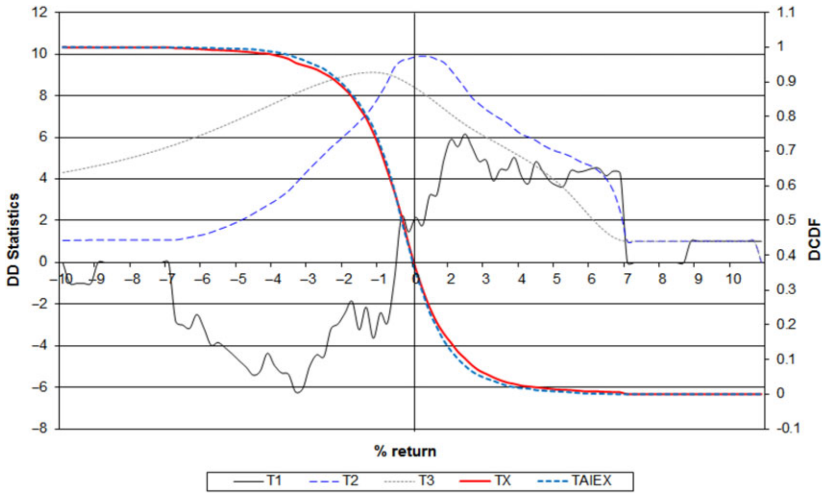
To determine the preferences for spot and futures by investors with S-shaped and reverse S-shaped utility functions, we examine the positive and negative domains of the return distributions separately. Table 2 reports the results of T_j^A in the positive and negative domains of the return distributions while Table 3 reports the results of T_j^D in the positive and negative domains of the return distributions. The results of ASD and DSD in both the positive and the negative domains are summarized in Table 4. Here, FASD, SASD, and TASD (FDSD, SDSD, and TDSD) refer to first-, second-, and third-order ASD (DSD) for risk averters (risk seekers) defined in Definition 1 (2). The component before the slash in each cell refers to the positive domain, while the component after the slash refers to the negative domain. Readers may refer to the note in Table 4 to find out how to read the table.

Table 4 was constructed as follows. To get SASD/SASD in the third column and third row of Table 4, we refer to Table 2 where we find that 30% of T_2^A is significantly positive and no T_2^A is significantly negative in the negative domain. This yields the right-hand side "SASD" in "SASD/SASD." For the left-hand side, we again refer to Table 2 where we find that 12% of the T_2^A is significantly positive and no T_2^A is significantly negative at the 5% level in the positive domain. This yields the left-hand side "SASD" in "SASD/SASD." To get SDSD/SDSD in the fourth column and second row, we refer to Table 3 where we find that 20% of T_2^D is significantly positive and no T_2^D is significantly negative in the negative domain. This yields the right-hand side "SDSD" in "SDSD/SDSD." On the other hand, from Table 3 again, we find that 31% of T_2^D is significantly positive and no T_2^D is significantly negative in the positive domain. This yields the left-hand side "SDSD" in "SDSD/SDSD."

The findings shown in Table 4 can be used to draw inference for investors with S-shaped and reverse S-shaped utility functions. Investors with S-shaped utility functions exhibit risk-averse behavior in the positive domain and risk-seeking

FIGURE 3

DCDF of Returns and DSD Statistics for Risk Seekers—Full Sample



Notes: TX and TAIEX are the first-order DCDFs of futures and spot returns, respectively. T_j is the *j*th order DSD test statistic for risk seekers, T_j^D ($j = 1$, 2, and 3) defined in Equation (3) with F_j^D and G_j^D denoting the *j*th order DCDFs of the results of futures and spot, respectively.

behavior in the negative domain. Thus, our findings from Table 4 imply that investors with S-shaped utility functions prefer spot to futures for the bull market regime when the returns of both spot and futures are positive. On the other hand, they prefer futures to spot for the bear market regime when the returns of both spot and futures are negative.

Similarly, the results of Table 4 can be used to draw inference for investors with reverse S-shaped utility functions. Investors with reverse S-shaped utility functions exhibit risk-averse behavior in the negative domain and risk-seeking behavior in the positive domain. Thus, the preferences of investors with reverse S-shaped utility functions with respect to futures and spot are opposite to those of investors with S-shaped utility functions. In other words, investors with reverse S-shaped utility functions prefer spot to futures for the bear market when the returns of both spot and futures are negative and futures to spot for the bull market when the returns of both spot and futures are positive.

E. Robustness Checking

Robustness Checking in Subperiods. We now turn to investigating the preferences for risk averters and risk seekers on the Taiwan stock index and its corresponding index futures in the two subperiods. We first discuss their relationship in the sense of MV analysis and thereafter in the sense of SD analysis.

Table 1 displays the descriptive statistics for the daily returns of spot and index futures. From the table, similar to the findings for the entire period, we note that both the means and the standard deviations of futures returns for the two subsample periods are higher than those of spot. Although the mean return of futures is larger (but insignificant) than that of spot, the *F* test shown in Table 1 reveals that the standard deviation of futures returns is significantly larger than that of spot returns at the 1% significance level in the two subsamples. Thus, according to the MV rule for risk seekers, the findings are the same as those for the full period (i.e., that risk seekers prefer futures to spot).

Table 2 indicates that 12% and 22% of T_1^A are significantly positive in the first and second subperiods, respectively, and all are in the negative domain. On the other hand, 17% and 23% of T_1^A are significantly negative for the first and second subperiods, respectively, and all are in the positive domain. We note that the results for both subperiods are similar to the ASD results for the entire period. These results lead us to reject the hypothesis that futures stochastically dominate spot or vice versa in the sense of FSD.

TABLE 3
Results of DSD Test for Risk Seekers

	FDSD		SDSD		TDSD	
	$\%T_1^D > 0$	$\%T_1^D < 0$	$\%T_2^D > 0$	$\%T_2^D < 0$	$\%T_3^D > 0$	$\%T_3^D < 0$
Full sample						
Total	28	22	51	0	75	0
Positive domain	28	0	31	0	27	0
Negative domain	0	22	20	0	48	0
$\left\|\max\left(T_j^D\right)\right\|$	6.163	6.229	9.901	0	9.133	0
Subsample 1						
Total	17	12	45	0	73	0
Positive domain	17	0	30	0	25	0
Negative domain	0	12	15	0	48	0
$\left\|\max\left(T_j^D\right)\right\|$	3.816	4.955	6.454	0	6.214	0
Subsample 2						
Total	23	22	73	0	99	0
Positive domain	23	0	49	0	49	0
Negative domain	0	22	24	0	50	0
$\left\|\max\left(T_j^D\right)\right\|$	5.390	3.917	8.747	0	7.190	0

Notes: This table summarizes the DSD test T_j^Dresults for risk seekers in which T_j^D ($j = 1$, 2, and 3) defined in Equation (3) with F_j^D and G_j^D denoting the *j*th order DCDFs of the results of futures and spot, respectively. The spot returns are calculated using TAIEX index. The DD test statistics are computed over a grid of 100 on the range of the empirical distributions of stock and futures returns. The table reports the percentage of DSD statistics that are significantly negative or positive at the 5% significance level, based on the asymptotic critical value generated from a bootstrap method proposed by Bai et al. (2011). The full sample covers July 21, 1998 to July 20, 2012. Subsample 1 covers July 21, 1998 to July 20, 2005 and subsample 2 covers July 21, 2005 to July 20, 2012.

Because the analysis of the FDSD is the same as that for the FASD, we skip the discussion of the FDSD analysis.

To check for higher orders of SD, we examine the T_j^A for the second and third orders in the first subperiod. Table 2 shows that 40% (62%) of the T_2^A (T_3^A) is significantly positive and no T_2^A (T_3^A) is significantly negative at the 5% level. In the second subperiod, Table 2 shows that 56% (90%) of the T_2^A (T_3^A) is significantly positive and no T_2^A (T_3^A) is significantly negative at the 5% level. The implication is that risk averters significantly prefer spot to futures in the sense of both SASD and TASD in both subperiods. This finding is the same as that obtained from testing the whole period.

Because there is no FDSD, we examine the T_j^D for the second and third orders. In the first subperiod, Table 3 shows that 45% (73%) of T_2^D (T_3^D) is significantly positive and no T_2^D (T_3^D) is significantly negative at the 5% level. In the second subperiod, Table 3 shows that 73% (99%) of T_2^D (T_3^D) is significantly positive and no T_2^D (T_3^D) is significantly negative at the 5% level. This implies that futures stochastically dominate spot in the sense of both SDSD and TDSD and risk seekers prefer futures to spot. In addition, the conclusion drawn from the results for the preferences of investors with S-shaped and reverse S-shaped utility functions for the subperiods shown in Table 4 is the same as that for the entire period. Thus, we skip a discussion of these results.

Robustness Checking for Spot Returns Including Dividends.[12] In the foregoing analysis, the spot price data in question comes from the TAIEX index, the only index available over the full sample period, but the corresponding spot returns do not include dividends. In this section, we examine whether the inclusion of dividends in the spot returns affect the results obtained above. For this robustness check, we use the TAIEX total return index to calculate the spot returns including dividends. Because TAIEX total return index was launched only on January 2, 2003, the sample period is shorter (i.e., January 2, 2003 to July 20, 2012) than our original sample period (i.e., July 21, 1998 to July 20, 2012). We

12. We appreciate one referee's suggestions to do this important robustness checking.

TABLE 4
Pairwise Comparisons of the Davidson–Duclos (DD) Tests between Futures and Spot for Both Risk-Aversion and Risk-Seeking Behaviors in the Negative and Positive Domains

		Futures	**Spot**
Full period	Futures		SDSD/SDSD
	Spot	SASD/SASD	
		Futures	**Spot**
Subperiod 1	Futures		SDSD/SDSD
	Spot	SASD/SASD	
		Futures	**Spot**
Subperiod 2	Futures		SDSD/SDSD
	Spot	SASD/SASD	

Notes: SASD refers to second-order ascending stochastic dominance (ASD) for risk averters defined in Definition 1, while SDSD refers to second-order descending stochastic dominance (DSD) for risk seekers defined in Definition 2. The left of the slash refers to the positive domain and the right of the slash refers to the negative domain. The table is read from left to right. For example, (1) in the third column and third row, we have SASD/SASD, and (2) in the fourth column and second row, we have SDSD/SDSD. These mean that in (1) spot dominates futures in the sense of SASD in both the positive and negative domains and in (2) futures dominates spot in the sense of SDSD in both the positive and negative domains. The full sample covers July 21, 1998 to July 20, 2012. Subsample 1 covers July 21, 1998 to July 20, 2005 and subsample 2 covers July 21, 2005 to July 20, 2012.

report the ASD and DSD test results in Tables 5 and 6, respectively.[13]

From Table 5, we find that, 38% (20%) of T_1^A is significantly positive (negative), which suggests that there is no FSD between spot and futures. Table 5 also shows that 58% (89%) of the T_2^A (T_3^A) is significantly positive and no T_2^A (T_3^A) is significantly negative at the 5% level. Hence, our finding implies that risk averters significantly prefer spot to futures in the sense of both SASD and TASD. Table 6 shows that 20% (38%) of T_1^D is significantly positive (negative), from which we can infer no dominance in FDSD. We also find that 53% (88%) of T_2^D (T_3^D) is significantly positive and no T_2^D (T_3^D) is significantly negative at the 5% level. This implies that futures stochastically dominate spot in the sense of both SDSD and TDSD and risk seekers prefer futures to spot to maximize their expected utilities. Therefore, our results using spot returns including dividends are qualitatively the same as those reported in previous subsections B and C using spot returns excluding dividends. In addition, following the same analytical procedures introduced in subsection D, we find that the inferences for the preference of investors with S-shaped and reverse S-shaped utility functions are exactly the same as what we reported in Table 4. Overall, using spot returns that include dividends does not change our previous findings.

F. Analysis of Investors' Preferences toward Diversification

It is interesting to examine preferences toward diversification in the spot and futures markets of both risk averters and risk seekers (Egozcue and Wong 2010; Samuelson 1967). To provide an answer to this question, we look into the dominance of spot or futures with respect to portfolios of the different convex combinations of spot and futures. More specifically, we compare the full 100% of index futures as one portfolio with another portfolio consisting of different weights of spot and futures from 10% to 90% (i.e., if the weight of the spot index is x%, then the weight of the index futures is $(100-x)$%). We also compare the full 100% of spot as one portfolio with another portfolio consisting of different weights of spot and futures from 10% to 90%. The corresponding DD test results for the whole sample are reported in Table 7.[14]

The second and fourth columns in Table 7 indicate that risk averters prefer spot to any convex combination of spot and futures, which in turn is preferred to futures. On the other hand, the third and fifth columns indicate that risk seekers prefer futures to any convex combination of spot and futures, which is then preferred over spot. In short, the diversification results in Table 7 are consistent with the preferences of spot and futures without diversification. This finding is consistent with the convex diversification theory developed by Fishburn (1989), Li and Wong (1999), Wong and Li (1999), and others.

VI. DISCUSSION AND CONCLUDING REMARKS

We note that our article is an empirical paper that admits the possibility of the existence of traders with heterogeneous utility functions, including risk averters and risk seekers, as well as those with S-shaped and reverse S-shaped

13. To save space, we do not show the diagrams plotting the ASD and DSD test statistics as well as the empirical ACDF and DCDF, but they are available upon request.

14. As a robustness check, we also conducted this analysis for the two subsamples. The results are qualitatively the same. To save space, we do not report the results here. However, they are available upon request.

TABLE 5
Robustness Checking Results of ASD Test for Risk Averters Using Spot Returns Including Dividends

	FASD		SASD		TASD	
	$\%T_1^A > 0$	$\%T_1^A < 0$	$\%T_2^A > 0$	$\%T_2^A < 0$	$\%T_3^A > 0$	$\%T_3^A < 0$
Full sample						
Total	38	20	58	0	89	0
Positive domain	0	20	17	0	51	0
Negative domain	38	0	41	0	38	0
$\left\lvert \max\left(T_j^A\right)\right\rvert$	4.723	5.616	9.657	0	8.610	0

Notes: This table reports the ASD test T_j^A results for risk averters in which $T_j^A (j = 1, 2, \text{and } 3)$ is defined in Equation (2) with F_j^A and G_j^A denoting the jth order ACDFs of the results of futures and spot, respectively. The spot returns are calculated using TAIEX total return index, which is launched on January 2, 2003. The ASD test statistics are computed over a grid of 100 on the range of the empirical distributions of stock and futures returns. The table reports the percentage of ASD statistics that are significantly negative or positive at the 5% significance level, based on the critical value generated from a bootstrap method proposed by Bai et al. (2011). The sample period covers January 2, 2003 to July 20, 2012.

TABLE 6
Robustness Checking Results of DSD Test for Risk Seekers Using Spot Returns Including Dividends

	FDSD		SDSD		TDSD	
	$\%T_1^D > 0$	$\%T_1^D < 0$	$\%T_2^D > 0$	$\%T_2^D < 0$	$\%T_3^D > 0$	$\%T_3^D < 0$
Full sample						
Total	20	38	53	0	88	0
Positive domain	20	0	40	0	39	0
Negative domain	0	38	13	0	49	0
$\left\lvert \max\left(T_j^D\right)\right\rvert$	5.616	4.723	8.320	0.457	7.102	0

Notes: This table summarizes the DSD test T_j^D results for risk seekers in which T_j^D $(j = 1, 2, \text{and } 3)$ defined in Equation (3) with F_j^D and G_j^D denoting the jth order DCDFs of the results of futures and spot, respectively. The spot returns are calculated using TAIEX total return index, which is launched on January 2, 2003. The DD test statistics are computed over a grid of 100 on the range of the empirical distributions of stock and futures returns. The table reports the percentage of DSD statistics that are significantly negative or positive at the 5% significance level, based on the asymptotic critical value generated from a bootstrap method proposed by Bai et al. (2011). The sample period covers January 2, 2003 to July 20, 2012.

utilities. However, it does not seek to determine directly whether or not these heterogeneous traders actually exist. That is a different approach that we have chosen not to follow. Our approach is to observe the preferences of the hypothesized traders and use the results to draw inference about market efficiency and/or whether or not they actually exist.

The foregoing findings based on SD rules can be used to draw inference on market efficiency and the existence of arbitrage opportunities. Where arbitrage is concerned, Jarrow (1986) and others have shown that, under certain conditions, FSD (FASD or FDSD) implies the existence of an arbitrage opportunity where investors can increase their expected wealth and utilities by shifting from the dominated to the dominant asset. Our results in Section V show that there is no FSD relationship between the Taiwan spot and futures markets. This is evidence that investors can increase neither their expected wealth nor their expected utilities by switching their investment from futures to spot or vice versa. Thus, our findings imply that there is no arbitrage opportunity between the Taiwan spot and futures markets. In the absence of arbitrage opportunities and the associated abnormal returns they imply, we can infer that the Taiwan spot-futures market is FSD efficient.

The situation is different when we look at higher orders of SD. Although higher orders of SD provide no information on wealth increasing arbitrage opportunities, they do provide information on market efficiency and opportunities for increasing utility. For example, Falk and Levy (1989), Shalit and Yitzhaki (1994), Clark

TABLE 7
Results of DD Test for the Portfolio of Spot and Futures

Percentage of Spot	100% Futures $\%T_2^A\left(T_3^A\right)>0$	100% Futures $\%T_2^D\left(T_3^D\right)>0$	100% Spot $\%T_2^A\left(T_3^A\right)>0$	100% Spot $\%T_2^D\left(T_3^D\right)>0$
10	47 (81)	56 (78)	42 (75)	50 (75)
20	46 (80)	55 (77)	42 (72)	51 (75)
30	46 (80)	55 (77)	40 (71)	50 (75)
40	46 (80)	54 (77)	39 (68)	50 (75)
50	45 (79)	54 (76)	37 (64)	49 (75)
60	45 (79)	53 (76)	34 (62)	48 (75)
70	44 (79)	53 (76)	33 (59)	48 (75)
80	44 (79)	52 (75)	32 (56)	48 (75)
90	43 (79)	51 (75)	31 (54)	48 (75)

Notes: The table reports the ASD and DSD test results for the second- (third-) order SD of the portfolios of spot and futures with spot or futures alone. The weight of spot in the portfolios is shown in the first column. The table summarizes the percentage of ASD and DSD statistics, which are significantly positive at the 5% significance level. In computing the ASD and DSD test statistics of the second and third columns, F and G in Equations (2) and (3) refer to 100% futures and a portfolio of spot and futures, respectively. In computing the ASD and DSD test statistics of the fourth and fifth columns, F and G in Equations (2) and (3) refer to a portfolio of spot and futures and 100% spot, respectively. The ASD and DSD test statistics are computed over a grid of 100 on the range of the empirical distributions of spot and futures returns. Full sample covers July 21, 1998 to July 20, 2012.

and Kassimatis (2012), and others have shown that, given two assets, X and Y, if an investor can increase his expected utility by increasing his holding of X and decreasing his holding of Y, the market is inefficient. In Section V, we have shown that spot dominates futures for risk averters and futures dominates spot for risk seekers, if they exist. We have also shown that there is no combination of futures and spot that is not dominated by spot for risk averters and by futures for risk seekers. Clark, Kassimatis, and Jokung (2011) have shown that in these conditions, given individual wealth composed of αS and $(1-\alpha)F$, a portfolio for risk averters composed of $\alpha = 100\%$ spot would be efficient.

These considerations raise several interesting questions of theoretical and practical importance. The first question is whether or not a futures market dominated by the spot market can exist if all investors are risk averse. The answer is yes if the futures market is a cheaper vehicle for hedging the risk associated with future portfolio rebalancing between cash and the risky spot index. Consider, for example, a risk-averse investor at time 0 who intends to increase his exposure to the spot index at time 1, but, because he is risk averse, wants to hedge the price he will pay. Two routes are possible. He can purchase a futures contract or he can borrow and purchase the spot index. Because we have shown that there is no arbitrage opportunity, the futures price at time 0 for delivery at time 1, denoted $F_{0,1}$, will be equal to the current spot price of the index, denoted S_0, multiplied by (1 + the one period risk-free interest rate): $F_{0,1} = S_0(1 + r_F)$.[15] If the investor purchases a futures contract, at maturity his outcome on the futures contract will be $S_1 - F_{0,1}$. In other words, he will have paid $F_{0,1}$ for what is now worth S_1. This outcome can be replicated if he borrows the amount S_0 at the risk-free rate and buys the index. At the loan's maturity, he owns the index worth S_1 and pays the loan of $F_{0,1} = S_0(1 + r_F)$. Because the payoffs are equivalent, the investor will choose the route that is the cheapest to follow. If purchasing the futures contract, which involves one transaction and one commission, is cheaper and less time consuming than organizing the loan and buying spot, which involves two transactions and two sources of cost, the futures market will be the route of choice.[16] The same type of comparison can be made if the investor intends to reduce his exposure to the risky spot index at time 1. He can replicate the outcome of the sale of a futures contract $F_{0,1} - S_1$ by selling the index spot and investing the proceeds in the risk-free asset. He receives the risk-free interest rate, but the time, effort, and transactions costs of organizing the loan and selling spot are also likely to be higher than the same considerations associated with a simple futures transaction.

15. For expository simplicity, we assume no dividend payouts over the period.

16. There is also the question of whether the investor will be able to borrow at the risk free rate. If he cannot borrow at the risk free rate, he will be better off by using the futures market.

Thus, if the costs associated with hedging on the futures market are lower than the costs associated with organizing the hedge on the spot market, the futures market will be the vehicle of choice for the risk-averse investor. When all investors are risk averse, the only advantage of the futures market is to reduce risk, and this comes at the expense of returns in the form of increased costs, which makes spot dominate futures. However, when risk seekers are present, futures can dominate spot. In this case, the expected price of the spot index must be larger than the current futures price, such that $E(S_1) > F_{0,1}$. In other words, forward parity does not hold, but the expected gain in returns is offset by the increased volatility that risk averters will pay to avoid by accepting a futures price lower than the expected spot price. The results in Section V show that futures do, in effect, dominate spot. This is a powerful argument that risk-seeking behavior is present in the Taiwan futures market. If this were not the case, how else, outside of some unexplained financial anomaly, could futures dominate spot?

Thus, we argue that both spot and futures markets can exist when only risk averters are present, but futures can dominate spot only if there is some risk-seeking behavior. This is evidence that some risk-seeking behavior does exist in the Taiwan futures market. However, risk seekers do not have to be numerically important. There only has to be enough of them to offset any disequilibrium between the risk averters using the futures market to hedge future purchases or sales of the spot index. Thus, the overall market could still be efficient even when there is SSD in the spot (futures) market. For example, in equilibrium, the number of trades made by risk averters, who go long in spot and/or short sell futures, would match the number of trades made by risk seekers, who go long in futures and/or short sell spot. In this situation, there is no upward or downward pressure on the price in the spot or futures market, and all different types of investors would be satisfied.

Our results contribute to the evidence on the existence of risk-seeking behavior. They add to the evidence from observed behavior such as purchasing lottery tickets, casino gambling, and bungee jumping and the clinical evidence, such as Holt and Laury (2002), who find that risk seekers do exist, although most subjects are risk averse. Furthermore, in practice, it has long been known that speculators who take on risk in return for a premium are powerful forces in the futures markets and that their behavior could be construed as risk seeking (Keynes 1930).

Our results also make it possible to draw some inference with regard to the existence of investors with S-shaped and reverse S-shaped utilities (e.g., Fishburn and Kochenberger 1979; Friedman and Savage 1948; Kahneman and Tversky 1979; Markowitz 1952). When we examine the positive and negative domains of the return distributions separately, our results are compatible with the existence of both S-shaped and reverse S-shaped utility functions. Investors with S-shaped utility functions prefer spot to futures in the bull market when the returns of both spot and futures are positive. They prefer futures to spot in the bear market when the returns of both spot and futures are negative. Investors with reverse S-shaped utility functions prefer spot to futures in the bear market when the returns of both spot and futures are negative and futures to spot in the bull market when the returns of both spot and futures are positive. These results add to those in the diversification puzzle of Statman (2004), Egozcue et al. (2011) where investors with S-shaped or reverse S-shaped utilities are compatible with the observed behavior of traders holding only a small number of stocks instead of the complete, diversified portfolios suggested in financial theory.

Thus, although we do not check whether risk averters, risk seekers, and investors with S-shaped and reverse S-shaped utility functions actually exist in the market, we do show that their existence is plausible.

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