An Efficient Retransmission Strategy for Multisource Multidestination Relay Networks over Rayleigh Flat Fading Channels

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***Abstract*—This paper considers the reliable transmission for wireless multisource multidestination relay networks where multiple sources want to distribute information to a set of destinations with the assistance of a relay. Simply, applying automatic repeat request (ARQ) protocols for retransmission of lost or erroneous packets may cause a considerable delay with a significantly increased number of retransmissions when multiple sources and multiple destinations are taken into consideration. To solve this problem, we propose a new ARQ protocol based on network coding (NC) to significantly reduce the number of retransmissions of the lost packets. In our proposed NC-based ARQ protocol, the relay detects packets from different sources, then combines and forwards the packets which are lost at the destinations using NC. In order to guarantee that all lost packets are retransmitted in an efficient way, we propose two packet-combination algorithms for the retransmissions at the relay and sources. Furthermore, we analyze the transmission bandwidth of different ARQ protocols over Rayleigh flat fading channels and provide the numerical results to demonstrate the superior performance of the proposed NC-based ARQ protocol over some existing schemes.**

I. INTRODUCTION

Recently, there has been a growing interest in relay techniques to extend the coverage and improve the reliability of wireless networks by exploiting the spatial diversity gains [1]. Relay communications have been investigated and included in Long Term Evolution (LTE)-Advanced and IEEE 802.16m candidates for the International Mobile Telecommunications (IMT)-Advanced (fourth generation [4G]) standards [2]. In basic relay scheme, packets traverse along the relays via a store-and-forward manner, and thus the use of relays does not increase network throughput. Dealing with this, network coding (NC) [3] was applied at the relays to dramatically improve the network throughput over some wireless network topologies such as unicast channels [4], multicast channels [5], and broadcast channels [6].

Generally, using automatic repeat request (ARQ) techniques [7], information can reliably be delivered over multicast or broadcast networks. However, the retransmission of lost packets using ARQ may cause a significant delay since the lost packets are retransmitted individually and the retransmission have to be repeated until every destination receives all packets correctly. To the best of our knowledge, the problem of designing a reliable transmission over multisource multidestination relay networks (MMRNs) [8] that can achieve a high network throughput efficiency has not been properly studied in literature.

In this paper, we propose a new ARQ protocol based on NC for MMRNs, in which the relay detects, combines, and sends the lost packets from different sources to the destinations. Moreover, to achieve the best performance, multiuser detection (MUD) techniques [9] are employed at the relay and destinations. With MUD, lost packets can be combined and retransmitted. Thus, the aim is to develop a new ARQ protocol to retransmit the lost packets in an efficient way. For simple representation of the lost packets at the relay and/or destinations, we classify the lost packets in MMRNs into two types: Type-I refers to the packets that are successfully received at the relay but lost at the destinations, and, Type-II refers to the packets that are lost at both the relay and destinations. Apparently, the retransmission of Type-II packets must be carried out by the sources. The problem of interest is how the relay retransmits Type-I packets with the smallest number of retransmissions. To cope with the retransmission problem, we propose an algorithm at the relay and an algorithm at the sources to retransmit Type-I and Type-II packets, respectively. Particularly, the algorithm for the retransmission at the relay is proposed based on an integration of NC and packet detection from two different sources.

As a further contribution of the paper, we compare our proposed NC-based ARQ protocol with other ARQ protocols for MMRNs by evaluating the transmission bandwidth over Rayleigh flat fading channels from both theoretical analysis and numerical results perspective. In fact, three protocols are taken into account for comparison: direct transmission (DT)[1], relaying transmission (RT)[2], and our proposed NC-based ARQ. As we show later in an example (Fig. 2), with

[1]DT protocol refers to the model in which multiple sources simultaneously transmit information to the destinations without using the relaying technique.

[2]RT protocol refers to the model in which the relay participates in the transmission but NC is not employed at the relay.

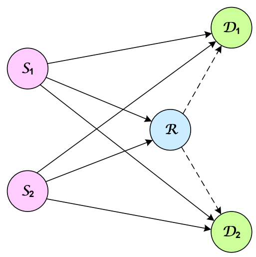

Fig. 1. Multisource multidestination relay network model.

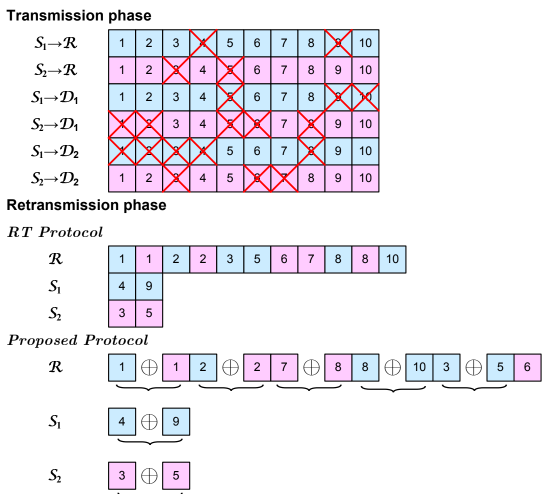

Fig. 2. Retransmission with RT and our proposed NC-based ARQ protocol.

our proposed ARQ protocol, the number of retransmissions in MMRNs is significantly reduced, compared with DT and RT protocols. This observation is finally confirmed through numerical results.

II. System Model and Transmission Protocols

In this paper, we consider a specific MMRN as shown in Fig. 1 where the data delivery from two sources $\mathcal{S}_1$ and $\mathcal{S}_2$ to two destinations $\mathcal{D}_1$ and $\mathcal{D}_2$ is assisted by one relay $\mathcal{R}$. We assume that the sources send data in the form of packets and each packet must be received correctly by all destinations after several transmissions and retransmissions, and the channel of the link $\mathcal{A} \to \mathcal{B}$, where $\mathcal{A} \in \{\mathcal{S}_1, \mathcal{S}_2, \mathcal{R}\}$, $\mathcal{B} \in \{\mathcal{R}, \mathcal{D}_1, \mathcal{D}_2\}$, $\mathcal{A} \neq \mathcal{B}$, is Rayleigh flat fading with a channel gain of $h_{\mathcal{AB}} \sim \mathcal{CN}(0, 1)$.

Receiving data from $\mathcal{S}_1$ and $\mathcal{S}_2$ along with feedback from $\mathcal{D}_1$ and $\mathcal{D}_2$, $\mathcal{R}$ knows what destinations are waiting for the lost packets to be retransmitted, and then decides how to combine and forward the data to the intended destinations. The role of retransmission protocols is to facilitate $\mathcal{R}$ to resend the lost packets to $\mathcal{D}_1$ and $\mathcal{D}_2$.

A. DT Protocol

In this protocol, $\mathcal{S}_1$ and $\mathcal{S}_2$ send data directly to $\mathcal{D}_1$ and $\mathcal{D}_2$. The transmission is carried out with the ARQ scheme and completes when both $\mathcal{D}_1$ and $\mathcal{D}_2$ receive the data correctly from both $\mathcal{S}_1$ and $\mathcal{S}_2$.

B. RT Protocol

This protocol is different from the DT protocol in that $\mathcal{R}$ participates in the transmission. When $\mathcal{D}_j, j = 1, 2$, fails to receive the packet from $\mathcal{S}_i, i = 1, 2$, whereas $\mathcal{R}$ receives this packet successfully, $\mathcal{R}$ can help $\mathcal{S}_i$ forward its correctly received packet to $\mathcal{D}_j$ in the next time slot. The retransmission at $\mathcal{R}$ will continue until its transmitted packet is received correctly by the intended $\mathcal{D}_j$. In the situation where $\mathcal{D}_j$ and $\mathcal{R}$ fail to receive the same packet from $\mathcal{S}_i$, it is obvious that $\mathcal{S}_i$ needs to resend that lost packet.

C. Our Proposed Protocol

Instead of resending the lost packet as soon as $\mathcal{D}_j, j = 1, 2$, fails to receive it, the retransmission in our proposed ARQ protocol will occur after N packets. A buffer of length N is required at $\mathcal{S}_i, i = 1, 2$, whereas a buffer of length $2N$ is required at $\mathcal{R}$ and $\mathcal{D}_j$ since they receive packets from two different sources. To improve the network throughput, $\mathcal{R}$ retransmits the packets of Type-I, and $\mathcal{S}_i$ deals with the retransmission of Type-II packets. What is different in our proposed ARQ protocol is that $\mathcal{R}$ can mix the packets from different data flows.

Let us illustrate our proposed protocol by an example as shown in Fig. 2, where $\mathcal{S}_i$ wishes to deliver $N = 10$ packets $\{s_i[1], s_i[2], \ldots, s_i[10]\}$ to both $\mathcal{D}_1$ and $\mathcal{D}_2$. In Fig. 2, the packets with a cross are lost or erroneously received. Without loss of generality, we assume that, for the data flow from $\mathcal{S}_1$, the erroneously received packets at $\mathcal{R}$, $\mathcal{D}_1$, and $\mathcal{D}_2$ are $\{s_1[4], s_1[9]\}$, $\{s_1[5], s_1[9], s_1[10]\}$, and $\{s_1[1], s_1[2], s_1[3], s_1[4], s_1[8]\}$, respectively. Similarly, the erroneously received packets at $\mathcal{R}$, $\mathcal{D}_1$, and $\mathcal{D}_2$ from $\mathcal{S}_2$ are assumed to be $\{s_2[3], s_2[5]\}$, $\{s_2[1], s_2[2], s_2[5], s_2[6], s_2[8]\}$, and $\{s_2[3], s_2[6], s_2[7]\}$, respectively.

As shown in Fig. 2, $\mathcal{R}$ needs to retransmit 11 packets by the RT protocol. For the packets $\{s_1[4], s_1[9], s_2[3], s_2[5]\}$ that are lost at $\mathcal{R}$ and also lost at $\mathcal{D}_1$ and/or $\mathcal{D}_2$, $\mathcal{S}_1$ and $\mathcal{S}_2$ need to retransmit $\{s_1[4], s_1[9]\}$ and $\{s_2[3], s_2[5]\}$, respectively. Thus, in total, 15 retransmissions are required by the RT protocol.

The number of retransmissions can be significantly reduced with our proposed NC-based ARQ scheme. According to our definition above, packets $\{s_1[1], s_2[1], s_1[2], s_2[2], s_1[3], s_1[5], s_2[6], s_2[7], s_1[8], s_2[8], s_1[10]\}$ are Type-I packets and packets $\{s_1[4], s_1[9], s_2[3], s_2[5]\}$ are Type-II packets. To improve the network throughput, in the retransmission phase, $\mathcal{R}$ forwards $\{s_1[1] \oplus s_2[1], s_1[2] \oplus s_2[2], s_2[7] \oplus s_2[8], s_1[8] \oplus s_1[10], s_1[3] \oplus s_1[5], s_2[6]\}$, and, $\mathcal{S}_1$ and $\mathcal{S}_2$ retransmit $\{s_1[4] \oplus s_1[9]\}$ and $\{s_2[3] \oplus s_2[5]\}$, respectively, where $\oplus$ denotes the bitwise XOR operator. The details of the combination algorithms at $\mathcal{R}$ and $\mathcal{S}_i, i = 1, 2$, are presented in Algorithm 1 and Algorithm 2, respectively (see below). This means that our proposed NC-based ARQ requires only 8 retransmissions, and thus saves 7 retransmissions compared with the RT scheme. $\mathcal{R}$, $\mathcal{S}_1$, and $\mathcal{S}_2$ retransmit these packets until they are received successfully by both $\mathcal{D}_1$ and $\mathcal{D}_2$. Then, the lost packets at $\mathcal{D}_j, j = 1, 2$, can be recovered by XORing the correctly received packets at $\mathcal{D}_j$ with the XORed packets received from $\mathcal{R}$ or $\mathcal{S}_i$.

Algorithm 1 Algorithm at $\mathcal{R}$ to retransmit Type-I packets

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1: Let $\mathfrak{G}_1$ and $\mathfrak{G}_2$ denote the ordered sets of correctly received packets at $\mathcal{R}$ transmitted from $\mathcal{S}_1$ and $\mathcal{S}_2$, respectively: $\mathfrak{G}_1 = \{s_1[i_1], s_1[i_2], \ldots, s_1[i_m]\}$, where $i_1 < i_2 < \cdots < i_m \in \{1, 2, \ldots, N\}$, $\mathfrak{G}_2 = \{s_2[j_1], s_2[j_2], \ldots, s_2[j_n]\}$, where $j_1 < j_2 < \cdots < j_n \in \{1, 2, \ldots, N\}$.
   Define $\Omega = \mathfrak{G}_1 \cup \mathfrak{G}_2$ and divide Ω into 3 groups as follows:
   - Group Ω_1 includes packets that $\mathcal{R}$ receives successfully from both $\mathcal{S}_1$ and $\mathcal{S}_2$, i.e., $\Omega_1 = \{(s_1[i], s_2[j]) \mid i = j\}$. In Fig. 2, Ω_1 = $\{(s_1[1], s_2[1]), (s_1[2], s_2[2]), (s_1[6], s_2[6]), (s_1[7], s_2[7]), (s_1[8], s_2[8]), (s_1[10], s_2[10])\}$.
   - Group Ω_2 includes packets that $\mathcal{R}$ receives successfully from $\mathcal{S}_1$ but fails to receive from $\mathcal{S}_2$, i.e., $\Omega_2 = \{s_1[i] \mid i \notin \{j_1, j_2, \ldots, j_n\}\}$. In Fig. 2, $\Omega_2 = \{s_1[3], s_1[5]\}$.
   - Group Ω_3 includes packets that $\mathcal{R}$ receives successfully from $\mathcal{S}_2$ but fails to receive from $\mathcal{S}_1$: $\Omega_3 = \{s_2[j] \mid j \notin \{i_1, i_2, \ldots, i_m\}\}$. In Fig. 2, $\Omega_3 = \{s_2[4], s_2[9]\}$.

2: For packets in Ω_1, if one packet is received correctly at $\mathcal{D}_1$ but lost at $\mathcal{D}_2$, while another packet is received correctly at $\mathcal{D}_2$ but lost at $\mathcal{D}_1$, we can combine these two packets. Thus, there are 3 possibilities: $s_1[k_1] \oplus s_2[k_2]$ or $s_1[m_1] \oplus s_1[m_2]$ or $s_2[n_1] \oplus s_2[n_2]$. Start from left to right in the group of packets in Ω_1, and choose the suitable XOR combination of packets (e.g., $s_1[1] \oplus s_2[1]$, $s_1[2] \oplus s_2[2]$, $s_2[7] \oplus s_2[8]$, and $s_1[8] \oplus s_1[10]$ in Fig. 2).

3: For packets in Ω_2 and Ω_3, similarly if one packet is received correctly at $\mathcal{D}_1$ but lost at $\mathcal{D}_2$, while another packet is received correctly at $\mathcal{D}_2$ but lost at $\mathcal{D}_1$, we can combine these two packets in only one way $s_1[m_1] \oplus s_1[m_2]$ (for Ω_2) or $s_2[n_1] \oplus s_2[n_2]$ (for Ω_3) (e.g., $s_1[3] \oplus s_1[5]$ in Fig. 2).

4: For the remaining lost packets at $\mathcal{D}_1$ and $\mathcal{D}_2$ that $\mathcal{R}$ receives successfully but cannot perform the combination, these are normally resent without using NC (e.g., $s_1[6]$ in Fig. 2).

III. Transmission Bandwidth Analysis

In this section, we derive the transmission bandwidth (TB)[3] of different transmission protocols in MMRNs consisting of $\mathcal{S}_1$, $\mathcal{S}_2$, $\mathcal{R}$, $\mathcal{D}_1$, and $\mathcal{D}_2$ over Rayleigh flat fading channels.

Over fading channels, the signal received at $\mathcal{B}$ from $\mathcal{A}$, where $\{\mathcal{A}, \mathcal{B}\} \in \{\mathcal{S}_1, \mathcal{S}_2, \mathcal{R}, \mathcal{D}_1, \mathcal{D}_2\}$, $\mathcal{A} \neq \mathcal{B}$, can be written as

$$\mathbf{y}_{\mathcal{AB}} = \sqrt{P_{\mathcal{AB}}} h_{\mathcal{AB}} \mathbf{x}_{\mathcal{AB}} + \mathbf{n}_{\mathcal{AB}}, \tag{1}$$

where $P_{\mathcal{AB}}$ is the transmission power of link $\mathcal{A} \to \mathcal{B}$, $\mathbf{x}_{\mathcal{AB}}$ is the binary phase shift keying (BPSK) modulated signal of

[3]Transmission bandwidth is defined as the average number of transmissions to successfully transmit two packets from two sources to two destinations.

Algorithm 2 Algorithm at $\mathcal{S}_i$ to retransmit Type-II packets

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1: Through the feedback from $\mathcal{D}_1$, $\mathcal{D}_2$, and $\mathcal{R}$, $\mathcal{S}_i$ determines the number and the position of remaining lost packets at destinations that $\mathcal{R}$ also fails in receiving them.

2: Combine the packets for retransmission by NC with the condition that only one packet in the combined packet should be received correctly by only one destination, similar to the combination performed for packets in Ω_2 and Ω_3 as explained in Algorithm 1. For example, in Fig. 2, $\mathcal{S}_1$ resends $s_1[4] \oplus s_1[9]$ and $\mathcal{S}_2$ resends $s_2[3] \oplus s_2[5]$.

3: For the remaining lost packets at $\mathcal{D}_1$ and $\mathcal{D}_2$ that $\mathcal{S}_i$ cannot perform the combination, these are resent without NC.

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the transmitted packet, and $\mathbf{n}_{\mathcal{AB}}$ is an independent circularly symmetric complex Gaussian (CSCG) noise vector with each entry having zero mean and variance of N_0. In this case, the bit error probability for signal transmission through link $\mathcal{A} \to \mathcal{B}$ is given by $P_b(E_{\mathcal{AB}}) = \phi(\gamma_{\mathcal{AB}})$ [10], where $\gamma_{\mathcal{AB}}$ is the average signal-to-noise ratio (SNR) given by $\gamma_{\mathcal{AB}} = P_{\mathcal{AB}}/N_0$ and $\phi(x) \triangleq \frac{1}{2}\left(1 - \sqrt{\frac{x}{1+x}}\right)$. Thus, the packet loss of the transmission link $\mathcal{A} \to \mathcal{B}$ can be calculated by

$$P_{\mathcal{AB}} = 1 - [1 - P_b(E_{\mathcal{AB}})]^{N_b} = 1 - [1 - \phi(\gamma_{\mathcal{AB}})]^{N_b}, \tag{2}$$

where N_b is the number of bits in a packet.

A. DT Protocol

Without $\mathcal{R}$ and NC, the TB of the DT protocol is given by

$$n_{DT} = \max\{n_{DT}^{(\mathcal{S}_1)}, n_{DT}^{(\mathcal{S}_2)}\}, \tag{3}$$

where $n_{DT}^{(\mathcal{S}_i)}, i = 1, 2$, denotes the average number of transmissions required for $\mathcal{S}_i$ to send a packet to both $\mathcal{D}_1$ and $\mathcal{D}_2$, and is calculated as [6]

$$n_{DT}^{(\mathcal{S}_i)} = \frac{1}{1 - P_{\mathcal{S}_i\mathcal{D}_1}} + \frac{1}{1 - P_{\mathcal{S}_i\mathcal{D}_2}} - \frac{1}{1 - P_{\mathcal{S}_i\mathcal{D}_1} P_{\mathcal{S}_i\mathcal{D}_2}}. \tag{4}$$

B. RT Protocol

With $\mathcal{R}$ and when no NC is applied, the average number of transmissions required to successfully transmit two packets from $\mathcal{S}_1$ and $\mathcal{S}_2$ to $\mathcal{D}_i, i = 1, 2$, is given by Eq. (5) (see below), where $n_{\mathcal{RD}_i}$ and $n_{RT}^{(\mathcal{S}_i, \mathcal{D}_j)}$ denote the average number of transmissions to successfully transmit a packet from $\mathcal{R}$ to $\mathcal{D}_i$ and from $\mathcal{S}_i$ to $\mathcal{D}_j$ with the help of $\mathcal{R}$, respectively. Thus, $n_{\mathcal{RD}_i}$ and $n_{RT}^{(\mathcal{S}_i, \mathcal{D}_j)}$ are computed by

$$n_{\mathcal{RD}_i} = \frac{1}{1 - P_{\mathcal{RD}_i}}, \tag{6}$$

$$n_{RT}^{(\mathcal{S}_i, \mathcal{D}_j)} = \frac{1 + P_{\mathcal{RD}_j} + P_{\mathcal{S}_i\mathcal{D}_j}(1 - P_{\mathcal{S}_i\mathcal{R}})}{(1 - P_{\mathcal{S}_i\mathcal{R}} P_{\mathcal{S}_i\mathcal{D}_j})(1 - P_{\mathcal{RD}_j})}. \tag{7}$$

Finally, the TB of this protocol is given by

$$n_{RT} = \max\{n_{RT}^{(\mathcal{D}_1)}, n_{RT}^{(\mathcal{D}_2)}\}. \tag{8}$$

$$n_{RT}^{(\mathcal{D}_i)} = \frac{1}{1 - P_{\mathcal{S}_1\mathcal{R}}P_{\mathcal{S}_2\mathcal{R}}P_{\mathcal{S}_1\mathcal{D}_i}P_{\mathcal{S}_2\mathcal{D}_i}}[1 + P_{\mathcal{S}_1\mathcal{R}}P_{\mathcal{S}_1\mathcal{D}_i}(1 - P_{\mathcal{S}_2\mathcal{D}_i})n_{RT}^{(\mathcal{S}_1,\mathcal{D}_i)} + P_{\mathcal{S}_2\mathcal{R}}(1 - P_{\mathcal{S}_1\mathcal{D}_i})P_{\mathcal{S}_2\mathcal{D}_i}n_{RT}^{(\mathcal{S}_2,\mathcal{D}_i)}$$
$$+ (1 - P_{\mathcal{S}_1\mathcal{R}})P_{\mathcal{S}_1\mathcal{D}_i}(1 - P_{\mathcal{S}_2\mathcal{D}_i})n_{\mathcal{R}\mathcal{D}_i} + (1 - P_{\mathcal{S}_2\mathcal{R}})(1 - P_{\mathcal{S}_1\mathcal{D}_i})P_{\mathcal{S}_2\mathcal{D}_i}n_{\mathcal{R}\mathcal{D}_i} + 2(1 - P_{\mathcal{S}_1\mathcal{R}})(1 - P_{\mathcal{S}_2\mathcal{R}})P_{\mathcal{S}_1\mathcal{D}_i}P_{\mathcal{S}_2\mathcal{D}_i}n_{\mathcal{R}\mathcal{D}_i} \quad (5)$$
$$+ (1 - P_{\mathcal{S}_1\mathcal{R}})P_{\mathcal{S}_2\mathcal{R}}P_{\mathcal{S}_1\mathcal{D}_i}P_{\mathcal{S}_2\mathcal{D}_i}(n_{\mathcal{R}\mathcal{D}_i} + n_{RT}^{(\mathcal{S}_2,\mathcal{D}_i)}) + P_{\mathcal{S}_1\mathcal{R}}(1 - P_{\mathcal{S}_2\mathcal{R}})P_{\mathcal{S}_1\mathcal{D}_i}P_{\mathcal{S}_2\mathcal{D}_i}(n_{\mathcal{R}\mathcal{D}_i} + n_{RT}^{(\mathcal{S}_1,\mathcal{D}_i)})],$$

C. Our Proposed Protocol

$\mathcal{R}$ in our proposed protocol combines the lost packets of different flows. Since a total of $2N$ packets is transmitted from $\mathcal{S}_1$ and $\mathcal{S}_2$, the TB is expressed as

$$n = \frac{n^{(1)} + n^{(2)} + n^{(3)}}{2N}, \quad (9)$$

where $n^{(i)}$, $i = 1, 2, 3$, denotes the average number of transmissions in the i-th step of the proposed protocol, including:

- Step 1. Each of $\mathcal{S}_1$ and $\mathcal{S}_2$ transmits N packets.
- Step 2. $\mathcal{R}$ retransmits Type-I packets.
- Step 3. $\mathcal{S}_1$ and/or $\mathcal{S}_2$ retransmit Type-II packets.

Thus, $n^{(1)}$, $n^{(2)}$, and $n^{(3)}$ are given by

$$n^{(1)} = 2N, \quad (10)$$

and Eqs. (11) and (12), where $E[.]$ denotes the expectation value and C_k^N denotes the total number of subsets consisting of k elements in a set of N elements. Here, K, L, and M denote three random variables used to represent the numbers of packets that $\mathcal{R}$ successfully receives in groups Ω_1, Ω_2, and Ω_3, respectively.

Given that $K = k$ packets are received successfully at $\mathcal{R}$ in Ω_1, the average number of transmissions at $\mathcal{R}$ based on the proposed algorithm (i.e., Algorithm 1) in the second step can be computed by

$$E[n^{(2)}|K = k] = \sum_{i=0}^{k}\sum_{j=0}^{k}\sum_{u=0}^{k}\sum_{v=0}^{k} C_i^k P_{\mathcal{S}_1\mathcal{D}_1}^i (1-P_{\mathcal{S}_1\mathcal{D}_1})^{k-i}$$
$$\times C_j^k P_{\mathcal{S}_2\mathcal{D}_1}^j (1 - P_{\mathcal{S}_2\mathcal{D}_1})^{k-j} \quad (13)$$
$$\times C_u^k P_{\mathcal{S}_1\mathcal{D}_2}^u (1 - P_{\mathcal{S}_1\mathcal{D}_2})^{k-u} C_v^k P_{\mathcal{S}_2\mathcal{D}_2}^v (1 - P_{\mathcal{S}_2\mathcal{D}_2})^{k-v}$$
$$\times [\min\{i + j, u + v\} n_{DT}^{(\mathcal{R})} + |(i + j) - (u + v)| n_{\mathcal{R}\mathcal{D}_a}],$$

where $n_{DT}^{(\mathcal{R})}$ is the average number of transmissions required at $\mathcal{R}$ to send a packet to both $\mathcal{D}_1$ and $\mathcal{D}_2$, and $n_{\mathcal{R}\mathcal{D}_a}$ is given by (6) with $a = 1$ if $i+j > u+v$, and $a = 2$ otherwise. Here, $n_{DT}^{(\mathcal{R})}$ can be similarly obtained as (4), i.e.,

$$n_{DT}^{(\mathcal{R})} = \frac{1}{1 - P_{\mathcal{R}\mathcal{D}_1}} + \frac{1}{1 - P_{\mathcal{R}\mathcal{D}_2}} - \frac{1}{1 - P_{\mathcal{R}\mathcal{D}_1}P_{\mathcal{R}\mathcal{D}_2}}. \quad (14)$$

For the packets in Ω_2 and Ω_3 in the second step, the average number of transmissions is calculated by

$$E[n^{(2)}|L = l] = \sum_{i=0}^{l}\sum_{j=0}^{l} C_i^l P_{\mathcal{S}_1\mathcal{D}_1}^i (1 - P_{\mathcal{S}_1\mathcal{D}_1})^{l-i} \quad (15)$$
$$\times C_j^l P_{\mathcal{S}_1\mathcal{D}_2}^j (1-P_{\mathcal{S}_1\mathcal{D}_2})^{l-j} [\min\{i, j\} n_{DT}^{(\mathcal{R})} + |i-j| n_{\mathcal{R}\mathcal{D}_a}],$$

$$E[n^{(2)}|M = m] = \sum_{i=0}^{m}\sum_{j=0}^{m} C_i^m P_{\mathcal{S}_2\mathcal{D}_1}^i (1 - P_{\mathcal{S}_2\mathcal{D}_1})^{m-i} \quad (16)$$
$$\times C_j^m P_{\mathcal{S}_2\mathcal{D}_2}^j (1-P_{\mathcal{S}_2\mathcal{D}_2})^{m-j} [\min\{i, j\} n_{DT}^{(\mathcal{R})} + |i-j| n_{\mathcal{R}\mathcal{D}_a}],$$

where $a = 1$ if $i > j$, and $a = 2$ otherwise.

In the third step where $\mathcal{R}$ fails to receive packets of the first group in the first step, $\mathcal{S}_1$ and $\mathcal{S}_2$ are required to retransmit these remaining lost packets with the average number of transmissions given by

$$E[n^{(3)}|K{=}k] = \sum_{i=0}^{N-k}\sum_{j=0}^{N-k}\sum_{u=0}^{N-k}\sum_{v=0}^{N-k} C_i^{N-k} P_{\mathcal{S}_1\mathcal{D}_1}^i (1-P_{\mathcal{S}_1\mathcal{D}_1})^{N-k-i}$$
$$\times C_j^{N-k} P_{\mathcal{S}_2\mathcal{D}_1}^j (1-P_{\mathcal{S}_2\mathcal{D}_1})^{N-k-j}$$
$$\times C_u^{N-k} P_{\mathcal{S}_1\mathcal{D}_2}^u (1-P_{\mathcal{S}_1\mathcal{D}_2})^{N-k-u} C_v^{N-k} P_{\mathcal{S}_2\mathcal{D}_2}^v (1-P_{\mathcal{S}_2\mathcal{D}_2})^{N-k-v}$$
$$\times [\min\{i + j, u + v\} n_{RT} + |(i + j) - (u + v)| n_{RT}^{(\mathcal{D}_a)}], \quad (17)$$

where $a = 1$ if $i + j > u + v$, and $a = 2$ otherwise. For the second group and the third group in the third step, the average numbers of transmissions are computed, respectively, through

$$E[n^{(3)}|L = l] = \sum_{i=0}^{N-k-l}\sum_{j=0}^{N-k-l} C_i^{N-k-l} P_{\mathcal{S}_1\mathcal{D}_1}^i$$
$$\times (1-P_{\mathcal{S}_1\mathcal{D}_1})^{N-k-l-i} C_j^{N-k-l} P_{\mathcal{S}_1\mathcal{D}_2}^i (1-P_{\mathcal{S}_1\mathcal{D}_2})^{N-k-l-j} \quad (18)$$
$$\times [\min\{i, j\} n_{RT}^{(\mathcal{S}_1)} + |i - j| n_{RT}^{(\mathcal{S}_1,\mathcal{D}_a)}],$$

$$E[n^{(3)}|M = m] = \sum_{i=0}^{N-k-l-m}\sum_{j=0}^{N-k-l-m} C_i^{N-k-l-m} P_{\mathcal{S}_2\mathcal{D}_1}^i$$
$$\times (1-P_{\mathcal{S}_2\mathcal{D}_1})^{N-k-l-m-i} C_j^{N-k-l-m} P_{\mathcal{S}_2\mathcal{D}_2}^j$$
$$\times (1-P_{\mathcal{S}_2\mathcal{D}_2})^{N-k-l-m-j} [\min\{i, j\} n_{RT}^{(\mathcal{S}_2)} + |i-j| n_{RT}^{(\mathcal{S}_2,\mathcal{D}_a)}], \quad (19)$$

where $a = 1$ if $i > j$, and $a = 2$ otherwise. In Eqs. (18) and (19), $n_{RT}^{(\mathcal{S}_i)}, i = 1, 2$, denotes the average number of transmissions to transmit packets from $\mathcal{S}_i$ to both $\mathcal{D}_1$ and $\mathcal{D}_2$ through $\mathcal{R}$ that can be computed by Eq. (20).

IV. Numerical Results

In this section, we compare the transmission bandwidth of the different protocols considered above over a Rayleigh flat fading channel. Fig. 3 plots the transmission bandwidth of the three ARQ protocols versus $\gamma_{\mathcal{S}_1\mathcal{R}}$, i.e., the SNR of the wireless link $\mathcal{S}_1 \to \mathcal{R}$. Both numerical and analytical results are included. The range of $\gamma_{\mathcal{S}_1\mathcal{R}}$ is set from 0 to 20 dB to characterize a wide range of wireless applications. To study the effect of the channels from the sources to the relay on the overall performance, we assume that $\gamma_{\mathcal{S}_1\mathcal{R}} = \gamma_{\mathcal{S}_2\mathcal{R}}$. The

$$
\begin{aligned}
n^{(2)} = &\sum_{k=0}^{N}\{C_k^N P_{\mathcal{S}_1\mathcal{R}}^{N-k}(1-P_{\mathcal{S}_1\mathcal{R}})^k P_{\mathcal{S}_2\mathcal{R}}^{N-k}(1-P_{\mathcal{S}_2\mathcal{R}})^k E[n^{(2)}|K=k] + \sum_{l=0}^{N-k}\{C_l^{N-k} P_{\mathcal{S}_1\mathcal{R}}^{N-k-l}(1-P_{\mathcal{S}_1\mathcal{R}})^l \\
&\times P_{\mathcal{S}_2\mathcal{R}}^{l}(1-P_{\mathcal{S}_2\mathcal{R}})^{N-k-l} E[n^{(2)}|L=l] + \sum_{m=0}^{N-k-l}\{C_m^{N-k-l} P_{\mathcal{S}_1\mathcal{R}}^{m}(1-P_{\mathcal{S}_1\mathcal{R}})^{N-k-l-m} P_{\mathcal{S}_2\mathcal{R}}^{N-k-l-m}(1-P_{\mathcal{S}_2\mathcal{R}})^m E[n^{(2)}|M=m]\}\}\},
\end{aligned}
\tag{11}
$$

$$
\begin{aligned}
n^{(3)} = &\sum_{k=0}^{N}\{C_k^N P_{\mathcal{S}_1\mathcal{R}}^{N-k}(1-P_{\mathcal{S}_1\mathcal{R}})^k P_{\mathcal{S}_2\mathcal{R}}^{N-k}(1-P_{\mathcal{S}_2\mathcal{R}})^k E[n^{(3)}|K=k] + \sum_{l=0}^{N-k}\{C_l^{N-k} P_{\mathcal{S}_1\mathcal{R}}^{N-k-l}(1-P_{\mathcal{S}_1\mathcal{R}})^l \\
&\times P_{\mathcal{S}_2\mathcal{R}}^{l}(1-P_{\mathcal{S}_2\mathcal{R}})^{N-k-l} E[n^{(3)}|L=l] + \sum_{m=0}^{N-k-l}\{C_m^{N-k-l} P_{\mathcal{S}_1\mathcal{R}}^{m}(1-P_{\mathcal{S}_1\mathcal{R}})^{N-k-l-m} P_{\mathcal{S}_2\mathcal{R}}^{N-k-l-m}(1-P_{\mathcal{S}_2\mathcal{R}})^m E[n^{(3)}|M=m]\}\}\},
\end{aligned}
\tag{12}
$$

$$
\begin{aligned}
n_{RT}^{(\mathcal{S}_i)} = &\frac{1}{1-P_{\mathcal{S}_i\mathcal{R}}P_{\mathcal{S}_i\mathcal{D}_1}P_{\mathcal{S}_i\mathcal{D}_2}}[1 + P_{\mathcal{S}_i\mathcal{R}}P_{\mathcal{S}_i\mathcal{D}_1}(1-P_{\mathcal{S}_i\mathcal{D}_2})n_{RT}^{(\mathcal{S}_i,\mathcal{D}_1)} + P_{\mathcal{S}_i\mathcal{R}}(1-P_{\mathcal{S}_i\mathcal{D}_1})P_{\mathcal{S}_i\mathcal{D}_2}n_{RT}^{(\mathcal{S}_i,\mathcal{D}_2)} \\
&+ (1-P_{\mathcal{S}_i\mathcal{R}})P_{\mathcal{S}_i\mathcal{D}_1}(1-P_{\mathcal{S}_i\mathcal{D}_2})n_{\mathcal{R}\mathcal{D}_1} + (1-P_{\mathcal{S}_i\mathcal{R}})(1-P_{\mathcal{S}_i\mathcal{D}_1})P_{\mathcal{S}_i\mathcal{D}_2}n_{\mathcal{R}\mathcal{D}_2} + (1-P_{\mathcal{S}_i\mathcal{R}})P_{\mathcal{S}_i\mathcal{D}_1}P_{\mathcal{S}_i\mathcal{D}_2}n_{DT}^{(\mathcal{R})}].
\end{aligned}
\tag{20}
$$

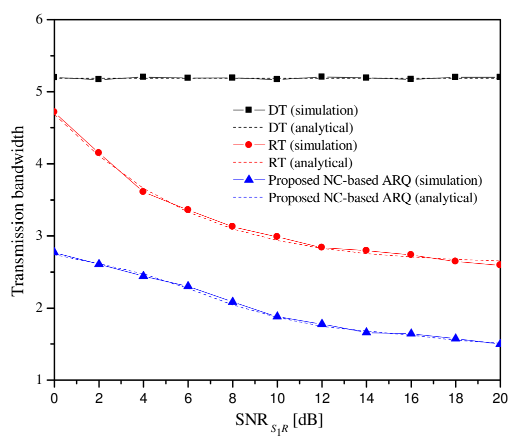

Fig. 3. Transmission bandwidth of different protocols over $\text{SNR}_{\mathcal{S}_1\mathcal{R}}$.

other SNRs are arbitrarily set as $\gamma_{\mathcal{S}_1\mathcal{D}_1} = \gamma_{\mathcal{S}_2\mathcal{D}_2} = 5$ dB, $\gamma_{\mathcal{S}_1\mathcal{D}_2} = \gamma_{\mathcal{S}_2\mathcal{D}_1} = 0$ dB, and $\gamma_{\mathcal{R}\mathcal{D}_1} = \gamma_{\mathcal{R}\mathcal{D}_2} = 10$ dB. We assume that the number of bits in a packet (i.e., N_b) is 10 bits and the length of the buffer at the sources (i.e., N) is 10 packets. It can be seen that our proposed NC-based ARQ protocol outperforms other two schemes since it can combine the lost packets from different flows in the retransmission phase. In particular, our proposed scheme shows a significant gain over the other ARQ methods. For packets in Ω_1, our proposed scheme can save a significant number of retransmissions by mixing the packets from different flows. Additionally, it can be seen that the analytical results are very closely matched with the simulation results.

V. Conclusion

In this paper, we have proposed a new reliable retransmission scheme for multisource multidestination relay networks based on network coding to significantly reduce the number of retransmissions. We have studied our proposed retransmission scheme for a specific case with two sources and two destinations. Specifically, we have presented two packet-combination algorithms to retransmit the lost packets and initially analyzed the transmission bandwidth of our protocol over Rayleigh flat fading channels. The efficiency of retransmission is increased because our algorithms distinguish different types of retransmission situations. Our proposed scheme is effective as it requires a significantly reduced transmission bandwidth compared to the relaying transmission and direct transmission protocols, for example, by over one and three retransmissions respectively. For future work, a more general network will be investigated with different numbers of sources, relays, and destinations.

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