Firefly Algorithm for Beamforming Design in RIS-aided Communications Systems

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***Abstract*—This paper studies a non-convex power minimization problem for reconfigurable-intelligent-surfaces-aided communication systems whose constraints are multivariate functions of two independent optimization variables, i.e., active and passive beamforming vectors. A widely adopted alternative optimization (AO) approach approximates the originally non-convex problem by two convex sub-optimization problems where each sub-optimization problem deals with one variable considering the other variable as a constant. The solution for the original problem is obtained by iteratively solving these sub-optimization problems. Although the AO approach converts the original NP-hard optimization problem to two convex sub-problems, the solutions attained by this method may not be the global optimal solution due to the approximation process as well as the inherent non-convexity of the original problem. To overcome the issue, this paper adopts a nature-inspired optimization approach and introduces a novel Firefly algorithm (FA) to simultaneously solve for two independent optimization variables of the originally non-convex optimization problem. Computational complexity analyses are provided for the proposed FA and the AO approaches. Simulation results reveal that the proposed FA approach prevails its AO counterpart in obtaining a better solution for the under-studied optimization problem with a similar computational complexity.**

***Index Terms*—Firefly algorithm, nature inspired optimization, transmit beamforming, reconfigurable intelligent surfaces.**

I. Introduction

In a reconfigurable-intelligent-surface-aided (RIS-aided) communication system [1], [2], active beamforming vectors, i.e., the beamforming vectors for the mobile users of a serving base station (BS), and a passive beamforming vector, i.e., the vector comprising of the phase-shift coefficients of the RIS's elements, are jointly designed. The design problems are normally posed as optimization problems. The objective function and/or constraints of such problems are functions of both active and passive beamforming vectors. They are independent variables yet need to be jointly optimized making their problems non-convex. The widely adopted approach for tackling the issue approximates the original problem by two convex sub-optimization problems [3]–[6], i.e., alternative optimization (AO) approach. In each of these two sub-optimization problems, one variable, i.e., either active or passive beamforming vector, is treated as a constant while solving for the other. By iteratively solving two sub-problems, the solution for the orignial problem is attained. Due to the approximation process and the inherent non-convexity character of the original problem, the resulting solutions for the passive and active beamforming vectors may not be the global solution.

Interior point methods (IPMs), a.k.a barrier methods, are gradient based algorithms being good at exploitation,[1] a.k.a., intensification, hence, they are regarded as effective methods to solve convex optimization problems [8]. On the other hand, solving non-convex optimization problems requires algorithms having better exploration[2] ability than that of the IPMs to avoid getting trapped in a local mode. Firefly algorithm (FA), i.e., a nature inspired algorithm, possesses both exploitation and exploration abilities. Consequently, FA is a good candidate for solving non-convex downlink beamforming problems. FA is a easy-to-implement, simple, and flexible algorithm based on the flashing characters and behaviour of tropical fireflies [7]. FA was first developed and published by Xin-She Yang, respectively, in late 2007 and in 2008 [7], [9] for optimization problems with objective and constrains being functions of a single optimization variable. Although FA has been widely applied to many applications [10], there has not been any significant work investigating the application of FA in solving transmit beamforming problems. To the best of the authors' knowledge, there was only one attempt to adopt FA for a throughput maximization problem in [11].

This paper considers a multivariate power minimization problem for a RIS-aided communication system. Particularly, passive and active beamforming vectors are, respectively, jointly designed for the RIS and the BS so that the total BS's transmit power is minimized while ensuring the signal-to-interference-plus-noise ratio (SINR) at each mobile user above a required level. The optimization is non-convex due to the fact that the SINR constraint is a function of two independent optimization variables, i.e., passive and active beamforming vectors. The paper proposes a novel FA method to find optimal passive and active beamforming vectors for the multivariate power minimization problem. Furthermore, the paper analyzes and compares the complexities of the AO and FA approaches. Finally, simulation results are obtained to evaluate the performance of the proposed FA approach.

Notation: Lower or upper case letter a or A: a scalar; bold-lower-case letter $\mathbf{a}$: a column vector; bold-upper-case letter $\mathbf{A}$: a matrix; $(\cdot)^T$: the transpose operator; $(\cdot)^H$: the complex-

[1]Exploitation is the ability of using any information from the problem of interest to form new solutions which are better than the current ones [7].

[2]Exploration is the ability of efficient exploring the search space to form new solutions with sufficient diversity and far from the existing ones [7].

conjugate-transpose operator; $||\cdot||$: the Euclidean norm; $\mathrm{Tr}(\cdot)$: the trace operator; $\mathbf{A} \succeq \mathbf{0}$: $\mathbf{A}$ is positive semidefinite; $\mathbf{I}_x$: an $x \times x$ identity matrix; O: the big O notation; $\mathbb{H}^{M_t \times M_t}$: the set of $M_t \times M_t$ Hermitian matrices; $\mathbb{C}^{M_t \times 1}$: the set of $M_t \times 1$ complex-element vectors; $a \sim \mathcal{CN}(0, \sigma^2)$: a is a zero-mean circularly symmetric complex Gaussian random variable with variance σ^2; $\mathrm{diag}(\mathbf{a})$: a diagonal matrix whose diagonal elements are the entries of vector $\mathbf{a}$; and finally $\mathrm{diag}(\mathbf{A})$: a vector whose entries are the diagonal elements of matrix $\mathbf{A}$.

II. Problem Formulation

A. Problem Formulation

Consider a communication system comprising of an M_t-antenna BS communicating with U single-antenna mobile users in which the direct communication links between the BS and its mobile users are blocked, e.g., because of high building etc., [3]. To circumvent the problem, an N_t-reflective-element RIS is utilized to support the communication. Let $\mathbf{H} = [\mathbf{h}_1, \ldots, \mathbf{h}_{N_t}] \in \mathbb{C}^{M_t \times N_t}$ represent the channel coefficients between the BS and the RIS and $\mathbf{g}_i = [g_{i1}, \ldots, g_{iN_t}]^T \in \mathbb{C}^{N_t \times 1}$ be the channel coefficients between the RIS and the i-th user.

Let x_i, i.e., $\mathbb{E}[|x_i|^2] = 1$, and $\mathbf{w}_i \in \mathbb{C}^{M_t \times 1}$, respectively, represent the data symbol and the active beamforming vector for the i-th user. Each reflective element of the RIS generates a phase shift to support the communication between the BS and the mobile users. Let θ_k be the phase shift at the k-th reflective element and let $\boldsymbol{\theta} = [\theta_1,\ \theta_2,\ \cdots,\ \theta_{N_t}]^T$ denote the phase-shift coefficients generated by the RIS with $|\theta_k| \leq 1$ and $\arg(\theta_k) \in [-\pi, \pi), \forall k = 1, \ldots, N_t$. Vector $\boldsymbol{\theta}$ is the passive beamforming vector for the RIS. The signal arrived at the i-th user is:

$$\begin{aligned} y_i &= \mathbf{g}_i^H \mathrm{diag}(\boldsymbol{\theta})^H \mathbf{H}^H \mathbf{w}_i x_i + \mathbf{g}_i^H \mathrm{diag}(\boldsymbol{\theta})^H \mathbf{H}^H \sum_{j=1, j\neq i}^{U} \mathbf{w}_j x_j + n_i, \\ &= \boldsymbol{\theta}^H \mathbf{G}_i^H \mathbf{w}_i x_i + \boldsymbol{\theta}^H \mathbf{G}_i^H \sum_{j=1, j\neq i}^{U} \mathbf{w}_j x_j + n_i, \end{aligned} \tag{1}$$

where $\mathbf{G}_i^H = \mathrm{diag}(\mathbf{g}_i^*)\mathbf{H}^H \in \mathbb{C}^{N_t \times M_t}$ and $n_i \sim \mathcal{CN}(0, \sigma^2)$ represents the additive noise measured at the i-th user. Furthermore, let $\{\mathbf{w}_i\} = \{\mathbf{w}_1, \mathbf{w}_2, \cdots, \mathbf{w}_U\}$ denote the set of active beamforming vectors, and $\mathrm{SINR}_i(\{\mathbf{w}_i\}, \boldsymbol{\theta})$ be the SINR at the i-th user. One can write:

$$\mathrm{SINR}_i(\{\mathbf{w}_i\}, \boldsymbol{\theta}) = \frac{|\boldsymbol{\theta}^H \mathbf{G}_i^H \mathbf{w}_i|^2}{\sum_{j=1, j\neq i}^{U} |\boldsymbol{\theta}^H \mathbf{G}_i \mathbf{w}_j|^2 + \sigma_i^2}. \tag{2}$$

The optimization is posed as follows:

$$\begin{aligned} \min_{\{\mathbf{w}_i\},\ \boldsymbol{\theta}} \quad & \sum_{i=1}^{U} \mathbf{w}_i^H \mathbf{w}_i \\ \text{s. t.} \quad & \mathrm{SINR}_i(\{\mathbf{w}_i\}, \boldsymbol{\theta}) \geq \eta_i, \forall i, \\ & |\theta_k| \leq 1, \forall k, \end{aligned} \tag{3}$$

where η_i is the required SINR level measured at the i-th user. Since the SINR constraint is a function of two optimization variables $\mathbf{w}_i$ and $\boldsymbol{\theta}$, problem (3) is non-convex.

B. Alternative Optimization Approach

For the sake of completeness, the widely adopted AO approach [3]–[6] is represented here as a baseline to solve (3). Let $\mathbf{F}_i = \mathbf{w}_i \mathbf{w}_i^H$ and $\boldsymbol{\Theta} = \boldsymbol{\theta}\boldsymbol{\theta}^H$, normalizing the SINR constraint by σ_i^2 with some manipulations, one can equivalently rewrite (3) as:

$$\begin{aligned} \min_{\{\mathbf{F}_i\},\ \boldsymbol{\Theta}} \quad & \mathrm{Tr}\left(\sum_{i=1}^{U} \mathbf{F}_i\right) \\ \text{s. t.} \quad & \left(1 + \frac{1}{\eta_i}\right) \mathrm{Tr}\left(\frac{\mathbf{G}_i \boldsymbol{\Theta} \mathbf{G}_i^H \mathbf{F}_i}{\sigma_i^2}\right) \\ & - \sum_{j=1}^{U} \mathrm{Tr}\left(\frac{\mathbf{G}_i \boldsymbol{\Theta} \mathbf{G}_i^H \mathbf{F}_j}{\sigma_i^2}\right) - 1 \geq 0,\ \forall i \in \{1, \cdots, U\}, \\ & \mathbf{F}_i \succeq \mathbf{0},\ \mathrm{rank}(\mathbf{F}_i) = 1, \forall i \in \{1, \cdots, U\}, \\ & \mathrm{diag}(\mathrm{diag}(\boldsymbol{\Theta})) \leq \mathbf{I}_{N_t},\ \boldsymbol{\Theta} \succeq \mathbf{0},\ \mathrm{rank}(\boldsymbol{\Theta}) = 1. \end{aligned} \tag{4}$$

Since the first constraint depends on of both $\mathbf{F}_i$ and $\boldsymbol{\Theta}$, problem (4) is still non-convex. As $\mathbf{F}_i$ and $\boldsymbol{\Theta}$ are two independent variables, they can be alternatively solved [3]–[6]. To that end, relaxing the rank-one constraint on $\mathbf{F}_i$ and beginning with any initial value of the reflecting coefficient matrix $\boldsymbol{\Theta}^{(0)}$, the following sub-problem will be solved at the p-th iteration:

$$\begin{aligned} \min_{\{\mathbf{F}_i\}} \quad & \mathrm{Tr}\left(\sum_{i=1}^{U} \mathbf{F}_i\right) \\ \text{s. t.} \quad & \left(1 + \frac{1}{\eta_i}\right) \mathrm{Tr}\left(\frac{\mathbf{G}_i \boldsymbol{\Theta}^{(p-1)} \mathbf{G}_i^H \mathbf{F}_i}{\sigma_i^2}\right) \\ & - \sum_{j=1}^{U} \mathrm{Tr}\left(\frac{\mathbf{G}_i \boldsymbol{\Theta}^{(p-1)} \mathbf{G}_i^H \mathbf{F}_j}{\sigma_i^2}\right) - 1 \geq 0,\ \forall i, \\ & \mathbf{F}_i \succeq \mathbf{0},\ \forall i \in \{1, \cdots, U\}. \end{aligned} \tag{5}$$

The reflecting coefficients $\boldsymbol{\Theta}^{(p)}$ is then updated from the optimal solution of (5) at p-th iteration, i.e., $\{\mathbf{F}_i^{(p)}\}$, by solving the following sub-problem [3]:

$$\begin{aligned} \min_{\boldsymbol{\Theta}} \quad & \mathrm{Tr}(\boldsymbol{\Theta}) \\ \text{s. t.} \quad & \left(1 + \frac{1}{\eta_i}\right) \mathrm{Tr}\left(\frac{\boldsymbol{\Theta} \mathbf{G}_i^H \mathbf{F}_i^{(p)} \mathbf{G}_i}{\sigma_i^2}\right) \\ & - \sum_{j=1}^{U} \mathrm{Tr}\left(\frac{\boldsymbol{\Theta} \mathbf{G}_i^H \mathbf{F}_j^{(p)} \mathbf{G}_i}{\sigma_i^2}\right) - 1 \geq 0,\ \forall i, \\ & \mathrm{diag}(\mathrm{diag}(\boldsymbol{\Theta})) \leq \mathbf{I}_{N_t}, \\ & \boldsymbol{\Theta} \succeq \mathbf{0}. \end{aligned} \tag{6}$$

The AO approach repetitively solves (5) and (6) in n_0 iterations to obtain the solution for (3).

Remark 1: It is worth noticing that the AO approach approximates the originally non-convex optimization (3) by two sub-problems (5) and (6). Although (5) and (6) are convex, the solutions to these sub-problems can be regarded as the upper bounds of the original problem (3) as these solutions may not be the global solution. Furthermore, the AO approach adopts the so-called semidefinite relaxation technique [12] in which the rank-one constraints on $\mathbf{F}_i$ and $\mathbf{\Theta}$ are relaxed. If solving (5) and/or (6) does not return rank-one matrices $\mathbf{F}_i$ and/or $\mathbf{\Theta}$, then a rank-one approximation or a Gaussian randomize procedure [13] is required to extract approximated rank-one solutions. Extracting the approximated solutions requires further computational resources yet only results in sub-optimal solutions.

Motivated by the above observations, we introduce a novel FA approach to simultaneously solve $\mathbf{w}_i$ and $\boldsymbol{\theta}$ for the original problem (3) in the following section.

III. Proposed FA Approach

The optimization (3) is equivalently expressed as follows:

$$\begin{aligned} \min_{\{\mathbf{w}_i\},\ \boldsymbol{\theta}} \quad & \sum_{i=1}^{U} \mathbf{w}_i^H \mathbf{w}_i \\ \text{s. t.} \quad & \Phi_i\left(\{\mathbf{w}_i\}\right) \le 0, \forall i, \\ & \varphi_k\left(\theta_k\right) \le 0, \forall k, \end{aligned} \tag{7}$$

where

$$\begin{aligned} \Phi_i\left(\{\mathbf{w}_i\}\right) \quad = \quad & \eta_i \frac{\sum_{j=1}^{U} \mathbf{w}_j^H \mathbf{G}_i \boldsymbol{\theta}\boldsymbol{\theta}^H \mathbf{G}_i^H \mathbf{w}_j}{\sigma_i^2} + \eta_i \\ & - \left(1+\eta_i\right) \frac{\mathbf{w}_i^H \mathbf{G}_i \boldsymbol{\theta}\boldsymbol{\theta}^H \mathbf{G}_i^H \mathbf{w}_i}{\sigma_i^2}, \end{aligned} \tag{8}$$

and

$$\varphi_k\left(\theta_k\right) = |\theta_k| - 1. \tag{9}$$

Adopting a penalty method [7], (7) can be written as:

$$\min_{\{\mathbf{w}_i\},\ \boldsymbol{\theta}} \quad \sum_{i=1}^{U} \mathbf{w}_i^H \mathbf{w}_i + P(\{\mathbf{w}_i\}, \boldsymbol{\theta}) \tag{10}$$

where $P(\{\mathbf{w}_i\}, \boldsymbol{\theta})$ is the penalty term given as:

$$\begin{aligned} P(\{\mathbf{w}_i\}, \boldsymbol{\theta}) \quad = \quad & \sum_{i=1}^{U} \lambda_i \max\left\{0, \Phi_i(\{\mathbf{w}_i\})\right\}^2 \\ & + \sum_{k=1}^{N_t} \rho_k \max\left\{0, \varphi_k(\theta_k)\right\}^2, \end{aligned} \tag{11}$$

with $\lambda_i > 0$ and $\rho_k > 0$ are penalty constants.

The FA was developed based on the following three idealized rules [7], [9]. First, any firefly attracts other fireflies regardless of its sex. Second, the attractiveness of any firefly to the other one is proportional to its brightness. Both attractiveness and brightness decrease as the distance between these two fireflies increases. Given two flashing fireflies, the darker firefly will move towards the brighter one. If a firefly does not find any brighter one, it will make a random move. Third, the brightness of a firefly depends on the landscape of the objective function.

The original FA approach [7], [9] was introduced for optimization problems with a single optimization variable only. In this paper, we propose a FA approach for an optimization problem for a RIS-aided communication system containing two independent optimization variables $\mathbf{w}_i$ and $\boldsymbol{\theta}$. To that end, let $\{\mathbf{W}_t, \boldsymbol{\theta}_t\}$ be the firefly t where $\mathbf{W}_t = \left[\mathbf{w}_1^t, \mathbf{w}_2^t, \cdots, \mathbf{w}_U^t\right]$. We initialize a population of N fireflies $\{\mathbf{W}_t, \boldsymbol{\theta}_t\}$, $t \in \{1, 2, \cdots, N\}$ and define the brightness, i.e., the light density, of the firefly t $\{\mathbf{W}_t, \boldsymbol{\theta}_t\}$ as:

$$I_t\left(\mathbf{W}_t, \boldsymbol{\theta}_t\right) = \frac{1}{\sum_{i=1}^{U} \mathbf{w}_i^H \mathbf{w}_i + P(\mathbf{w}_i, \boldsymbol{\theta})}. \tag{12}$$

For any fireflies t and l amongst the population, if $I_t\left(\mathbf{W}_t, \boldsymbol{\theta}_t\right) > I_l\left(\mathbf{W}_l, \boldsymbol{\theta}_l\right)$ then the firefly l will move toward the firefly t as:

$$\mathbf{W}_l^{(n+1)} \quad = \quad \mathbf{W}_l^{(n)} + \beta_0 e^{-\gamma\left(r_{w,tl}^{(n)}\right)^2} \left(\mathbf{W}_t^{(n)} - \mathbf{W}_l^{(n)}\right) + \alpha^{(n)}\mathbf{V}, \tag{13}$$

$$\boldsymbol{\theta}_l^{(n+1)} \quad = \quad \boldsymbol{\theta}_l^{(n)} + \beta_0 e^{-\gamma\left(r_{\theta,tl}^{(n)}\right)^2} \left(\boldsymbol{\theta}_t^{(n)} - \boldsymbol{\theta}_l^{(n)}\right) + \alpha^{(n)}\mathbf{v}, \tag{14}$$

where $r_{w,tl}^{(n)} = \|(\mathbf{W}_t^{(n)} - \mathbf{W}_l^{(n)}\|$ and $r_{\theta,tl}^{(n)} = \|(\boldsymbol{\theta}_t^{(n)} - \boldsymbol{\theta}_l^{(n)}\|$ are the Cartesian distances, β_0 is the attractiveness at $r_{w,ij}^{(n)} = 0$ and $r_{\theta,tl}^{(n)} = 0$, γ presents the variation of of the attractiveness. The second terms of (13) and (14) capture the attractions while the third terms of (13) and (14) are randomization comprised of randomization factor $\alpha^{(n)}$, $\mathbf{V} \in \mathbb{C}^{M_t \times U}$ and $\mathbf{v} \in \mathbb{C}^{M_t \times 1}$. The factor $\alpha^{(n)}$, the elements of $\mathbf{V}$ and $\mathbf{v}$ are drawn from either an uniform or a Gaussian distribution. The proposed FA for solving optimization problem (3) is summarized in Algorithm 1 on the next page.

IV. Complexity analysis

In this section, we provide the computational complexity analyses for the AO approach and the proposed FA approach, i.e., Algorithm 1. We start by the following definition.

Definition 1: At a given $\varepsilon > 0$, $\mathbf{F}_t^{\varepsilon}$ is defined as the ε-solution of problem (5), i.e., an acceptable solution with the accuracy of ε, if:

$$\sum_{t=1}^{U} \mathrm{Tr}\left(\mathbf{F}_t^{\varepsilon}\right) \le \min_{\mathbf{F}_t} \sum_{t=1}^{U} \mathrm{Tr}\left(\mathbf{F}_t\right) + \varepsilon. \tag{15}$$

Moreover, $\mathbf{\Theta}^{\varepsilon}$ is defined as the ε-solution of problem (6) if:

$$\mathrm{Tr}\left(\mathbf{\Theta}\right) \le \min_{\mathbf{\Theta}} \mathrm{Tr}\left(\mathbf{\Theta}\right) + \varepsilon. \tag{16}$$

The complexity of the AO approach is described in the following lemma.

Lemma 1: The computational complexity of AO approach is on the order of:

$$n_o\left(\tau_1 + \tau_2\right), \tag{17}$$

where n_0 is the number of iterations to obtain the ε-solution,

$$\begin{aligned} \tau_1 = \ln\left(\varepsilon^{-1}\right) \sqrt{U(M_t+1)} \Big[& (M_t^2+1)U \\ & + UM_t^2(M_t^2+M_t) + M_t^4 \Big] M_t^2, \end{aligned} \tag{18}$$

$$\tau_2 = \ln\left(\varepsilon^{-1}\right) \sqrt{U + 2N_t} \left[(N_t^2+1)(U+2N_t^2) + N_t^4\right] N_t^2. \tag{19}$$

Proof: The proof is similar to [14, Section V-A] and [15, Section IV-C]. ■

Next, the complexity of the proposed FA approach is stated as follows.

Algorithm 1 Firefly Algorithm for solving (3)

Input: Channel matrices $\mathbf{H}$; $\mathbf{g}_i, \forall i$; Noise variance σ_i^2; required SINR η; population size N; maximum generation T; λ_i; ρ_n; β_0; γ;
Randomly generate N populations $\{\{\mathbf{W}_1, \boldsymbol{\theta}_1\}, \{\mathbf{W}_2, \boldsymbol{\theta}_2\}, \cdots, \{\mathbf{W}_N, \boldsymbol{\theta}_N\}\}$;
Evaluate the light intensities of N populations as (12);
Rank the fireflies in a descending order of $I_t(\mathbf{W}_t, \boldsymbol{\theta}_t)$;
Define the current best solution: $I^\star := I_1(\mathbf{W}^\star, \boldsymbol{\theta}^\star)$; $\{\mathbf{W}^\star, \boldsymbol{\theta}^\star\} := \{\mathbf{W}_1, \boldsymbol{\theta}_1\}$;
for $n = 1 : T$ **do**
  for $l = 1 : N$ **do**
    for $t = 1 : N$ **do**
      if $I_l(\mathbf{W}_l, \boldsymbol{\theta}_l) > I^\star$ **then**
        $I^\star := I_l(\mathbf{W}_l, \boldsymbol{\theta}_l)$; $\{\mathbf{W}^\star, \boldsymbol{\theta}^\star\} := \{\mathbf{W}_l, \boldsymbol{\theta}_l\}$;
      end if
      if $I_t(\mathbf{W}_t, \boldsymbol{\theta}_t) > I^\star$ **then**
        $I^\star := I_t(\mathbf{W}_t, \boldsymbol{\theta}_t)$; $\{\mathbf{W}^\star, \boldsymbol{\theta}^\star\} := \{\mathbf{W}_t, \boldsymbol{\theta}_t\}$;
      end if
      if $I_t(\mathbf{W}_t, \boldsymbol{\theta}_t) > I_l(\mathbf{W}_l, \boldsymbol{\theta}_l)$ **then**
        Move firefly l towards firefly t as (13) and (14);
      end if
      Attractiveness varies with distances via $e^{-\gamma\left(r^{(n)}_{w,tl}\right)^2}$ and $e^{-\gamma\left(r^{(n)}_{\theta,tl}\right)^2}$;
      Evaluate new solutions and update light intensity as (12);
    end for
  end for
  Rank the fireflies in a descending order of $I_t(\mathbf{W}_t, \boldsymbol{\theta}_t)$;
  Update the current best solution: $I^\star := I_1(\mathbf{W}^\star, \boldsymbol{\theta}^\star)$; $\{\mathbf{W}^\star, \boldsymbol{\theta}^\star\} := \{\mathbf{W}_1, \boldsymbol{\theta}_1\}$;
end for
Post-process results and visualization;
return $\mathbf{W}^\star, \boldsymbol{\theta}^\star$.

Lemma 2: The computational complexity of the FA approach is on the order of:

$$\begin{aligned} &TN^2\left[M_t^2 + N_t + N\left(UM_t + U(N_t^2 + M_tN_t) + N_t\right)\right] \\ &+TN\log N + NM_tU + N_tN + N\log N \\ &+N\left[UM_t + U(N_t^2 + M_tN_t) + N_t\right]. \end{aligned} \tag{20}$$

Proof: Due to space limitation the proof is omitted here. It will be provided in a full report of this work. ■

Remark 2: When the numbers of antennas M_t and N_t are large, letting $N_t = M_t$ in (17), the dominant term of the complexity to attain ε-solution to (3) is $N_t^{6\frac{1}{2}}$. On the other hand, the dominant term of (20) is N_t^5 when assuming $N = N_t = M_t$.

V. Simulation Results

We simulate a RIS-aided communication system which consists of one BS, one RIS, and two users, i.e., $U = 2$. The distance between the BS and the RIS is 10 m. Users are randomly distributed with a distance of 6 m from the RIS.

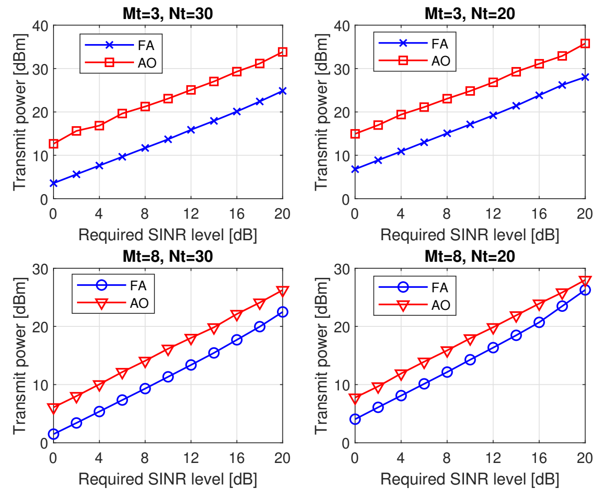

Fig. 1: The total BS's transmit power versus the required SINR level with different numbers of BS's antennas and RIS's reflective elements. The firefly population is $N = 120$. The number of maximum generations $T = 50$.

The pathloss exponents of both wireless links from the BS to the RIS and from the RIS to users are set to be 2.2 with the signal attenuation at the reference distance of 1 m being 30 dB [16], i.e., the large-scale fading coefficient is modeled as $-30 - 22\log_{10}(d)$ dB where d is the distance between the BS to RIS or RIS to a user. The noise variance at each user is -124 dBm. Monte Carlo simulations are carried over 100 channel realizations. Each channel realization is associated with a random user location and a random fading coefficient.

CVX package [17], i.e., a Matlab based modeling system for disciplined convex programs, is utilized to obtain the solution for the AO approach with $n_0 = 10$ iterations. The setup parameters for FA are as follows. The variation of the attractiveness γ is set at 1. The penalty constants are set equal but they dynamically vary as $\lambda_i = \rho_k = n^2$, $\forall i, k$ where n is the generation index in Algorithm 1. The attractiveness at zero distance is $\beta_0 = 1$. Finally, the initial randomization factor is $\alpha^{(0)} = 0.9$ and its value at the n-th generation is $\alpha^{(n)} = \alpha^{(0)}0.9^n$.

Fig. 1 illustrates the total BS's transmit power versus the required SINR level with different numbers of BS's antennas and RIS's reflective elements. The results indicate that the proposed FA prevails the AO approach in terms of lower power consumption. The superior performance of the FA approach over its AO counterpart can be explained as follows. As the AO approach approximates non-convex problem (3) by two convex sub-problems (5) and (6), the solution obtained by the AO approach is not necessary the global optimal solution of the original problem (3). On the other hand, the proposed FA possessing both exploitation and exploration abilities can effectively handle such non-convex problem and obtain much better solution than its counterpart.

We now compare the computational complexities of the AO and FA approaches for the experiments presented on Fig. 1. As N_t is larger than M_t, from Lemma 1 one can show that

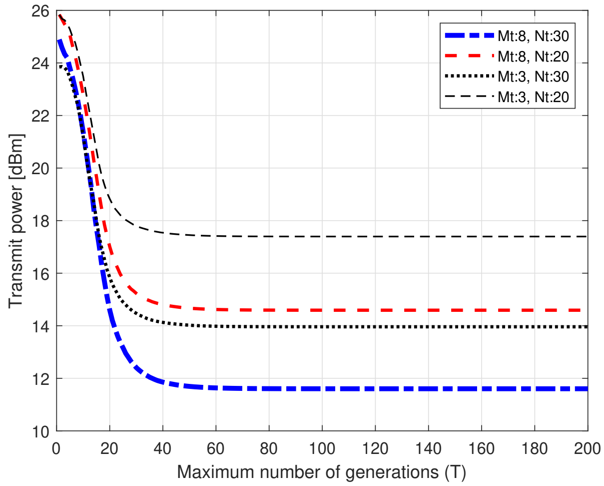

Fig. 2: The total BS's transmit power versus the number of maximum generations with different numbers of BS's antennas and RIS's reflective elements. The firefly population is $N = 120$. The required SINR level is 10 dB.

the dominant term of the complexity of the AO approach is $n_0 N_t^{6\frac{1}{2}}$. Similarly, from Lemma 2 one can conclude that the dominant term of the complexity of the FA approach is $TN^3N_t^2$. Substituting for $N_t = 30$, $n_0 = 10$, $N = 120$ and $T = 50$, we can arrive at the fact that the computational complexities of the AO and FA approaches are on the same order of $\mathcal{O}\left(10^{10}\right)$.

In Fig. 2, the total BS's transmit power is plotted versus the maximum of generation T used in the FA in Algorithm 1 with different BS's antennas and RIS's elements. The results indicate that the proposed FA require around 50 to 60 generations to attain the optimal solution for all setups.

VI. Conclusion

We have proposed a novel FA to solve a non-convex optimization problem comprising constraints as multivariate functions of independent optimization variables. The proposed FA approach outperforms the alternative optimization approach in offering better solution with similar computational complexity. This verifies the effectiveness of the proposed FA approach in handling multivariate and non-convex problems.

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