Analysis of Synaptic Weight Distribution in an Izhikevich Network

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Abstract. Izhikevich network is a relatively new neuronal network, which consists of cortical spiking model neurons with axonal conduction delays and spike-timing-dependent plasticity (STDP) with hard bound adaptation. In this work, we use uniform and Gaussian distributions respectively to initialize the weights of all excitatory neurons. After the network undergoes a few minutes of STDP adaptation, we can see that the weights of all synapses in the network, for both initial weight distributions, form a bimodal distribution, and numerically the established distribution presents dynamic stability.

Key words. Izhikevich network, STDP, uniform distribution, Gaussian distribution, bimodal distribution, dynamic stability

1 Introduction

Since Izhikevich's neuronal network [1] was presented some 6 years ago, it has been used efficiently in simulating the activities of human brain [2]. Specifically, the network exhibits cortical-like dynamics, including delta and gamma oscillations which correspond to deep sleep and extreme anxiety states, respectively. One thousand cortical spiking neurons with axonal delays [3] and spike-timing dependent plasticity (STDP) can make up a minimal phenomenological spiking network. These constituent neurons can be described by the simple Izhikevich neuronal model [4, 5] for the spiking behaviors like regular spiking (RS) and fast spiking (FS). Unlike the classical Hodgkin-Huxley spiking neuron model and the leaky integrate-and-fire neuron model, Izhikevich spiking neuronal model combines biological plausibility and computational efficiency, and can simulate some 20 different types of neuronal activities by simply modifying its 4 parameters.

In this work, we modify the original Izhikevich network [1] by using two typical distributions, i.e., uniform distribution and Gaussian distribution, respectively, to initialize the weights of all excitatory neurons in Izhikevich network. In the process of simulation, for both initial weight distributions, we find that the network approaches a similar bimodal distribution. Numerical experiments show the bimodal distribution is dynamically stable. Another important thing is to observe this interesting procedure in order to find the differences and similarities between these two cases.

The paper is organized as follows. The next section introduces the model of Izhikevich's spiking neuron and relevant knowledge of the uniform and Gaussian

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distributions. Section 3 includes the results of simulation with analysis. Section 4 presents the concluding remarks.

2 Model

2.1 Architecture

The Izhikevich network [1] is a minimal spiking neuronal model that consists of 1000 randomly connected spiking neurons with conduction delays and STDP [6], and is sparse with 0.1 probability of connection between any two neurons with the random connection strength normalized in the range of (0,10). Among all these neurons, there are 800 excitatory neurons showing regular spikes (RS) and 200 inhibitory neurons showing fast spikes (FS). Although there are more excitatory neurons in the network, their firing rate is relatively lower than the inhibitory neurons'. As a result, the network exhibits a state with balance of excitation and inhibition.

The Izhikevich spiking neuron [4] is represented by the following two-dimensional system of ordinary differential equations [7]:

$$v' = 0.04v^2 + 5v + 140 - u + I$$
$$u' = a(bv - u)$$

After the spike reaches its apex (+30mV), the membrane voltage (v) and the recovery variable (u) are reset according to the following equation:

$$\text{if} v \geq 30mv$$
$$v = c \quad u = u + d$$

Where a, b, c and d are dimensionless parameters. Different configurations of {a, b, c, d} reproduce different kinds of neuronal dynamics. In our work, if a neuron is an excitatory one, it has {a, b, c, d} = {0.02, 0.2, -65, 8}; if a neuron is an inhibitory one, it has {a, b, c, d} = {0.1, 0.2, -65, 2}. The former one presents RS neurons and the latter one presents FS neurons.RS and FS neurons are the major class of excitatory and inhibitory neurons, respectively. The main phenomenological difference between RS and FS is that RS fires in low frequency and FS fires in high frequency.

At every time step of simulation, the network has three tasks: firstly, to assign a random thalamic input to a random neuron; then to use the Izhikevich spiking neuron to detect the happening of spike in each neuron; finally, to carry out STDP learning.

STDP [8, 9, 10, 11] is a kind of unsupervised learning [12] reflecting the causal relation of spike generation by two neurons interconnected by a synapse. Long-term potentiation (LTP) of the synapse occurs if the pre-synaptic action potentials precede post-synaptic firing by no more than 50ms; long-term depression (LTD) occurs if the pre-synaptic action potentials follow post-synaptic spikes by the similar time window. In this work we use the hard bound STDP to adapt the excitatory synapses in Izhikevich network (for a review of hard and soft bound STDP see [13]).

2.2 Uniform and Gaussian distribution

In Izhikevich neuronal network, weights of excitatory neurons range from 0 to 10 and weights of inhibitory neuron are -5 all the time. So we lay emphasis on the excitatory neurons and allocate the synapses with the weight interval (0-10) into 20 bins.

Before simulating the network, we have to do some preliminary works. First of all, the synaptic weights of excitatory neurons are initialized. In this work, we choose two kinds of distributions as mentioned in section 1. Then, the weight interval (0-10) is divided into 20 bins, from (0-0.5) to (9.5-10).

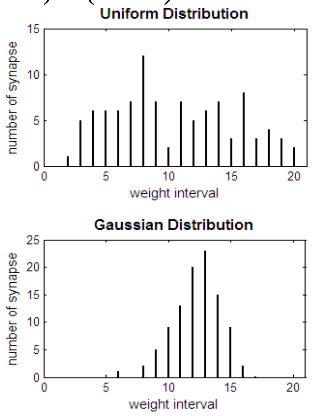

Fig. 1: Initial weight distribution. The top one is uniform weight distribution and the bottom one is Gaussian weight distribution. The X-axis shows the bin index and the Y-axis shows the number of synapses in a certain weight bin.

In the top of Fig. 1, we can see that the excitatory synaptic weights are distributed randomly in different bins with a uniform distribution and that is relevant to the modification of Izhikevich Network by using a uniform function. In the bottom of Fig. 1, the excitatory synaptic weights form a classic bell curve in a Gaussian distribution, which is expressed as $N = (\mu, \sigma^2)$ with $\mu = 6.5$ and $\sigma = 1$. In this configuration we can see the 13th bin has the maximum value, and other values decrease in the neighboring bins.

Despite of the initial state of excitatory neurons' weights in either uniform or Gaussian distribution, the synaptic weights will evolve with the number of synapses in both end bins of the whole weight interval increasing gradually, and decreasing in the middle bins. This dynamics is mediated by STDP learning. The final state of the weight distribution is represented by a dynamic bimodal distribution as we can see in the next section.

3 Simulation Results

We run the Izhikevich neuronal network which is configured with the aforementioned distributions and random delays normalized in a range between 0 and 20 milliseconds, for several minutes with STDP.

Here we take the uniform weight distribution as an example (as shown in Fig. 2, Left). The synaptic weight distributions gradually present a bimodal mode when the simulation carries on. After some time of simulation, the difference of bin value becomes small in the middle of weight bins, while the difference of bin value is large for the first and the last bins. If the first bin value is large, then the last bin value is small, and vice versa.

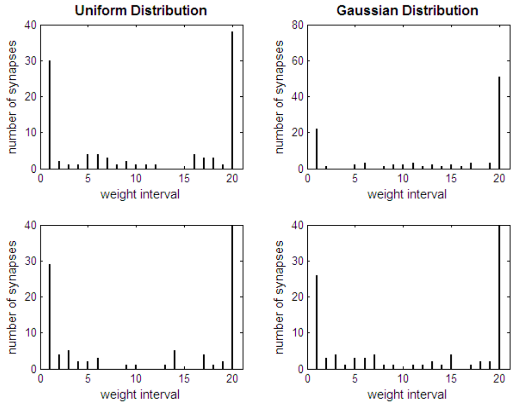

Fig. 2: Bimodal distribution of the excitatory synaptic weight distribution of a single excitatory neuron after STDP mediated simulation in an Izhikevich network. Left column (initially uniform weight distribution) has two plots showing the dynamic bimodal distributions of excitatory synaptic weights after 100 (upper) and 120 (lower) seconds of simulation time. Right column (initially Gaussian weight distribution) shows a dynamic bimodal distribution as well, after simulation time of 120 (upper) and 130 (lower) seconds, respectively. In actual, we can see the network can reach dynamic stability by using two time spans in both distributions. The X-axis shows the bin index and the Y-axis shows the number of synapses in a certain bin.

This phenomenon reflects dynamic stability of synaptic weight distribution after an initial STDP adaptation period when the distribution remains similar in shape. We observe that, in the first bin of weight, although some synapses which were allocated to that bin can leave due to STDP adaptation, some other synapses with the equivalent number of leaving ones will join that bin. And we can say that Izhikevich neuronal network reaches dynamic stability under this circumstance. The first, middle and last weight bins are significant, since the numbers of synapses obey bimodal distribution.

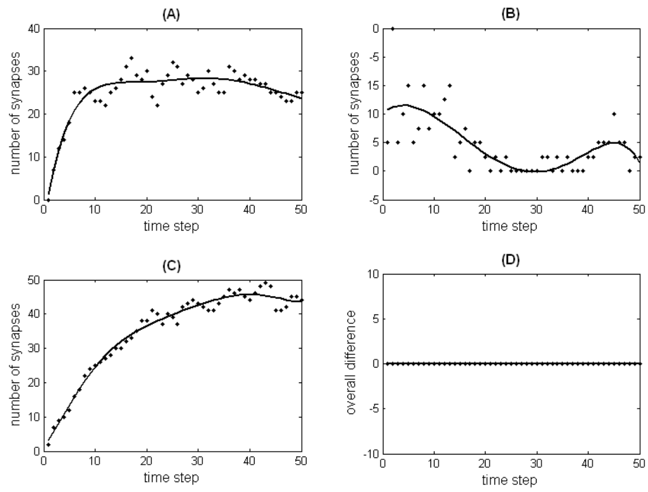

Fig. 3: Variation tendency of the number of synapses in three bins, i.e., the two ending bins and the middle one, and the overall synaptic difference of an excitatory neuron in simulation with an initial uniform weight distribution. (A)In the first weight bin, the number of synapses increases until the network reaches dynamic stability. (B) In the middle weight bin, the number of synapses decreases until the network reaches dynamic stability. (C) In the last weight bin, the number of synapses increases until the network reaches dynamic stability. (D) shows that, although each synaptic weight is changing in simulation, the sum of all synaptic weights of an excitatory neuron is a constant value. The X-axis shows time step (each time step presents 5s, that is to say, the largest simulating time is 250s according to the last time step) and the Y-axis shows the number of synapses in a certain time step.

No matter what the initial weight distribution is (uniform or Gaussian distribution), variation tendency of the number of synapses in three bins mentioned above and the overall synaptic difference is similar to some extent, as shown in Fig. 3 and Fig. 4. If the network reaches dynamic stability, variation of weight distribution remains but is much smaller than the initial stage. In Fig. 4(B), for instance, there is an obvious downtrend as the initial weight distribution is Gaussian distribution.

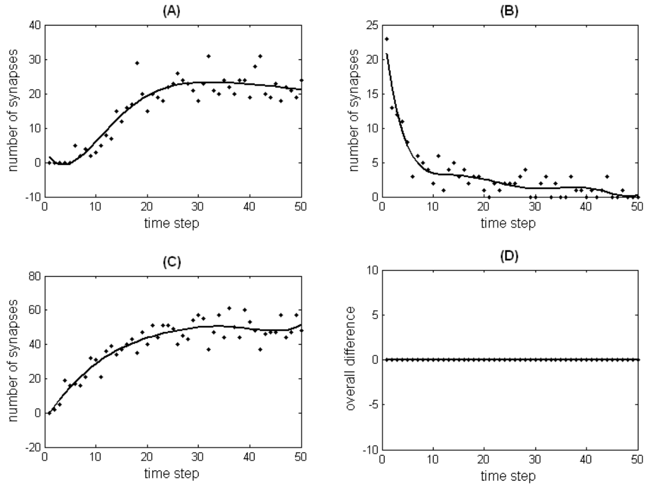

Fig. 4: Variation tendency of the number of synapses in three bins and the overall synaptic difference of an excitatory neuron in simulation with an initial Gaussian weight distribution (the plot descriptions are similar to Fig. 3). In (B), the middle weight bin has the maximum synapse number in the initial state, which decreases until the network reaches dynamic stability.

The choice of the length of the simulation time is critical to reflect the dynamic stability of the system adapted by STDP. A short simulation time cannot reveal the dynamic stability. In contrast, a long time can not only show the stability but also the details of the change of synapse number in corresponding bins.

4 Concluding remarks

In this paper, we use two kinds of distributions to initialize the state of excitatory neurons' weights, and then to train Izhikevich neuronal network to achieve dynamic stability and to observe the distribution of the number of synapses of an arbitrary neuron under STDP adaptation.

The Izhikevich network undergoes an unsupervised, hard bound STDP learning for several minutes, and reaches a state with some synapses strengthened and some others weakened. If the learning time is too short, the network will be unstable. If the learning time is long enough, the network will be stable dynamically and a typical bimodal distribution of synapses of an arbitrary excitatory neuron will emerge.

Through network simulation, data analysis and figures comparison, we can come to the conclusion that, when the network reaches dynamic stability after STDP adaptation for a period of time, the weight of synapses forms a bimodal distribution for each excitatory neuron and the established distribution numerically presents dynamic stability.

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