# Joint Design of Groupwise STBC and SIC based Receiver

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*Abstract*—In this letter, we propose a simple groupwise spacetime block code (GSTBC) which can be easily applied to a large number of transmit antennas and be effectively decoded by a low complexity successive interference cancellation (SIC) based receiver. The proposed GSTBC and SIC based receiver are jointly designed and the diversity repetition in GSTBC is used to induce the dimension expansion that can be exploited by the SIC based receiver to suppress interfering signals as well as to obtain diversity gain. Our proposed scheme provides a near maximum likelihood (ML) performance while keeping a reasonably low complexity at the receiver.

*Index Terms*—MIMO system, layered array processing, successive interference cancellation (SIC), groupwise STBC.

## I. INTRODUCTION

Various space-time codes (STC) have been proposed to improve data rate and bit error rate (BER) performance of multiple input multiple output (MIMO) systems [1]-[4]. Unfortunately, the decoding complexity of STC at the receiver usually grows exponentially with the number of transmit antennas. Thus, the use of STC for a large MIMO system becomes impractical due to prohibitively high decoding complexity. Groupwise STC was then considered in [5]-[7] for a trade-off between performance and complexity, where transmit antennas are divided into multiple groups to transmit smaller-size STC symbol blocks. In general, a groupwise interference canceller is required at the receiver.

In this letter, we propose a groupwise space time block code (GSTBC) in conjunction with a successive interference cancellation (SIC) based receiver. The Alamouti code in [2] becomes a building block for GSTBC. This approach allows us to build an STC for a large number of transmit antennas. The repetition of Alamouti symbol blocks is used to facilitate the SIC that can reduce the complexity at the receiver. A simple GSTBC is designed in a layered manner to apply to the MIMO system of a large number of transmit antennas. The advantage of the proposed approach is two-fold: i) the receiver exploits the dimension expansion, which is available from the GSTBC design, to suppress interference across layers using a linear minimum mean square error (MMSE) filter and, at the same time, obtains the coding/diversity gains of the GSTBC; ii) the

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layered GSTBC can be decodable by a low complexity SIC based receiver.

*Notations*:  $(\cdot)^{*}$ ,  $(\cdot)^{T}$  and  $(\cdot)^{H}$ - complex conjugation, transpose and Hermitian transpose, respectively;  $Diag(x)$  - diagonal matrix whose diagonal is vector  $x$ ; Rank $(\cdot)$  - rank of matrix.

# II. JOINT GSTBC AND SIC BASED DETECTION

In this section, we propose a GSTBC using an example for MIMO systems with that of 4 transmit and 2 receive antennas. The diagonal repetition in the proposed GSTBC plays a key role in deriving a low complexity SIC based receiver that can effectively detect signals using the dimension expansion. We assume flat fading channels. In addition, the channel state information is assumed to be perfectly known at the receiver.

## *A. Layered GSTBC design*

Assume that the four transmit antennas are divided equally into two groups (indexed groups 1 and 2 hereafter). Each group conveys a conventional Alamouti symbol block over two transmit antennas and two time slots [2]. Our code design is illustrated in Fig. 1, where three Alamouti symbol blocks  $S_1$ ,  $S_2$  and  $S_3$  are transmitted over four transmit antennas and four time  $slots<sup>1</sup>$ . Note that

$$
\mathbf{S}_u = \begin{bmatrix} s_{u,1} & s_{u,2} \\ -s_{u,2}^* & s_{u,1}^* \end{bmatrix}, \ u = 1,2,3,
$$
 (1)

where  $s_{u,1}$  and  $s_{u,2} \in S$  are equally likely data symbols. Here,  $S$  denotes the symbol alphabet. For convenience, we assume that  $E[|s_{u,i}|^2] = 1$ . The code constitutes two layers in which Layer 1 contains  $S_1$  and its repetition while  $S_2$  and S<sub>3</sub> form Layer 2. Stacking four consecutive received signals, the received signal vector  $\mathbf{v}_i = [v_{i,1}, v_{i,2}, v_{i,3}, v_{i,4}]^T$  at the *i*th receive antenna  $(i = 1, 2)$  becomes

$$
\mathbf{v}_{i} = \begin{bmatrix} \mathbf{S}_{1} & \mathbf{S}_{2} \\ \mathbf{S}_{3} & \mathbf{S}_{1} \end{bmatrix} \begin{bmatrix} \mathbf{h}_{i,1} \\ \mathbf{h}_{i,2} \end{bmatrix} + \mathbf{n}_{i}, \tag{2}
$$

<sup>1</sup>The proposed GSTBC can be extended to different numbers of antennas. The general case with K transmit antennas will include  $K/2$  layers and  $K-1$ symbol blocks, where  $K$  is an even number. Each diagonal in the codeword matrix will contain one symbol block and its repetition. Layer 1 is the main diagonal while Layer  $k$  ( $2 \le k \le K/2$ ) is formed by the two kth symmetric diagonals.

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where  $\mathbf{h}_{i,j} = [h_{i,j,1}, h_{i,j,2}]^T$  is the channel vector over two consecutive time slots from the transmit antennas of the  $j$ th group ( $j = 1, 2$ ) to the *i*th receive antenna, and  $n_i$  is the white zero-mean Gaussian noise vector with  $E[\mathbf{n}_i \mathbf{n}_i^H] = \sigma_n^2 \mathbf{I}$ . Letting

$$
\mathbf{r}_{1} = [v_{1,1}, -v_{1,2}^{*}, v_{2,1}, -v_{2,2}^{*}]^{T},
$$
\n
$$
\mathbf{r}_{2} = [v_{1,3}, -v_{1,4}^{*}, v_{2,3}, -v_{2,4}^{*}]^{T},
$$
\n
$$
\mathbf{H}_{i,j} = \begin{bmatrix} h_{i,j,1} & h_{i,j,2} \\ -h_{i,j,2}^{*} & h_{i,j,1}^{*} \end{bmatrix},
$$
\n
$$
\mathbf{H}_{j} = [\mathbf{H}_{1,j}^{T}, \mathbf{H}_{2,j}^{T}]^{T},
$$
\n
$$
\mathbf{G}_{1} = [\mathbf{H}_{1}^{T}, \mathbf{H}_{2}^{T}]^{T},
$$
\n
$$
\mathbf{G}_{2} = \begin{bmatrix} \mathbf{H}_{2} & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_{1} \end{bmatrix},
$$
\n(3)

after some re-arrangements the received signal vector  $r =$  $[\mathbf{r}_1^T, \mathbf{r}_2^T]^T$  can be written as

$$
\mathbf{r} = \mathbf{G}_1 \mathbf{x}_1 + \mathbf{G}_2 \mathbf{x}_2 + \mathbf{w},\tag{4}
$$

where  $\mathbf{x}_1 = \mathbf{s}_1$  and  $\mathbf{x}_2 = [\mathbf{s}_2^T, \mathbf{s}_3^T]^T$  are the transmitted signal vectors in Layers 1 and 2, respectively. Here  $\mathbf{s}_u = [s_{u,1}, s_{u,2}]^T$ for  $u = 1, 2, 3$ . In (4), the noise vector w has the same statistical characteristics as those of the vector  $[\mathbf{n}_1^T, \mathbf{n}_2^T]^T$ .

From (4), we observe that the signal from one layer is seen as the interference to the other layer. To detect the signal in each layer, the interference signal from the other layer must be cancelled or mitigated. In our GSTBC design, the repetition of  $S_1$  in the diagonal of the codeword matrix is used as shown in Fig. 1, which is called the diagonal repetition, to induce the dimension expansion through  $G_1$  as shown in (4). This dimension expansion can be exploited by the SIC based receiver to suppress interfering signals using a linear filtering as well as to obtain diversity gain in Layer 1 as two  $S_1$ 's are transmitted by different channels. Using the diagonal repetition, we can reduce the error in the detection of Layer 1 and, as a result, the error propagation is mitigated in Layer 2. The proposed GSTBC is inspired by [8] where a layered transmission can achieve the channel capacity with an SIC based receiver.

## *B. Layered SIC based detection*

For decoding, we derive an SIC based receiver. The decoding operation consists of two steps. In the first step for the detection of Layer 1, the MMSE filtering is used to estimate  $S_1$ . Then, in the second step for the detection of Layer 2,  $S_2$  and  $S_3$  are detected after (soft) cancelling  $S_1$ . The SIC process can be extended to the second iteration where the soft estimates of  $S_2$  and  $S_3$  obtained from the first iteration are utilized for the interference cancellation in Layer 1. For convenience, we describe the SIC-based detection for the first iteration only.

*1) Soft estimation of signals in Layer 1:* As mentioned earlier, the repetition of  $S_1$  induces the dimension expansion. From this, the MMSE detection can be applied to the detection of  $S_1$  with suppressing the interfering signals,  $S_2$  and  $S_3$ . The output of the MMSE filter, denoted by  $\tilde{\mathbf{x}}_1 = [\tilde{s}_{1,1}, \tilde{s}_{1,2}]^T$ , is

$$
\tilde{\mathbf{x}}_1 = \mathbf{G}_1^H \mathbf{R}^{-1} \mathbf{r},\tag{5}
$$

where  $\mathbf{R} = \mathbf{G}_1 \mathbf{G}_1^H + \mathbf{G}_2 \mathbf{G}_2^H + \sigma_n^2 \mathbf{I}$ .

Due to the diagonal repetition, the MMSE detection exploits the diversity gain and suppress interfering signals through the dimension expansion. As the detection of  $S_1$  can be reliable, there would be much less error propagation error in the detection of Layer 2.

Lemma 1: The MMSE detection exploiting dimension expansion does not have an error floor in the asymptotic case where the noise variance approaches zero if the number of receive antennas is not less than two.

*Proof*: For any N receive antennas,  $H_j$  in (3) becomes  $\mathbf{H}_{j} = [\mathbf{H}_{1,j}^{T}, \mathbf{H}_{2,j}^{T}, \cdots, \mathbf{H}_{N,j}^{T}]^{T}, j = 1, 2, \text{ and } \mathbf{H}_{j} \text{ of size}$  $2N \times 2$  is a full column rank matrix. Note that for any N,

$$
Rank(\mathbf{H}_j\mathbf{H}_j^H) = Rank(\mathbf{H}_j) = Rank(\mathbf{H}_{i,j}) = 2.
$$

Since the size of  $H_j H_j^H$  is  $2N \times 2N$ , it is full-rank only if  $N = 1$ . In the other cases (i.e.,  $N \ge 2$ ),  $\mathbf{H}_j \mathbf{H}_j^H$  is not full-rank. When  $\sigma_n^2 \to 0$ , the covariance matrix of the noiseplus-interference vector (i.e.,  $\mathbf{G}_2\mathbf{x}_2+\mathbf{w}$ ) in (4) (assuming that  $E[\mathbf{x}_2] = \mathbf{0}$ ) becomes

$$
\mathbf{C} = \mathbf{G}_2 \mathbf{G}_2^H = \begin{bmatrix} \mathbf{H}_2 \mathbf{H}_2^H & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_1 \mathbf{H}_1^H \end{bmatrix} .
$$
 (6)

Since the linear MMSE filter can perfectly suppress the interference if C is not full rank, we can see that there is no error floor as  $\sigma_n^2$  approaches zero if  $\mathbf{H}_j \mathbf{H}_j^H$  is not full rank from (6). As a result, the linear MMSE filter does not have the error floor if  $N \geq 2$ . This completes the proof.

It is well-known that the distribution of the MMSE filter output can be well approximated by a Gaussian distribution [9]. We therefore make the following approximation:

$$
\tilde{s}_{1,j} = \mu_j s_{1,j} + \eta_j,\tag{7}
$$

where  $j = 1, 2, \mu_j$  is the equivalent amplitude of the jth symbol and  $\eta_j$  is a zero-mean complex Gaussian noise variable with  $E[\eta_j \eta_j^*] = \nu_j^2$ . We can show that

$$
\mu_j = \mathbf{g}_{1,j}^H \mathbf{R}^{-1} \mathbf{g}_{1,j}, \n\nu_j^2 = \mu_j - |\mu_j|^2,
$$

where  $g_{1,j}$  is the j<sup>th</sup> column of  $G_1$ . Under the Gaussian assumption, the log-likelihood ratio (LLR) can be found as a soft-decision.

*2) SIC based detection of signals in Layer 2:* Once a soft decision of  $s_1$ , denoted by  $\bar{s}_1$ , is obtained, the SIC based detection is applied to  $s_2$  and  $s_3$  separately in Layer 2. The received signal vectors after cancelling the interference become

$$
\bar{\mathbf{r}}_1 = \mathbf{r}_1 - \mathbf{H}_1 \bar{\mathbf{s}}_1 = \mathbf{H}_2 \mathbf{s}_2 + \bar{\mathbf{w}}_1, \tag{8}
$$

$$
\overline{\mathbf{r}}_2 = \mathbf{r}_2 - \mathbf{H}_2 \overline{\mathbf{s}}_1 = \mathbf{H}_1 \mathbf{s}_3 + \overline{\mathbf{w}}_2, \tag{9}
$$

where  $\bar{w}_1 = H_1(s_1 - \bar{s}_1) + w_1$ ,  $\bar{w}_2 = H_2(s_1 - \bar{s}_1) + w_2$ and  $[\mathbf{w}_1^T, \mathbf{w}_2^T]^T = \mathbf{w}$ . Here  $\bar{\mathbf{w}}_i(i = 1, 2)$  is assumed to be a Gaussian vector with  $E[\bar{w}_i] = 0$  and  $E[\bar{w}_i \bar{w}_i^H] =$  $\mathbf{H}_i \mathbf{Q} \mathbf{H}_i^H + \sigma_n^2 \mathbf{I}$ , where  $\mathbf{Q} = E[(\mathbf{s}_1 - \bar{\mathbf{s}}_1)(\mathbf{s}_1 - \bar{\mathbf{s}}_1)^H] =$ Diag( $[1 - |\bar{s}_{1,1}|^2, 1 - |\bar{s}_{1,2}|^2]$ ). Then, we apply the MMSE filtering to estimate  $s_2$  and  $s_3$ .

As the MMSE filtering is used for the signal estimation in each layer, the overall complexity of the SIC based detection is mainly dominated by the matrix inversion of  $\bf{R}$ . Assume that two iterations are required, the complexity of the SIC detection would be of order  $\mathcal{O}(160N^3)$ , while that of the conventional MMSE detector is of  $\mathcal{O}(64N^3)$ . Note that the number of receive antennas, N, is usually 2 for the proposed GSTBC. If the maximum likelihood (ML) detection is employed with the same GSTBC structure, the complexity would be of order  $\mathcal{O}(24NM^6)$ , where M is the size of the symbol alphabet. Obviously, in terms of the complexity, the SIC based receiver is not too expensive compared to the MMSE detector and is much more efficient than the ML approach.

### III. SIMULATION RESULTS

Suppose that there are 4 transmit and 2 receive antennas. Each path gain from a transmit antenna to a receive antenna is modeled as a circularly symmetric complex zero-mean Gaussian variable with variance one (i.e., Rayleigh fading is assumed). The Alamouti [2] and quasi orthogonal STBC (QOSTBC) [10] schemes are considered for comparison purpose. The MMSE detection is employed in the QOSTBC with the equivalent complexity. Note that for the same transmission rate, we use 8-phase shift keying (PSK) for the Alamouti and QOSTBC schemes, while quadrature phase shift keying (QPSK) is used for the proposed GSTBC.

Fig. 2 shows BER simulation results. The proposed GSTBC scheme with SIC (denoted by GSTBC-SIC which converges after two iterations) outperforms the Alamouti and QOSTBC schemes. The performance improvement results from the MMSE detection which utilizes the dimension expansion in Layer 1. As we then apply the SIC to Layer 2, the reliable detection in Layer 1 or reliable cancellation in Layer 2 additionally results in a better overall BER performance. Note that the performance gap between the QOSTBC and GSTBC-SIC schemes decreases with  $E_b/N_0$  (the bit energy to noise spectral density) as the QOSTBC has a higher diversity gain. With the same GSTBC structure, the SIC based detection (i.e., GSTBC-SIC) provides a near ML performance and outperforms the conventional MMSE detector. As mentioned previously, the proposed GSTBC can be extended to a larger MIMO system. Fig. 3 shows the BER performance of the GSTBC with 8 antennas and 2 receive antennas (i.e., with 4 layers and 7 symbol blocks). The similar result is observed when comparing to the ML and MMSE detectors.

## IV. CONCLUSION

This letter proposed a simple but effective GSTBC in conjunction with an SIC based receiver. By designing the GSTBC in a layered manner with diversity repetition, the MMSE detector exploiting the dimension expansion can be used to suppress the interfering signals from the other layers effectively. It was shown that the proposed scheme has about 1.5 dB gain at a BER of  $10^{-3}$  over the QOSTBC scheme and stands a reasonable complexity thanks to the SIC based detection processes across the layers.



0 2 4 6  $\frac{8}{E_b/N_0}$  (dB) 12 14 16

Fig. 2. BER performance comparison with different schemes.

10−5

10−4

10−3

BER

10−2

10−1

 $10<sup>0</sup>$ 



Fig. 3. BER performance of the proposed GSTBC (different detectors) with 8 transmit and 2 receive antennas.

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