When to Stop - A Cardinal Secretary Search Experiment

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Abstract

The cardinal secretary search problem confronts the decision maker with more or less candidates who have identically and independently distributed values and appear successively in a random order without recall of earlier candidates. Its benchmark solution implies monotonically decreasing sequences of optimal value aspirations (acceptance thresholds) for any number of remaining candidates. We compare experimentally observed aspirations with optimal ones for different numbers of (remaining) candidates and methods of experimental choice elicitation: "hot" collects play data, "warm" asks for an acceptance threshold before confronting the next candidate, and "cold" for a complete profile of trial-specific acceptance thresholds. The initially available number of candidates varies across elicitation methods to obtain more balanced data. We find that actual search differs from benchmark behavior, in average search length and success, but also in some puzzling qualitative aspects.

Keywords: **Behavioral OR, Optimal Stopping, Secretary Problem, Sequential Search Mechanism**

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1 Introduction

In commercial and private life, one is often individually responsible to search for a qualified candidate to fill a certain need. Since one usually interviews candidates successively, due to time and monetary constraints, one may not be able to assess the quality - in the following we refer to individual qualities as values - of all potential candidates. On the other hand the impossibility to keep candidates waiting for long questions recall. Without recall, past candidates are lost. Waiting to check the last candidate is thus very risky. Consider the example of trying to find an apartment, an employee, or a life partner; one must often act immediately or otherwise risk to lose an attractive option. This highlights the relevance of asking and answering "When to stop searching?".

The modified version of the secretary search paradigm, based on successively appearing and a priori identical candidates and no recall, has an elegant benchmark solution. Its optimal stopping rule does not maximize the probability to hire the best possible candidate[1] but maximizes expected quality which requires cardinal values of candidates. The decision maker confronts potential candidates sequentially, each with quantifiable quality (an unambiguously recognizable monetary value). Candidates become unavailable when not accepted immediately. While stylized, this captures rather realistically the example of searching for a house in a popular area with many competitors, provided that the values of the various properties can be assessed unambiguously and quickly.

One important stylized aspect of this search type is that having already met more or less candidates is completely uninformative about the random values of later ones. This avoids, at least theoretically, engaging in Bayesian updating and allows to focus instead on how the number of remaining candidates affects search behavior. Behaviorally, qualities of past candidates may nevertheless matter, e.g. due to the Gambler's fallacy, anxiety or regret when running out of candidates,

[1] The original secretary search paradigms focus on finding the single-best candidate out of $n(\geq 2)$ possible ones. The potential candidates are interviewed one by one in random order and the decision maker has to decide whether to keep each candidate immediately after encountering it. Before making the decision, the decision maker is only aware of the candidate's rank among all candidates encountered so far, but does not know values of yet to be seen ones. Once rejected, a candidate cannot be recalled.

especially if more recent candidates turn out worse than prior ones.

In field situations, previous experiences with search episodes could account for heterogeneity in aspiration formation and adaptation. A controlled stylized lab experiment helps to limit theoretically confounding effects of past experiences.[2] Participants confront the various search tasks more than once which vary in the known initial number of a priori identical candidates. Although, theoretically, the number of remaining candidates is the only state variable, behaviorally such pure dependency of behavior on the state variable seemed questionable. Instead, we expected the quality of past rejected candidates and the difference between the initial and remaining number of candidates to matter and that larger differences will let one think that it is time to stop.

Optimality in dynamic decision tasks with finite horizon relies on backward induction or dynamic programming (see Bellman, 2013). Since the remaining number of candidates is the only state variable, the optimal strategy is a complete profile of first more and later less ambitious acceptance thresholds, derived in appendix A.[3] Although one could have experimentally induced risk neutrality via binary lottery incentives (the value of the accepted candidate determines linearly the probability of earning the larger rather than the smaller monetary amount),[4] we let participants successively confront several search tasks, each with possibly many chance moves, to reduce the variance of earnings across all tasks.

In view of the stochastic complexity of the search tasks, optimal behavior of participants would be *explanandum* rather than *explanans* - but we do not confirm optimality. So optimal aspiration profiles are just benchmarks for analyzing actual search behavior of, at best, boundedly rational participants. The choice data, in part, directly reveal the success aspirations of participants and how they are adapted. This sheds light on the core concepts of bounded rationality theory like aspiration formation and adaptation (see originally Lewin, 1926; Hoppe, 1930; Lewin and Denbo,

[2]Cox and Oaxaca (2000) confront constant priors about options with known priors for discrete values but also differ from our setup by assuming that after stopping successive dividends are collected till the end with 0-termination value. An even more systematic variation of conditions is provided by Schotter and Braunstein (1981) who base their hypotheses on various (theoretical) studies.

[3]We gratefully acknowledge the support of Alessandro Arlotto and Marco Scarsini who supplied the recursive formula and its algorithm which they, furthermore, adjusted to the discrete distribution, used in our experiment (See Moser, 1956; Sakaguchi, 1961; Karlin, 1962).

[4]Thus, expected utility requires only that the larger monetary premium is better than the smaller one and that probabilities are calculated properly.

1931; Lewin et al., 1944; Heckhausen, 1955; Simon, 1955, and specifically for aspiration adaptation Sauermann and Selten, 1962). If at all, individual differences in search behavior can be attributed to idiosyncratic characteristics of participants,[5] like regret inclinations, analytic capability, etc.[6]

In the experiment participants accept or reject the successively revealed value v_t in trial t directly after seeing it without recall (when rejecting v_t in trial t this realization is lost and cannot be retrieved). If the last value is reached, it is automatically accepted. Since all values v_t for $t = 1, \ldots, n$ (with $n(\geq 2)$ denoting the initial number of candidates) are randomly and independently generated according to the uniform density distribution, concentrated on the interval [0,1], rejected values do not inform about future ones. So the number of remaining candidates $n - t$ is all what theoretically matters[7] when deciding whether to accept or reject v_t in trial t.

The experiment relies on many integer realizations. We have shifted up and enlarged the interval from 0 to 1 by allowing for all integer values v from 24 to 123 which are all equally probable. The number n of candidates is either 5, 10, or 15. Figure 1 illustrates the optimal profile of aspirations or acceptance thresholds which decrease with the remaining number of trials, i.e. the number of remaining candidates (since the last candidate must be accepted).

We vary the initial number n (> 1) of candidates within subjects (participants confront all three n-tasks) and between subjects only whether n increases or decreases. The other between subjects variation of choice elicitation is more psychologically grounded. In "cold", participants are asked for complete strategies, i.e. a complete pattern of acceptance thresholds, before the first trial.[8] For boundedly rational participants this presupposes an awareness that nothing can be inferred from the values of past rejected candidates and that more remaining candidates are better than less. Data of this condition allows to assess how actual and optimal aspirations differ. For example, whether actual aspirations are less often adapted than optimal ones when many candidates still remain to be seen.

[5] This corresponds to the so-called full information condition of Palley and Kremer (2014), based on the optimality analysis of Karlin and Carr (1962).

[6] In later research one should also employ personality questionnaires whose effects we expect to be minor compared to the striking effects of choice elicitation methods.

[7] This would be different when regret, possibly measured by the positive difference between the so far highest rejected value and the present one, would matter.

[8] Like for optimal strategies participants can condition only on the number of remaining candidates.

The choice data in "hot" provide only the sequences of so far rejected values and the finally accepted one. When confronting early on a rather high value in "hot", one may feel compelled to accept it in ways resembling the well known endowment effects. This likely differs from just imagining such a high early value in "cold". In the hot "marshmallow delay" task[9] children would likely predominantly wait when choosing "coldly" but would often fail to wait in "hot". In our view, this suggests, on average, earlier stopping in "hot".

The intermediate "warm" condition asks for trial-specific acceptance thresholds as in "cold" but only before encountering that trial, respectively its candidate. Thus one successively states acceptance thresholds in "warm" being aware of the rejected values so far. This allows for regret in "warm", possibly measured by how many candidates have been lost and how far the present value is below the best former rejected one. Both aspects one can only anticipate when deciding in "cold". We expected sharper declines of acceptance thresholds in "warm" due to an acute awareness of lost options. Behaviorally, post-decisional regret and how many opportunities have been lost could affect the next stated acceptance threshold in "warm", similar to how it may affect it in "hot".

In view of the crucial stochastic uncertainty of the iid-cardinal secretary search tasks we have abstained from adding another random event via experimentally inducing risk neutrality. Instead we promote risk neutrality via "cumulative pay", i.e. participants are paid for all successive tasks to reduce their variance of earnings when viewing the entire experiment holistically. Using (risk neutral) optimality (RN-optimality from now on) as the benchmark, we partly focus on deviations from RN-optimality and how elicitation method, the $n-$sequence, the number of remaining candidates and past experiences shape them.

Participants substantially deviate from RN-optimal search, especially when there are still many candidates to be seen. These deviations overwhelmingly let them stop too early although the opposite can also be observed. Not only the elicitation method matters but also whether the number n of initially available candidates increases or decreases. It seems that participants perceive the six successive rounds rather holistically, i.e. as a single comprehensive task what seems to

[9] When delaying consumption of a given marshmallow by 30 minutes a child can eat two marshmallows, what is claimed to be positively correlated with professional success in adulthood (see Mischel et al. 2010 for a thorough review).

justify that cumulative pay for several successive tasks reduces risk sensitivity.

In view of experimental methodology, cardinal secretary search tasks are interesting as they directly reveal success aspirations and their dependence on the remaining number of candidates but also the considerable heterogeneity in human psychology and cognition. Furthermore, they are suitable paradigms to shed new light on the debate among experimentalists whether to use the "cold" strategy method or to elicit "hot" play data which so far has largely neglected individual sequential choice making. One wonders how the debate so far could concentrate (Sonnemans, 2000, is an exception) on comparing elicitation methods for social and strategic interaction experiments without a profound decision theoretic foundation. For the latter we convincingly confirm that elicitation method and task sequencing matter crucially.

Regarding field relevance, one restriction is the known number of (remaining) attempts. In the field this may arise due to idiosyncratic characteristics of decision makers, for example, due to them being seriously time constrained. In the animal kingdom, an already starving predator has fewer attempts to hunt, much like somebody urgently searching for an apartment. Financial markets with stationary random-walk assets whose traders have to invest immediately could also be similar to our setup. Unlike Güth and Weiland (2011) we have neglected competition in search.

Section 2 informs about the related literature. Section 3 describes the experimental protocols. The data and main findings are described and statistically validated in section 4 before the final discussion in section 5.

2 On the literature

Our setup is rather specific in multiple ways: rather than trying to hire the best candidate, as in the classic secretary search task, we rely on the familiar expected profit motive in neo-classical economics; rather than inferring aspirations from sequential search data, we directly observe in "cold" and "warm" the stated aspiration levels which we can compare with the optimal ones. We will diagnose "anti-monotonicity" as one crucial aspect which clearly signals a much richer motivation of search behavior and can reject optimality simply by exploring how its only state variable fails to account alone for the aspiration profiles chosen by participants. Additionally, unlike the general literature on stopping theory, our main focus is whether and how different elicitation methods and sequencing of different n-tasks shape search behavior.

Regarding reviews of the broader stopping literature, e.g. Freeman (1983), Ferguson (1989), Samuels (1991), Szajowski (2008), and Wu and He (2016) who provide informative overviews, let us be rather brief. A standard text on search in economics is Phelps (1968). An influential paradigm, which we also employ, is the so-called secretary-search task (for specific surveys see Todd and Miller, 1999; and Stein et al., 2003) which one often relates to specific applications like search for an employee, durable consumption goods like housing, right product design, minable land.

Even when paying the value of the accepted candidate, the decision maker, DM, often decides to stop or not based on additional information, e.g. whether or not the present candidate is the best one so far, see Bearden (2006) and experimentally Bearden et al. (2006). A field example could be a head hunter (institution) who can rank candidates relatively but not assess the firm-specific usefulness of the candidates for the hiring firm. Altogether the theoretical and experimental literature is still strongly influenced by the traditional incentive to hire the best, partly by paying only when actually hiring single best candidates so far (Ferenstein and Krasnosielska, 2009).

The empirical literature often explores satisficing by employing, in analogy to the revealed preference approach of empirical neoclassical economics, the revealed aspiration approach: one infers success aspirations from observed search data relying - in analogy to the as-if optimality in revealed preference theory - on as-if satisficing. Güth and Weiland (2011) let participants state (binding) aspirations first for sampling size and later for acceptance. Although aspirations can be freely chosen, this presupposes satisficing since the aspirations are binding.

Other studies include essential search costs (Kogut, 1990; Moriguti 1993; Seale and Rapoport, 1997 and 2000; Stein et al., 2003), e.g. for random numbers of candidates (Presman and Sonin, 1972; Ferenstein and Krasnosielska, 2009), or investigate reasons for (too little or too much) consumer search (Zwick et al., 2003). Bayesian secretary search tasks with known priors for monetary values as in our own study, partly named “full-information" tasks, are studied by Cox and Oaxaca (2000), and Schotter and Braunstein (1981), often via comparisons with other tasks which for instance keep participants less informed. Compared to this we confine ourselves to full-information" tasks where, theoretically, all what matters is the number of remaining candidates. So in the following we focus on directly exploring and testing satisficing in search and relating it to optimality, which requires “full information".

3 The choice tasks and (risk neutral) optimality

The setup features a situation where DM confronts a known number $n(\geq 2)$ of a priori identical candidates of whom DM has to hire one. What renders hiring difficult is that the quality or value for all candidates is randomly determined by the same independent and identical random move. Furthermore, candidates show-up sequentially and can be hired only when revealing their randomly selected value at their trial without recall (one cannot go back to former candidates in the sequence).

A strategy in such a search task is a complete profile of acceptance thresholds, one for each subgame where a subgame is defined by the sequence of so far rejected candidates. According to the risk neutral benchmark solution, RN-optimality, acceptance thresholds depend monotonically only on the remaining number of candidates and not on the value sequence of candidates seen so far. In game theoretic jargon this means to impose subgame consistency (subgames with the same number of remaining candidates are isomorphic and should have the same solution) which, of course, may be questionable behaviorally.

Figure 1 illustrates the pattern of optimal acceptance thresholds which with fewer remaining candidates first decline rather weakly but more steeply towards the end. The integer k counts the number of remaining candidates when rejecting the present one: so for $n = 15$ rejecting the first, second,..., candidate leaves 14, 13, ... remaining candidates. The reason for using k is that RN-optimal thresholds depend on k only, whereas the numbers $t = 1, ..., n-1$ of decision trials vary with n.

Participants in "cold" and "hot" are restricted to trial-specific aspirations or acceptance thresholds[10] for all possible trials: in a given trial one rejects only random values below this level. Specifically, in "cold" we elicit complete aspiration profiles; participants state, before the first trial, a complete profile of aspiration levels for all possible trials which can condition only on the number of remaining candidates. They are free to reduce aspirations when fewer candidates remain, as suggested by RN-optimality, but can also increase them.[11] Thus one has to distinguish two types of monotonicity, namely the monotonicity imposed via acceptance thresholds similar to acceptance thresholds of responder participants in ultimatum experiments, and the monotonicty of acceptance thresholds across trials which, theoretically, should decrease with fewer candidates (see Figure 1). So our setup guarantees the former monotonicity but allows participants to violate the latter one. In the following we can therefore restrict ourselves to the latter type of (anti-)monotonicity, i.e. refer to higher aspirations for fewer left candidates than for more as anti-monotonicity.[12]

"Warm" elicits acceptance thresholds in the same way but only sequentially, i.e. trial by trial, till acceptance. DM states the trial-specific acceptance threshold before learning the value realization of that trial. So one does not have to state aspirations for trials after acceptance, i.e. the first value which is not below the trial-specific aspiration. This obviously allows comparing "cold" and "warm" acceptance thresholds till acceptance in "warm".

"Hot" implements the actual search dynamics. Participants confront the first value and may stop by accepting it or continue searching till a candidate is finally accepted. So what one observes is a shorter or longer sequence of random values of which all preceding the acceptance trial are rejected. This illustrates the difficulties when wanting to infer aspirations from search data in the tradition of as-if satisficing.

One can compare RN-optimality with "hot" play data by analysing acceptance of non-RN-acceptable candidates and rejection of RN-acceptable ones. Due to the less informative choice data in "hot"

[10]In other choice-tasks one may accept a lower value but reject a higher one which can be rationalized (see Cox and Oaxaca, 2000) when one can infer an unknown prior from observed values.

[11]In view of the large multiplicity of sub-games applying an unrestricted strategy method in the search tasks at hand is practically impossible.

[12]Like acceptance thresholds of responders in ultimatum games value aspirations impose monotonic responses (one rejects below and accepts otherwise). Here anti-monotonicity instead relies on larger acceptance thresholds for fewer remaining candidates.

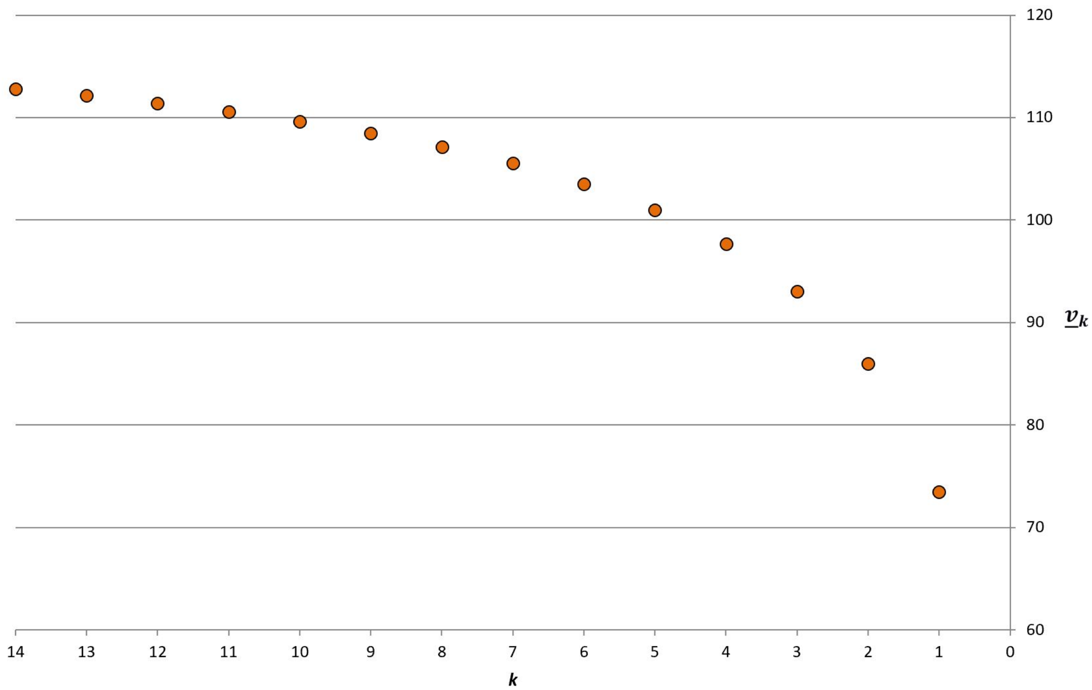

Figure 1: RN-optimal acceptance thresholds for the number, $k = 14 - k$, of remaining candidates when rejecting the present one, k. The possible values v of the candidates range from 24 to 123.

we will mainly consider outcome variables like length of search and payoff when comparing the data of "hot" with those of the other elicitation methods. To gather more of its less informative data "hot" always relies on the largest number of initially available candidates ($n = 15$).

How can elicitation method influence search behavior? In "hot" awareness of what the present value yields may trigger something close to the well known endowment effect which likely is enhanced by the certainty of the payoff one would earn in case of acceptance.[13] Knowing for sure what acceptance yields, might trigger an even earlier acceptance when the present value is the upper value range. Prospect theory could suggest the value of the so far best rejected value as the reference point, meaning to view later lower (higher) value realizations as losses (gains). This could trigger and explain path dependent behavior due to the robust confirmed higher evaluation of losses over gains. "Cold" and "warm" differ in how they trigger feelings of regret, actual ones in "warm" and only anticipated ones in "cold", as well as in urgency, like feelings of stress in "warm", especially when few trials remain. We expected similar patterns for early trials of "cold"

[13]When employing binary-lottery incentives, acceptance of a known value would still imply only an uncertain expectation and question the latter argument.

and "warm" but lower ones in "warm" in later trials due to an acute awareness of running out of candidates.

Our design does not only vary the elicitation method but also the sequence of n-tasks. Whereas a larger n allows for more trials and in all likelihood implies longer search length, the smaller $n-$tasks might be viewed as more stressful since, for instance, the probability of the first candidate being best is much higher for $n = 5$ than for $n = 15$. So experiencing $n = 15$ before $n = 10$ and then $n = 5$ may imply a more relaxed attitude when later on encountering shorter n-tasks compared to facing $n = 5$ before $n = 10, 15$. Since a larger n should yield a larger expected payoff (compare in Figure 1 the optimal aspiration for 14, 9, respectively, 4 remaining candidates[14]), participants perceiving the whole experiment holistically might let them feel deprived in case of a decreasing n and more eager to stop when confronting a satisfactory option early, i.e. they would stop on average earlier when encountering a given n after larger ones. "Cold-increasing" and "cold-decreasing" are therefore especially suited for analysing $n-$sequence effects. We expected some sequence effects, but were surprised by how large they are.

The three protocols ("cold", "warm" and "hot") require different instructions (see Appendix B for the "Cold-increasing" instructions). Participants obtained a show-up fee of 5 Euro, plus the sum of earnings of all the six rounds. The experiment was run at the CESARE lab in Rome, lasted on average 1 hour and 40 minutes. In total 194 participants participated; 48 in "cold-increasing", 47 in "cold-decreasing", 47 in "warm", and 52 in "hot". The four between-subjects conditions, listed in Table 1, inform about the sequence of tasks in the six successive rounds of each condition and the elicitation mode.

Since in "hot" one does not observe success aspirations but would have to infer them from (non) accepted values in the tradition of the revealed aspiration approach, we try to compensate for that by always employing $n = 15$ in "hot" (a larger n triggers higher aspirations initially and thereby longer expected search, see Figure 1).

[14]The optimal aspirations are the values when searching optimally, i.e. the (discrete) value function.

Table 1: Condition design

	"Cold-Increasing"	"Cold-Decreasing"	"Warm"	"Hot"
Round 1	n=5	n=15	n=5	n=15 -"hot"
Round 2	n=5	n=15	n=5	n=15 -"hot"
Round 3	n=10	n=10	n=10	n=15 -"hot"
Round 4	n=10	n=10	n=10	n=15 -"hot"
Round 5	n=15	n=5	n=15	n=15 -"warm"
Round 6	n=15	n=5	n=15	n=15 -"cold"

Notes: Columns feature between subjects conditions with partly varying n across rounds

In all conditions (except "hot") participants repeat the same task once to distinguish pure learning when confronting the same number n again from adapting to new numbers n of initially available options (see Table 1 for details). "Hot" is run for $n = 15$ only: in the first four rounds as required by "hot", followed by one round of "warm" and "cold" in rounds 5 and 6, respectively, again based on $n = 15$, to possibly compare with the fifth, respectively sixth round of "warm" and "cold-increasing", respectively.

In summary, our experimental attempt is to answer "when to stop searching?" by trying to avoid confounding effects of sequential search[15] like having

- first to sample candidates before being able to form value aspiration for candidates (Güth and Weiland, 2011, elicit sampling aspirations when initially little is known),
- to update beliefs about the likely values of future candidates.

Additionally we focus on an environment for which it can be assessed how far at best boundedly rational search behavior deviates from RN-optimality and explore how aspects like

- elicitation method of choice data,
- sequencing of search tasks

with no relevance for benchmark behavior may matter behaviorally. The partly systematic non-monotonicity of aspirations across search in "cold" was unexpected. If expected, we might have run a "cold" condition with imposed monotonicity to learn whether this aligns actual aspirations

[15] Once again we could have experimentally induced risk neutrality via employing binary-lottery incentives (see Di Cagno et al., 2017, for a simpler choice task). Here we were afraid to cognitively overburden participants.

closer to optimal ones.

4 Data analysis

Due to less informative data in "hot", we compare between-subjects conditions mainly via outcome data like average stopping times, payoffs, and standard deviations for $n = 15$ when distinguishing only whether $n = 15$ has been experienced first, respectively last (see Table 1). For $n = 15$ one can also compare across "cold-increasing", "warm", and "hot" the frequencies of accepted but RN-unacceptable and rejected but RN-acceptable values till acceptance, i.e. omitting later evidence of such deviations after acceptance in "cold". We, however, mainly focus on choice data of "cold" and "warm" for which one can compare trial-specific acceptance thresholds. For these conditions we first analyze the surprising anti-monotonicity, i.e. when later acceptance thresholds exceed earlier ones in the same search task.

4.1 Anti-monotonicity

When considering anti-monotonicity, we focus on rounds 3 & 4 which are most suitable for its comparison (due to $n = 10$ in rounds rounds 3 & 4 in all conditions which elicit acceptance thresholds). Anti-monotonicity[16] is defined by at least one such observation in rounds 3 & 4. It is a decisive advantage of "cold" choice elicitation that one can assess "anti-monotonicity", irrespective of when it occurs. In our view, consciously behaving anti-monotonically indicates rather clearly that behavior is not driven by trial-specific expected payoffs but by other motives like anticipated regret, e.g. in the form of trying to gain more than the highest rejected value so far, or risk seeking after a loss. Such motives suggest rare occurrence of "anti-monotonicity". We therefore distinguish participants whether they behave anti-monotonically at least once or not at all.

According to Table 2 the share of anti-monotonic participants depends strongly on the format of choice elicitation: it is naturally lowest for "warm" which provides fewer opportunities to reveal anti-monotonicity, compared to "cold". In "cold" we distinguish between increasing and

[16] The other form of anti-monotonicity, namely to accept smaller values than rejected ones or to reject larger values than accepted ones in the same trial, has been excluded by eliciting trial-specific acceptance thresholds.

decreasing sequence of n. Of all participants in "cold-in(de)creasing" 27.08% (42.55%) reveal anti-monotonicity in rounds 3 and 4, compared to 6.38% in "warm" in the same two rounds. The difference between the share of anti-monotonic participants in "warm" significantly differs from the share of "cold-increasing" ($z = -2.681$, $p = 0.008$) as well as "cold-decreasing" ($z = -4.057$, $p = 0.000$) using the two-tailed Wilcoxon Rank-sum Test (henceforth WRST). The difference in anti-monotonicity shares between "cold-increasing" and "cold-decreasing" is just short of being (marginally) significant ($z = 1.575$, $p = 0.115$, two-tailed WRST).

Since participants in "warm" have fewer opportunities to reveal potential anti-monotonicity, it is more appropriate to compare anti-monotonicity shares across "cold" and "warm" when balancing their frequencies for observing monotonicity violations. We have randomly simulated shorter acceptance profiles for "cold-increasing" (with the lower anti-monotonicity share and same sequence of n) according to actual search lengths in "warm".[17] When taking into account only the randomly simulated shorter profiles (based on the stopping times in "warm") the anti-monotonicity rate in "cold-increasing" drops to 14.5% but is still higher than "warm" with 6.38%, which is marginally significant ($z = -1.295$, $p = 0.097$, two-tailed WRST).

Result 1 "Cold", increasing and decreasing, triggers significantly higher shares of anti-monotonicity than "warm" in rounds 3 and 4.

Table 2: Percentage of anti-monotonic participants

	"Cold-increasing" n=10	"Cold-decreasing" n=10	"Warm" n=10
End-game Anti-monotonic (%)	6.25	17.02	0.00
Other Anti-monotonic (%)	20.83	25.53	6.38
Total (%)	27.08	42.55	6.38

Notes: Total number of participants: 48 in "cold-increasing", 47 in "cold-decreasing", 47 in "warm"

One might have expected a lower share of monotonicity violations for "cold" due to the obvious intuition that fewer remaining candidates are worse. On the other hand, anticipating a long

[17] We randomly matched "cold" and "warm" search profiles of same n sequence and shortened the "cold" profiles based on their matched "warm" profile's actual stop times. Given that in "warm" one has fewer opportunities to reveal anti-monotonicity, looking at the acceptance thresholds of search profiles of same length makes them more comparable.

unsuccessful search in “cold” could trigger attempts to avoid possible failures by a final high aspiration which, when satisfied, would render a too long and possibly even the longest search successful. We identify such (end-game) anti-monotonicity in “cold” elicitation methods by an increase of acceptance thresholds when only one or two candidates remain (see top row of Table 2 whose 0-share for “warm” is due to missing data). The substantial share of end-game anti-monotonic participants in “cold” seems closely related to becoming (more) risk loving after losing (see Kahnemann and Tversky, 1984). A participant in “cold” stating last aspirations seems to reason like: “I may search too long and be unlucky, but I can still try to make it a real success".

Result 2: “Cold”, especially “cold-decreasing”, let many participants (17.02%) adapt their last aspirations upwards.

So the format of choice elicitation crucially affects anti-monotonicity. Many participants seem to view fewer remaining candidates as a loss and reveal risk tolerance via anti-monotonicity, especially in “cold-decreasing” whose participants may already consider the decrease of n between rounds as a loss.

4.2 Acceptance Data

We compare the four between-subjects conditions of Table 1 by outcome data, separately for monotonic and anti-monotonic participants although this is more selective in “cold”. According to Table 3, which shows average search length of all and participants without monotonicity[18], average search is longest for monotonic participants in “cold-increasing”. The shorter search in “cold-decreasing” is mainly due to its shorter $n = 15$ plays (in round 1 & 2) by inexperienced participants. The positive difference between “cold-increasing” and “cold-decreasing” is significant for all participants and marginally significant for participants with no anti-monotonicity ($z = 2.376$, $p = 0.018$ for all participants, $z = 1.707$, $p = 0.088$ when considering participants with no anti-monotonicity, two-tailed WRST’s).

While search length is comparable across all elicitation methods, the chances of revealing at least

[18] Anti-monotonic participants in “Hot” are assessed based on “Hot-Warm” and “Hot-Cold” data in rounds 5 and 6, respectively.

one anti-monotonicity in "warm" and "hot" are fewer since they depend on search length. Average lengths of search in "cold-increasing" and in "warm" are closest (5.05 versus 5.01 for all participants, 5.51 versus 4.91 for only monotonic ones) suggesting that n-sequence affects search length more than the elicitation format.

Table 3: Stopping Time - Average number of participants drawn (seen) before accepting for $n = 15$

	Cold-Increasing		Cold-Decreasing		Warm		Hot	
	Mean	Sdt. Dev.	Mean	Sdt. Dev.	Mean	Sdt. Dev.	Mean	Sdt. Dev.
All	5.052	3.985	3.936	3.704	5.011	3.947	4.051	3.391
No Anti-Monotonicity	5.614	3.987	4.426	3.456	4.909	3.891	4.149	3.478

Result 3 *(i)* In "cold" participants search significantly longer when n is larger. "Warm" relies on the same n-sequence and triggers a similar average search length as "cold-increasing".

(ii) "Hot" induces significantly shorter plays than "cold-increasing" and "warm", though the difference is partially marginally significant ($p = 0.027$ when comparing "hot" with "cold-increasing" and $p = 0.059$ when comparing "hot" with "warm"; two-tailed WRST). "Cold-decreasing" with its high anti-monotonicity share seems strikingly different.

(iii) When combining all elicitation methods participants without anti-monotonicity (in rounds 3 and 4) search longer than anti-monotonic ones ($z = 3.417$, $p = 0.000$, two-tailed WRST).[19] Their longer search is aligned with more intuitive behavior, and suggests that monotonicity in aspiration formation goes hand in hand with more rationality.

Based on the range of possible iid-value realizations, for each $n-$task, we have calculated the theoretically expected RN-optimal search length. The cumulative distributions of actual as well as theoretically expected stopping times (based on RN-optimal acceptance thresholds), for each $n-$task, are illustrated by Figures 10, 11 and 12 in Appendix C.

[19] We couldn't do this analysis by conditions as the share of anti-monotonic participants is low for some conditions; see Table 2.

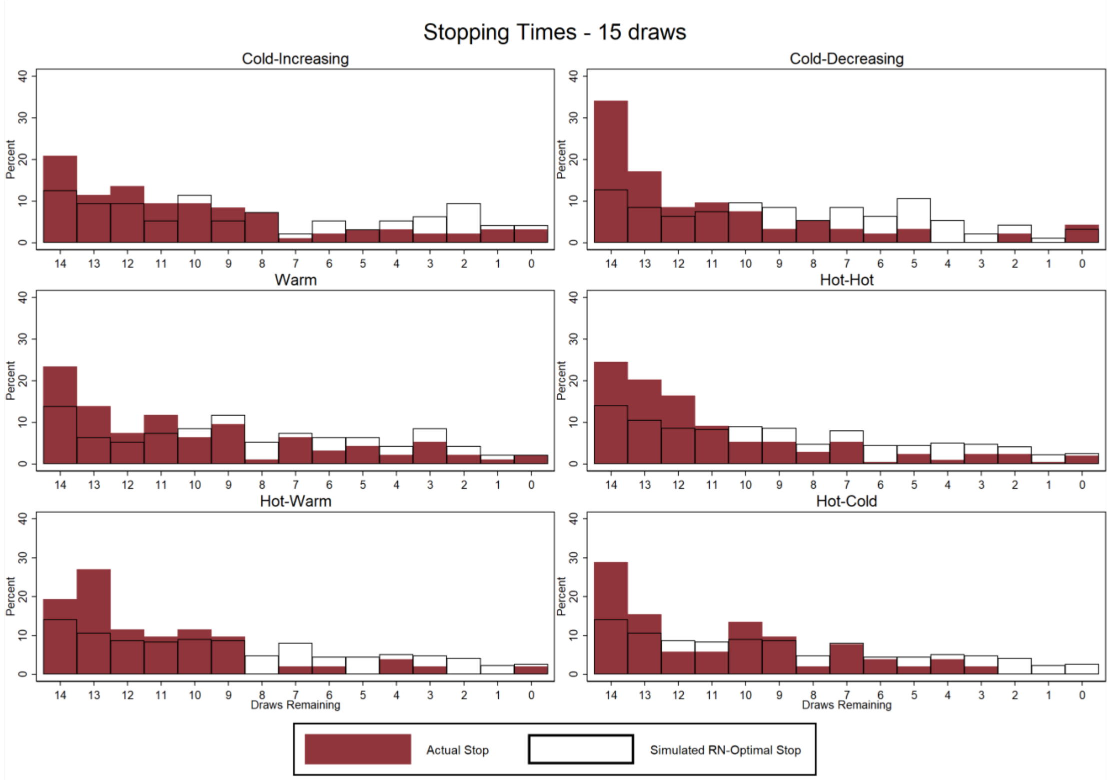

Figure 2: Stopping time for $n = 15$

Additionally, when comparing RN-optimal search length we can exploit that for each $n-$task and each participant the software generated a random sequence of n successive iid-value realizations. This allows to assess for each n-task and each participant the RN-optimal search length, based on RN-optimal play and actual iid-value realizations (RN-optimal stop would occur in the first instance in which the RN-optimal acceptance threshold is below the actual iid-value realization), which can be confronted with the actual search length of participants (based on both actual participant play and the same iid-value realizations).

Figure 2 visualizes the distributions of actual and RN-optimal stopping times for $n = 15$. Immediate, first trial, stopping is more frequent in "cold decreasing" than in "cold increasing" due to participants confronting n=15 earlier what triggers the largest difference of actual and RN-optimal stopping. Immediate acceptance in non-cold elicitation methods may be explained by participants naively comparing the first value with its expectation and stopping when the former is larger. Overall, participants in all cases, on average, stop significantly earlier than suggested by

RN-optimality ($p = 0.000$ for "cold increasing", "cold decreasing", "warm" and "hot-hot", $p = 0.004$ for "hot-warm", and $p = 0.001$ for "hot-cold" using a two-tailed Wilcoxon sign-rank test).

For $n = 15$, "cold increasing" and and "warm" have the longest search, yet stopping in the first two draws occurs in around 33% of cases in "Cold increasing" and around 38% of cases in "warm" but are much higher than predicted by RN-optimal stopping (See Figure 2). For $n = 10$ and $n = 5$ there exist only three comparisons each (see Figure 3). Differences across between-subject conditions are minor due to fewer candidates. Part (ii) of Result 3 suggests a possible important advice, namely to rely on self-delegation via committing to a complete aspiration profile already before the first trial. One could describe this as self-nudging (see Thaler and Sunstein, 2009). Rather than engaging trial-by-trial sequential choice making and becoming stressed and emotionally upset when running out of candidates; one should decide for all trials before beginning to search, i.e. when still being patient.

Figure 3 provides the same information for $n = 10$ and $n = 5$. "Cold increasing" and "cold decreasing" in case of $n = 5$ reveal a striking similarity of actual and RN-optimal stopping. These two conditions are the only ones for which the null hypothesis of no difference between actual and RN-optimal stopping cannot be rejected (using a two-tailed Wilcoxon sign-rank test).

We readily admit that information about individual characteristics, for example, elicited by post-experimental personality questionnaires and possibly complemented by cognitive reflection tasks could be helpful when wanting to account for heterogeneity in stopping. Here the focus has been on whether and how elicitation mode and $n-$sequencing affects search. In a follow-up study we want to induce risk-neutrality (and pay one random round[20]) and let participants answer post-experimental questionnaires suitable to shed light on the reasons of more or less heterogeneity. When doing so one may want to avoid the striking effects of elicitation format and n-sequence and focus instead on the condition with the highest (end game) anti-monotonicity share for which heterogeneity in idiosyncratic characteristics may be more crucial.

[20]This would have the additional advantage that both stopping and continuing to search yield expected payoffs. In the field, stopping often offers only stochastic payoff.

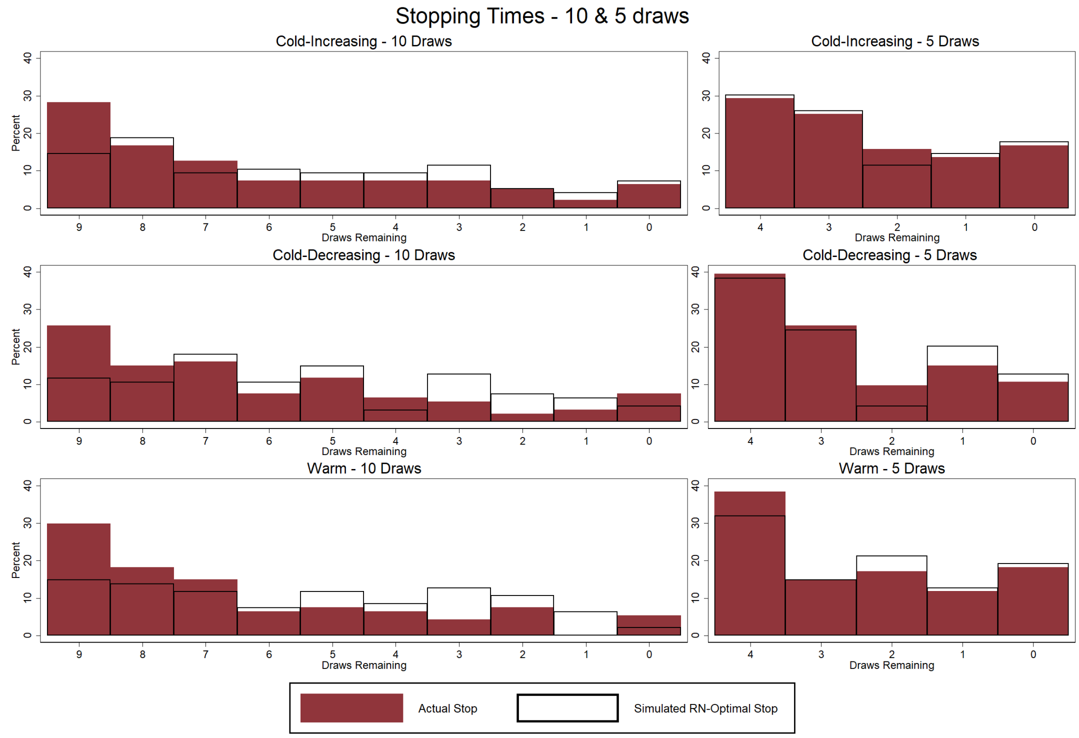

Figure 3: Stopping time for $n = 10$ and $n = 5$

In three of the four conditions $n = 10$ is played in the 3rd and 4th round which avoids the effect of differences in experienced number of rounds. Table 4 compares the stopping times of these rounds. There is no significant difference when considering all participants. When excluding the anti-monotonic, however, "cold-increasing" leads to significantly longer search than "warm" ($z = 2.132$, $p = 0.033$, two-tailed WRST). The difference between "cold-decreasing" and "warm" is larger, however just marginally significant due to the low frequency of participants with no anti-monotonicity in "cold-decreasing" ($z = 1.576$, $p = 0.115$, two-tailed WRST).

Table 4: Stopping Time - Average number of participants drawn (seen) before accepting in rounds 3 & 4 when $n = 10$

	Cold-Increasing			Cold-Decreasing			Warm		
	Mean	Std. Dev.	Freq.	Mean	Std. Dev.	Freq.	Mean	Std. Dev.	Freq.
No Anti-monotonicity	4.129	2.854	70	4.241	2.642	54	3.386	2.539	88
All	3.823	2.828	96	3.872	2.791	94	3.543	2.691	94

4.3 Payoff Comparisons

Average payoffs are visualised by the kernel densities in Figures 4 and 5 for participants without any anti-monotonicity (for all participants see Figure 9 in Appendix C). “Hot” payoffs are higher due to 6 rounds with $n = 15$. To account for this difference, the left panel of Figure 5 presents the kernel densities of payoffs only for the first two rounds of “cold-decreasing” and “hot”, and the right panel of Figure 5 for the last two rounds of “cold-increasing”, “warm” and “hot”, all relying on n=15. Table 5 lists for participants without anti-monotonicity the average payoffs and their standard deviations, separately for $n = 15, 10$, and 5. Table 6 includes all participants. The obvious increase in payoffs, due to more candidates, is missing in “cold-decreasing" (see the difference between $n = 15, 10$, and 5 in Tables 5 and 6) even when considering only participants without anti-monotonicity. Altogether, repeating the same n-task, avoiding anti-monotonicity and increasing numbers n of candidates enhance payoffs at best slightly.

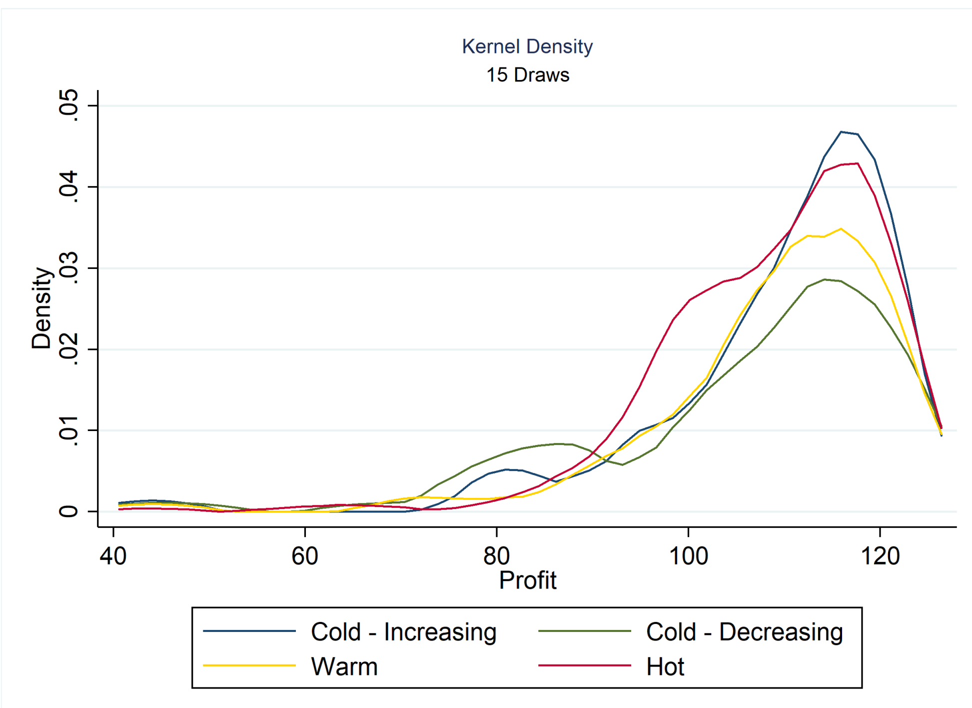

Figure 4: Kernel density of profits ($n = 15$) - monotonic participants only

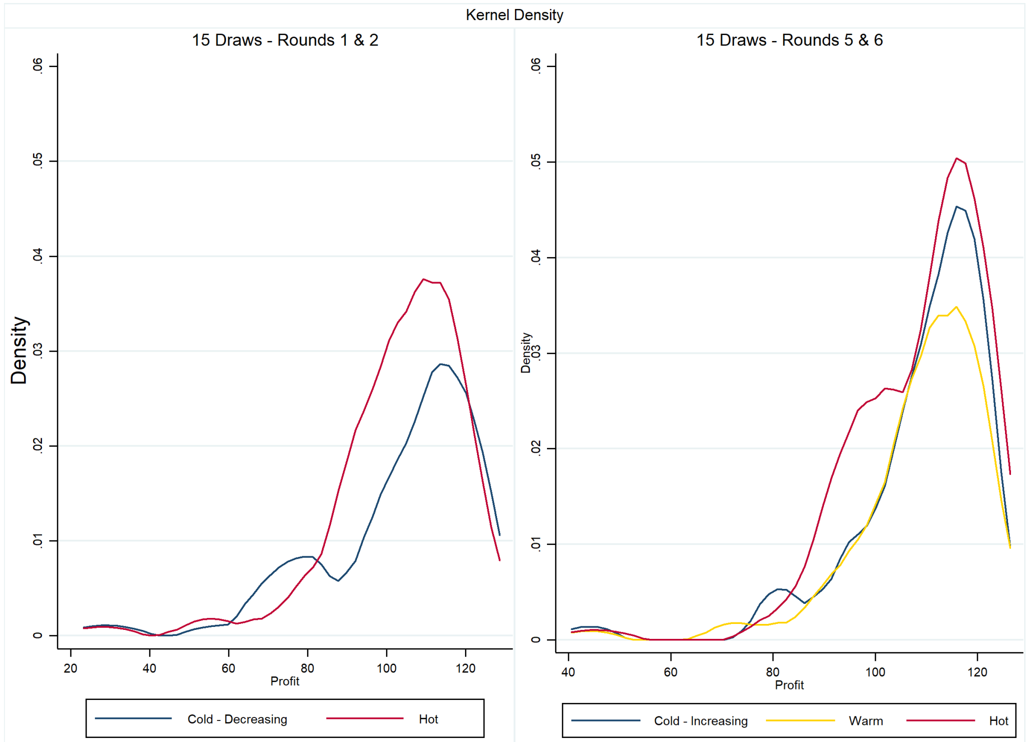

Figure 5: Kernel density of profits ($n = 15$) in the first two rounds of "cold-decreasing" and "hot" (left panel), respectively of "cold-increasing", "warm" and "hot" for the last two rounds (right panel) - monotonic participants only

Based on the simulated RN-optimal search we can compute for each participant and each task what (s)he has actually earned and what (s)he would have earned by RN-optimality (see the Figures 13 and 14 in Appendix C for actual average and RN-optimal individual earnings). As expected, actual earnings in most tasks are below RN-optimal ones. Except for "cold-increasing" and "cold-decreasing" in the case of $n = 5$ and "hot-hot" tasks, participants in all other tasks earn significantly less than they would have under RN-optimal search.[21]

[21] Using a 2-tailed Wilcoxon sign rank test, $p = 0.060$ for "cold-increasing" when n=5, $p = 0.000$ for "cold-increasing" when n=10, $p = 0.005$ for "cold-increasing" when n=15, $p = 0.556$ for "cold-decreasing" when n=5, $p = 0.001$ for "cold-decreasing" when n=10, $p = 0.000$ for "cold-decreasing" when n=10, $p = 0.000$ for "warm" when n=5, $p = 0.003$ for "warm" when n=10, $p = 0.000$ for "warm" when n=15, $p = 0.000$ for "hot-hot", $p = 0.002$ for "hot-warm", $p = 0.175$ for "hot-cold".

Table 5: Payoff per round - participants without anti-monotonicity only

	Cold-Increasing			Cold-Decreasing			Warm			Hot ($n = 15$)		
Play	5	10	15	5	10	15	5	10	15	Hot	Warm	Cold
1st	96.91 (19.13)	102.14 (17.87)	108.49 (15.18)	102.00 (21.55)	111.11 (8.84)	106.96 (11.49)	92.36 (26.35)	107.11 (15.97)	105.55 (17.47)	106.45 (15.38)	112.13 (11.09)	113.66 (8.04)
2nd	99.34 (20.8)	104.51 (19.94)	110.06 (11.23)	105.19 (17.42)	101.93 (21.03)	108.67 (11.29)	92.89 (23.47)	105.73 (11.58)	108.14 (11.87)	108.72 (8.20)		
3rd										109.45 (8.99)		
4th										109.15 (8.25)		

Table 6: Payoff per round - all participants

	Cold-Increasing			Cold-Decreasing			Warm			Hot ($n = 15$)		
Play	5	10	15	5	10	15	5	10	15	Hot	Warm	Cold
1st	94.92 (21.02)	100.42 (19.23)	107.98 (15.61)	98.32 (23.15)	102.47 (19.59)	101.83 (21.83)	92.83 (25.82)	106.87 (15.55)	106.15 (17.07)	105.17 (15.85)	109.81 (15.43)	112.02 (11.40)
2nd	99.42 (20.13)	103.50 (19.27)	106.04 (19.09)	100.04 (22.16)	98.53 (22.19)	106.60 (14.97)	93.91 (23.12)	105.66 (11.34)	107.91 (11.54)	108.72 (8.20)		
3rd										108.69 (9.71)		
4th										109.65 (8.06)		

Result 4 *(i)* Repeating the same n-task in round $t = 2, 4, 6$ after round $t - 1$ with the same n, does not significantly increase payoffs across conditions ($p = 0.232$ in “cold-increasing"; $p = 0.994$ in “cold-increasing"; $p = 0.479$ in “warm"; and $p = 0.924$ in the first 2 rounds of “hot", when comparing round $t = 2, 4, 6$ with $t - 1$ for all n in a treatment using a two-tailed WRST).

(ii) Anti-Monotonicity reduces payoffs in all conditions except “warm”, possibly due to its low share of anti-monotonic participants ($p = 0.088$ in “cold-increasing", $p = 0.000$ in “cold-decreasing", and $p = 0.003$ in “hot” using a two-tailed WRST), though for “cold-increasing" the reduction is only marginally significant.

(iii) For participants without anti-monotonicity, larger n yields larger average payoffs ($p = 0.001$ when comparing $n = 10$ with $n = 5$, and $p = 0.018$ when comparing $n = 15$ with $n = 10$, pooling across conditions and using a two-tailed WRST).

Altogether anti-monotonicity leads to shorter search for which one pays by lower average payoffs, similar to the lower success of children (in life income) who behave myopically in the marshmallow task.

4.4 Choice Data

We begin with graphical illustrations. Figure 6 plots for the number of remaining candidates (on the abscissis) the average aspirations (acceptance thresholds) for $n = 15$ across the 14 trials of "hot" (based on "hot-cold" data from round 6 only), "cold-increasing" and "cold-decreasing" and confronts them with RN-optimal ones (dotted).[22] Playing "cold" after "hot" play triggers average aspirations closest to RN-optimality. Except for overshooting at the end, "cold-decreasing" differs most from RN-optimality with "cold-increasing" in between. The striking difference between "cold-increasing" and "cold-decreasing" (see Figures 6 and 8) confirms, in our view, that a decreasing n is perceived as more stressful and as a deprivation to which one reacts by shorter search what, in turn, lowers average payoffs.

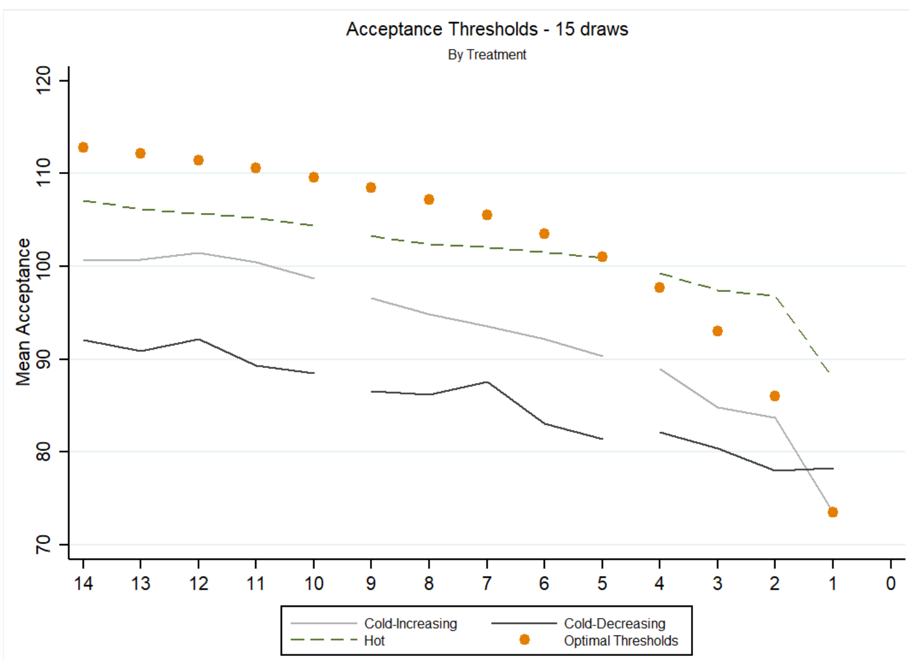

Figure 6: Acceptance thresholds for $n = 15$

Result 5 *(i)* for $n = 15$ average "hot" aspirations are closest to RN-optimal ones, when neglecting their overshooting in the last four draws. "Cold-increasing" average aspirations for $n = 15$ are

[22]"Warm", which does not ask for a complete sequence of aspirations in any of the six rounds, is not included in Figures 6, 7, and 8 since its successive means would rely on fewer threshold data.

closer to RN-optimality than those for "cold-decreasing" (Figure 6),[23]

(ii) Comparing average choice behavior for $n = 15$ between "cold-increasing" and "cold-decreasing" reveals a striking and significant sequence effect ($p = 0.000$ using a two-tailed WRST) whereas average aspirations for $n = 5$ and $n = 10$ do not differ significantly between the two "cold" conditions (see Figures 6, 7, and 8). In our view, playing the more rewarding $n = 15$ tasks first when being still inexperienced, lets one stop on average too early via lower $n = 15$-acceptance thresholds.

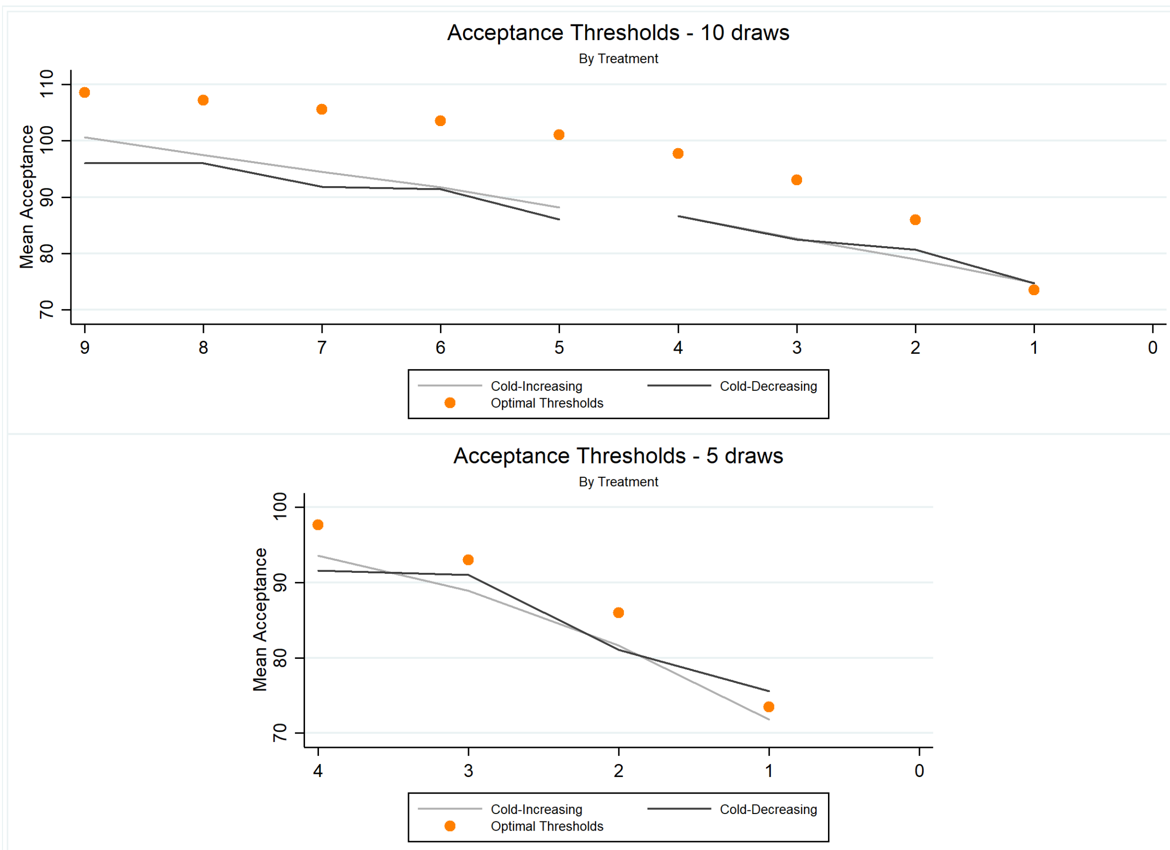

Figure 7: Acceptance thresholds for $n = 10$ and $n = 5$

In "cold-increasing", "cold-decreasing", and "warm" participants experience the $n = 10$ task in rounds 3 and 4. We compare the acceptance thresholds in rounds 3 and 4 in Table 7 which neglects the differences in experience before round 3. It reveals a surprising stationarity of average

[23]This result also holds when only regarding round six for "cold-increasing", see Table 9 in Appendix C. We have done this to avoid confounding condition differences in aspirations with experience (in "hot" participants state complete profiles with 14 aspirations only in round six).

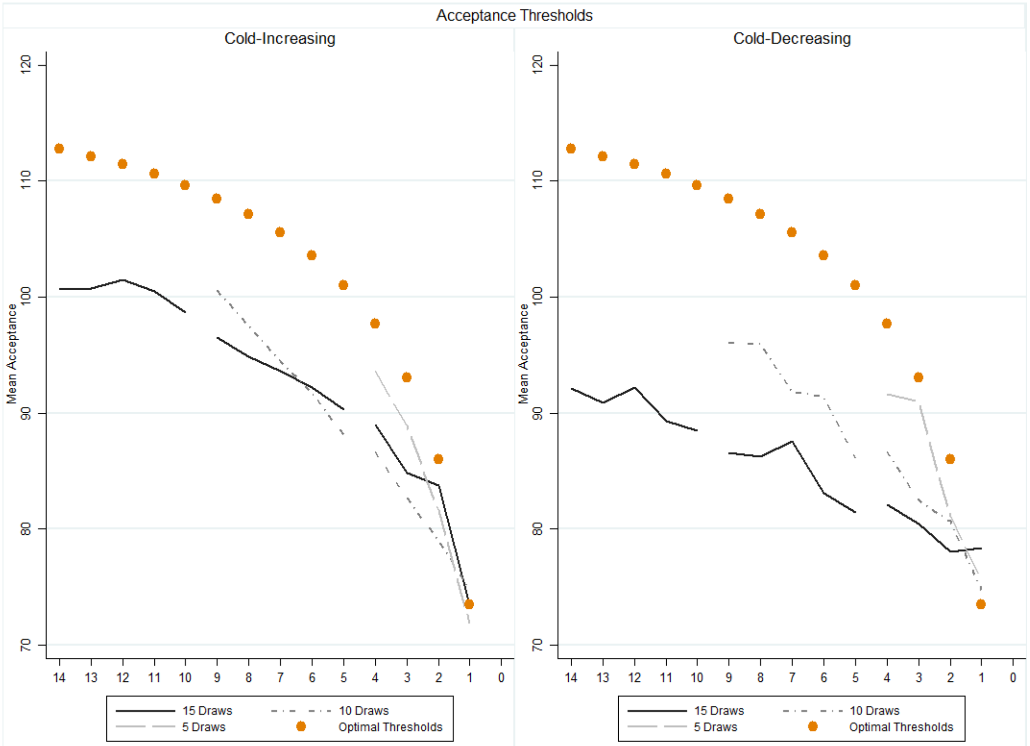

Figure 8: Acceptance thresholds with cold elicitation methods

acceptance thresholds in "warm" whereas they steadily decrease for both "cold" conditions. This, again, highlights the importance of relying on self-delegation or self-nudging via committing to an aspiration profile at the beginning rather than trial-by-trial sequential aspiration formation which leads to overshooting (higher than optimal thresholds) when the number of remaining candidates gets small.

Table 7: Acceptance thresholds in rounds 3 & 4 (when $n = 10$), and RN-optimal thresholds.

Number of remaining trials		Cold-Increasing	Cold-Decreasing	Warm	RN-Optimal
	Mean	100.594	95.989	99.000	108.480
9	Std. Dev.	20.271	27.419	15.414	
	Freq.	96	94	94	
	Mean	97.479	95.968	100.197	107.143
8	Std. Dev.	18.523	22.983	14.217	
	Freq.	96	94	66	
	Mean	94.458	91.798	98.102	105.528
7	Std. Dev.	19.513	23.625	16.224	
	Freq.	96	94	49	
	Mean	91.719	91.394	98.029	103.535
6	Std. Dev.	19.349	19.804	15.905	
	Freq.	96	94	35	
	Mean	88.135	86.074	96.207	101.007
5	Std. Dev.	20.282	22.504	20.897	
	Freq.	96	94	29	
	Mean	86.615	86.660	94.818	97.671
4	Std. Dev.	19.177	21.829	20.953	
	Freq.	96	94	22	
	Mean	82.677	82.479	94.938	93.030
3	Std. Dev.	21.251	24.606	20.978	
	Freq.	96	94	16	
	Mean	79.021	80.649	95.583	86.000
2	Std. Dev.	22.653	22.662	17.059	
	Freq.	96	94	12	
	Mean	74.771	74.755	98.000	73.500
1	Std. Dev.	25.513	24.811	19.235	
	Freq.	96	94	5	

Table 8 looks at the difference between stated acceptance thresholds and optimal thresholds in both "cold" conditions, separated by monotonicity of each sequence.[24] It confirms that the decreasing sequence triggers a larger share of anti-monotonicity: the largest difference occurs for $n = 15$ (though the monotonicity share is in "cold-increasing" larger for all three $n-$values). The average acceptance thresholds of monotonic sequences are, for all $n-$parameters, lower than the optimal ones. While monotonic sequences of "cold-increasing" are on average closer to RN-optimal ones than "cold-decreasing", the difference is not statistically significant. This allows us to conclude that the differences, reported in Result 5, are largely due to the higher frequency of non-monotonic plays in "cold-decreasing".

[24]Unlike in Section 4.1, here we differentiate between (anti-) monotonic participants for each of the six sequences (rounds) asking participants for acceptance thresholds.

Table 8: Differences between actual and optimal thresholds in Cold conditions - by monotonicity

		Cold-Increasing			Cold-Decreasing		
		n=5	n=10	n=15	n=15	n=10	n=5
Non Anti-Monotonic	Mean	-1.909	-7.533	-8.396	-10.825	-6.092	-2.497
	Std. Dev.	13.317	13.223	9.996	15.580	12.666	15.230
	Freq.	74	70	66	46	54	67
Anti-Monotonic	Mean	-10.405	-14.561	-12.962	-23.382	-16.486	-4.269
	Std. Dev.	16.472	17.829	16.309	21.617	20.807	23.092
	Freq.	22	26	30	48	40	27

Table 9 reports the share of participants with lower acceptance thresholds than RN-optimal ones during the first three draws. For "cold" conditions the first aspiration is more frequently below the RN-optimal one when n is larger ($p = 0.008$ when comparing $n = 5$ & $n = 10$, $p = 0.009$ when comparing $n = 10$ & $n = 15$, and $p = 0.000$ when comparing $n = 5$ & $n = 15$, using a logit regression and pooling both conditions).

Table 9: Percentage of aspirations profiles with three initial aspirations below the optimal ones

	Cold-Increasing			Cold-Decreasing			Hot		
	Draw 1	Draw 2	Draw 3	Draw 1	Draw 2	Draw 3	Draw 1	Draw 2	Draw 3
n=5	49.74%	43.75%	30.21%	39.36%	35.11%	25.53%			
n=10	56.25%	86.05%	68.12%	54.26%	65.96%	74.47%			
n=15	68.75%	72.92%	73.96%	68.09%	76.60%	77.66%	71.15%	76.92%	75.00%

Result 6 Initial aspirations below RN-optimal ones occur more often for $n = 15$ than for $n = 10$ and least often for $n = 5$.

Stopping too early (compared to RN-optimality) had to be expected as RN-optimality neglects regret concerns. Anticipating that one may have rejected a better option earlier only to accept a worse one later lets participants anticipate regret and conclude "I should have stopped earlier!" what strongly discourages to state initial aspirations which are as ambitious as the RN-optimal ones. Even when regret has to be coldly anticipated it strongly affects decision making.

5 Conclusions

Let us begin by what can be learned from our analysis, especially in view of the different results due to variations in experimental choice elicitation (see first paragraph of Section 2). Clearly the familiar motive of neo-classical economics, expected profit, cannot account alone for how participants form value aspirations and adjust them across trials. This is obviously true quantitatively (see the striking differences between various conditions) but also partly qualitatively as revealed by the partly substantial and significant shares of aspiration profiles with at least one anti-monotonicity. In our view, these rather systematic deviations from RN-optimality are often due to anticipation of regret and loss perception.

Although we expected elicitation method ("cold", "warm" and "hot") and sequencing of n-tasks ("increasing" versus "decreasing") to matter, the large effects are surprising. Many participants seemingly viewed the experiment with its six rounds rather holistically, whereas RN-optimality neglects how a specific n-task is embedded in a sequence of n-tasks and also how many candidates were initially available. The theoretical neglect of path dependence with its conditioning only on the state variable, the number of remaining candidates, is the main reason for behaviorally rejecting RN-optimality.

The cardinal iid-secretary search task avoids complications like unknown priors, costs of search, future dividends, and competition in search. It thus limits confounding aspects like belief updating, learning across trials, etc. Theoretically, there is nothing to learn from the past: all what should matter is the number of remaining candidates. In spite of this our data analysis identifies and confirms purely behavioral effects of path and context dependence, temperature of choice elicitation, experience, and how these affect the proximity of behavior to the benchmark prediction. Although choice elicitation methods leave optimality intact, they as expected were shown to trigger different emotions and aspirations.

The "cold" data especially allow to compare optimal and actual conditioning on the number of remaining candidates. Another advantage of the "cold" and "warm" conditions is rendering aspiration formation and adaptation directly observable whereas "hot" instead provides only values of rejected, respectively accepted candidates. To infer aspirations from the latter choice data in the tradition of the revealing aspiration approach would have to assume satisficing, similar to presupposing optimality in revealed preference theory.

RN-optimal search behavior describes optimal satisficing (when accepting risk neutrality). Satisficing as such does not require optimality but is experimentally imposed in "cold" and "warm" via eliciting binding acceptance thresholds. Such induced satisficing is more rational when avoiding anti-monotonicity of aspirations across trials. There is striking heterogeneity in individual behavior: participants partly form more and partly less ambitious aspirations than RN-optimal ones where the latter dominates. Other surprising findings, also varying interpersonally, are the strong n-sequence effect for "cold" and the widely differing degrees of anti-monotonicity shares across elicitation modes. Attributing anti-monotonicity to noise is questioned by the high anti-monotonicity shares even in later rounds, and by the considerable sequence and elicitation effects on anti-monotonicity, specifically on end-game anti-monotonicity (see Table 2 and its discussion). An important conclusion of comparing "cold" and "hot" is that self-delegation or self-nudging is prolonging search and slightly improves average payoffs.

In future research we plan to employ binary lottery incentives what requires to pay only for one randomly selected round to experimentally induce risk neutrality. We will probably focus on "cold-decreasing" with its high shares of anti-monotonicity to investigate whether and how heterogeneity can be attributed to idiosyncratic characteristics.

One could also elicit choice behavior via employing multiple choice elicitations. One possibility is using all three via employing the CWH-method: participants state, as in "cold", a complete aspiration profile before the first trial and additionally, trial after trial as in "warm", an acceptance threshold which possibly differs from the one chosen "coldly". Then as in "hot", they learn the value and decide whether to accept the known value at the given trial or not. Incentivizing the CWH-method could rely on positive probabilities for all three choices being applied randomly trial by trial. So, when wanting to stop as in "hot", one stops with the probability for "hot" but may continue the search with the complementary probability when the choices in "cold", respectively "warm" each with the respective probability suggest it. The CWH-method obviously allows to control intra-personally, rather than inter-personally as in this paper, how elicitation formats affect search behavior.

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Appendix A - RN Optimal Strategy

The rational and risk neutral decision maker (DM) is sequentially presented with n "secretaries" with independent, non-negative qualities $X_1, X_2, \ldots, X_n$ with common continuous distribution F. At any trial t, $1 \leq t \leq n$, DM sees the quality realization $X_t = v_t$ and decides whether to recruit this "secretary" and stop searching or to reject the secretary and continue searching. Rejected "secretaries" cannot be recalled. DM maximizes the expected quality of the recruited "secretary". Thus, DM's objective when n secretaries are available is given by

$$\mathbb{E}_n = \sup_{1 \leq \tau \leq n} \mathbb{E}[X_\tau], \tag{1}$$

where $\mathbb{E}$ denotes the expectation operator, τ the stopping time with respect to the increasing sequence of σ-fields $\mathcal{F}_t = \sigma\{X_1, \ldots, X_t\}$ and the trivial σ-field $\mathcal{F}_0$.

The optimal stopping problem (1) can be solved by dynamic programming. To do so set $\underline{v}_0 = 0$, and let $\mathbb{E}_k$ denote the optimal expected quality of the recruited secretary when there are k secretaries still to be seen. The principle of optimal dynamic programming says that $\mathbb{E}_k$ obeys the following recursion

$$\mathbb{E}_k = \int_0^\infty \max\{v, \mathbb{E}_{k-1}\}\, dF(v) = \mathbb{E}[X_1] + \int_0^{\mathbb{E}_{k-1}} F(v)\, dv, \qquad \text{for } 1 \leq k \leq n. \tag{2}$$

The left term (2) $\mathbb{E}[X_1]$ is the pay-off that the decision maker obtains by recruiting the secretary currently under evaluation with quality $X_{n-k+1} = v_{n-k+1}$, while the right term is the pay-off for rejecting the current secretary and continuing to search optimally in the next trial $k-1$. Due to our special choice task the RN-optimal payoffs, when continuing search, coincide with the RN-optimal acceptance threshold, i.e. acceptance thresholds are also optimal aspirations. We also see from (2) that the "secretary" inspected, at time t with quality X_t, is an optimal recruit at time t if and only if

$$X_t > \underline{v}_{n-t} \qquad \text{for all } 1 \leq t \leq n-1$$

and that the sequence of optimal thresholds is monotonically increasing in the number of (remaining) trails:

$$0 = \underline{v}_0 \leq \underline{v}_1 \leq \underline{v}_2 \leq \cdots \leq \underline{v}_n.$$

Moreover, if the random qualities $X_1, X_2, \ldots, X_n$ have the uniform distribution on $[0,1]$, then $F(v) = v$ for all $v \in [0,1]$ and

$$\underline{v}_0 = 0 \qquad \text{and} \underline{v}_k = \frac{1}{2}(1 + \underline{v}_{k-1}^2), \text{ for } 1 \leq k \leq n.$$

The experiment actually uses discrete integer qualities and one derives (2) when the cumulative distribution function F has discrete support. Specifically, we choose two integers $a \geq 0$ and $J \geq 0$ and suppose that the random qualities $X_1, X_2, \ldots, X_n$ have support on the integers $\{a+1, a+2, \ldots, a+J\}$. Setting $\underline{v}_0 = a$ and for any $\underline{v}_{k-1} \in [a, a+J]$ we have the recursion

$$\underline{v}_k = \mathbb{E}[\max\{X_{n-k+1}, \underline{v}_{k-1}\}] = \mathbb{E}[X_1] + (\underline{v}_{k-1} - \lfloor \underline{v}_{k-1} \rfloor)F(\lfloor v_{k-1} \rfloor) + \sum_{j=a+1}^{\lfloor v_{k-1} \rfloor - 1} F(j).$$

When F is the discrete uniform distribution on $\{a+1, a+2, \ldots, a+J\}$, then $F(j) = (\lfloor j \rfloor - a)/J$ for $a+1 \leq j \leq a+J$ and the right hand side becomes

$$\underline{v}_k = a + \frac{1}{2}(J+1) + \frac{1}{2J}(\lfloor \underline{v}_{k-1} \rfloor - a)(2\underline{v}_{k-1} - \lfloor \underline{v}_{k-1} \rfloor - a - 1), \qquad \text{for } 1 \leq k \leq \tag{3}$$

Is optimal to accept the secretary, inspected at time t, if and only if

$$X_i > \underline{v}_{n-t}$$

The optimal thresholds (3) when $n = 15$, $a = 24$ and $J = 99$ are graphically shown in Figure 1 in the main text.

Appendix B - Instructions: Cold-Increasing

Welcome! This is an experiment on how individuals make decisions. We are only interested in your choices. Be careful how you take your decisions as your behavior will determine the amount of money that you will receive and which will be paid out at the end of the experiment. In addition, you will gain an amount of 5(€) as a show up fee.

The following instructions will explain what choices you make and the gains associated with each choice.

Expected gains from this experiment are defined in ECUs (Experimental Currency Unit), converted at the following rate:

1 ECU = 2cents (€)

This experiment is computerized and is based on individual decisions. All decisions will be taken anonymously through the computer in front of you. It is forbidden to communicate in any way with other participants for the duration of the experiment. At the end of the experiment there will be a questionnaire. At the end of the questionnaire you will be called individually to receive the final payment. Please wait in silence until the experimenters call your number.

After reading the instructions by the experimenter you'll have some minutes to read: If something is not clear p

Please do not disturb other participants during the experiment.

Design

In this game you will play 6 rounds. In each round there will be made a number of draws, namely:

- In round 1 and round 2 there will be up to 5 draws

- In round 3 and round 4 there will be up to 10 draws

- In round 5 and round 6 there will be up to 15 draws

We'll call the drawn values (v): each value v is randomly drawn by the computer from a range of integers between 24 to 123, where all possible 100 integers are equally likely, i.e. every value v is chosen with probability 1/100. In round 1 and round 2 a maximum of 5 values (v) will be drawn, values v_1, v_2, v_3, v_4, v_5, in round 3 and 4 a maximum of 10 values (v) will be drawn, that is, v_1, v_2, ..., v_9, v_{10} etc..

The round ends when a draw is accepted (so if you accept the first draw, remaining draws for that round will not be carried out); in case no extraction is accepted then the round will end at last draw for that round. The procedure for acceptance will be explained in detail below.

In each round your gain is defined by the accepted draw, or the last draw if no earlier draw has been accepted. In particular the ECU's (Experimental Currency Unit) is equal to the accepted value v in that particular round (so, in each round you can earn minimum 24ECU, and maximum 123ECU).

Acceptance and choosing acceptance thresholds

At the beginning of each round, before the draws begin, you will be asked to define your acceptance thresholds (t) for all potential draws that round. The acceptance threshold t is the value you choose, from 24 to 124, in order to define what value you are willing to accept for each draw. For example in rounds 1 and 2:

Draw	Acceptance Threshold
1	t_1
2	t_2
3	t_3
4	t_4

In particular: in round 1 and round 2 you define four acceptance thresholds t_1, t_2, t_3, t_4 for the first 4 draws (the last value, value v_5 is automatically accepted if no other draw has been accepted). In round 3 and round 4 you will have to define nine acceptance thresholds t_1, t_t, ..., t_8, t_9 for the first 9 draws, and in rounds 5 and 6 you will have to define fourteen acceptance thresholds t_1, t_2, ..., t_{13}, t_{14} for the first 14 draws.

After you define all the acceptance thresholds $t_1 - t_{n-1}$, the draw of the first value v1 is made and:

a) If the extracted value v1 is greater or equal than the threshold of acceptance ($v_1 \geq 5_1$), the draw is accepted and the round ends;

b) If the extracted value is below the threshold of acceptance ($v_1 < 5_1$), the draw is not accepted and you move on to the second draw for which you have already defined an acceptance threshold t_2. Since you may want to reject any value of a certain draw, we included the acceptance threshold $t = 124$ which automatically rejects any possible value drawn. $t = 24$, on the other hand, accepts every possible drawn value.

Please take your time in making your decisions, there is no point in rushing as the next round starts only when all participants concluded their decision and extractions.

<u>The final gain from this experiment</u>

Your final gain of the experiment will be determined by the sum of earnings for each round and the amount that you are paid for your participation, specifically:

- 5€ro as a fixed participation fee;

- the gain equal to the value v of the accepted draw (value v of the last draw if no earlier is accepted) in round 1;

- the gain equal to the value v of the accepted draw (value v of the last draw if no earlier is accepted) in round 2;

- the gain equal to the value v of the accepted draw (value v of the last draw if no earlier is accepted) in round 3;

- the gain equal to the value v of the accepted draw (value v of the last draw if no earlier is accepted) in round 4;

- the gain equal to the value v of the accepted draw (value v of the last draw if no earlier is accepted) in round 5;

- the gain equal to the value v of the accepted draw (value v of the last draw if no earlier is accepted) in round 6;

Appendix C - Additional Tables and Figures

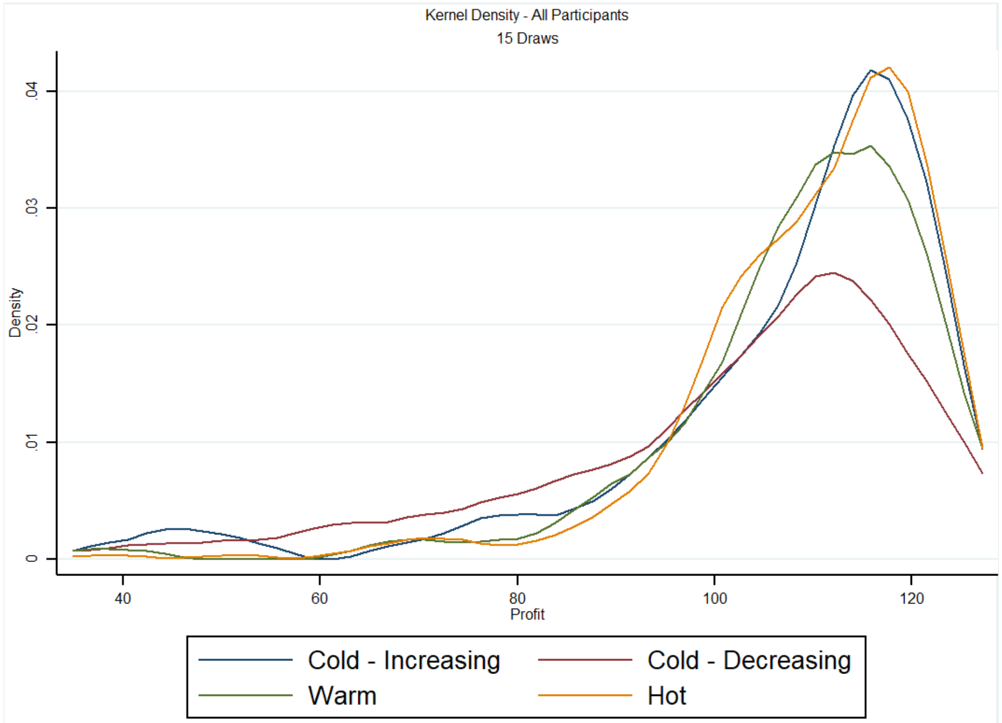

Figure 9: Kernel density of profits ($n = 15$) - all participants

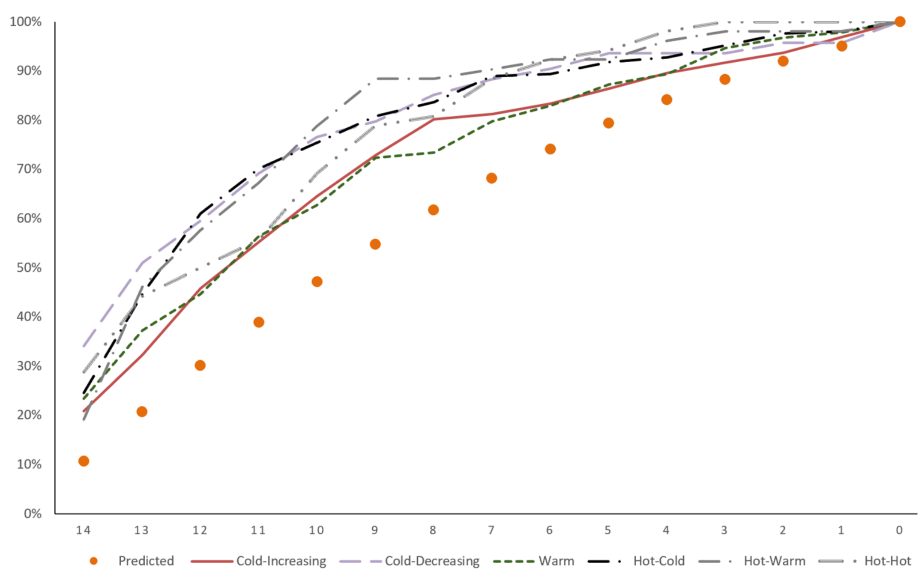

Figure 10: Cumulative actual vs. theoretically expected RN-optimal stopping times ($n = 15$)

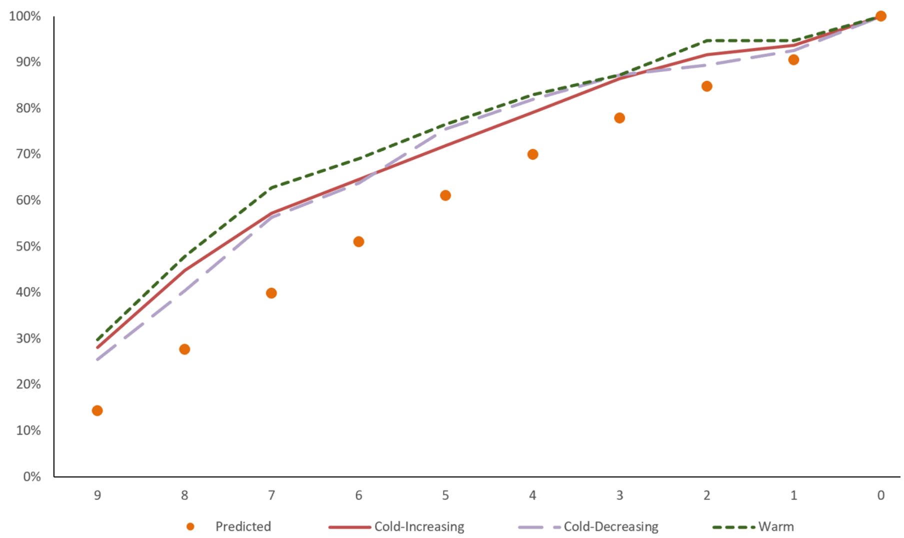

Figure 11: Cumulative actual vs. theoretically expected RN-optimal stopping times ($n = 10$)

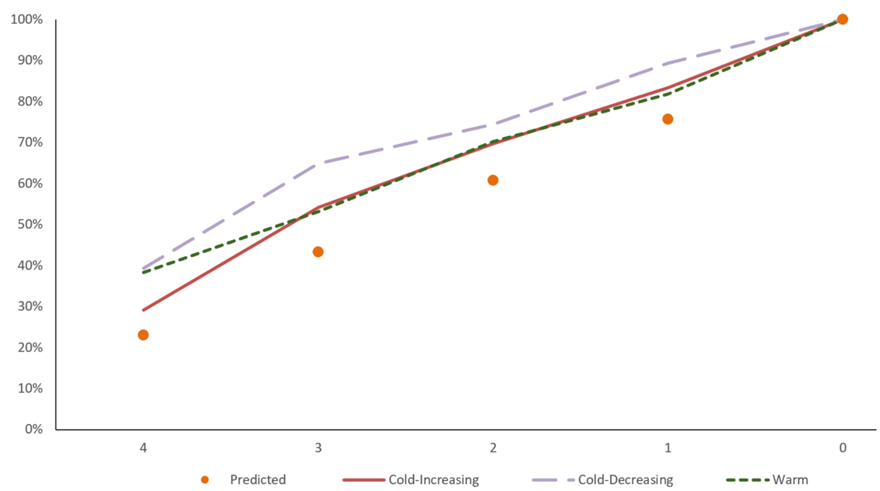

Figure 12: Cumulative actual vs. theoretically expected RN-optimal stopping times ($n = 5$)

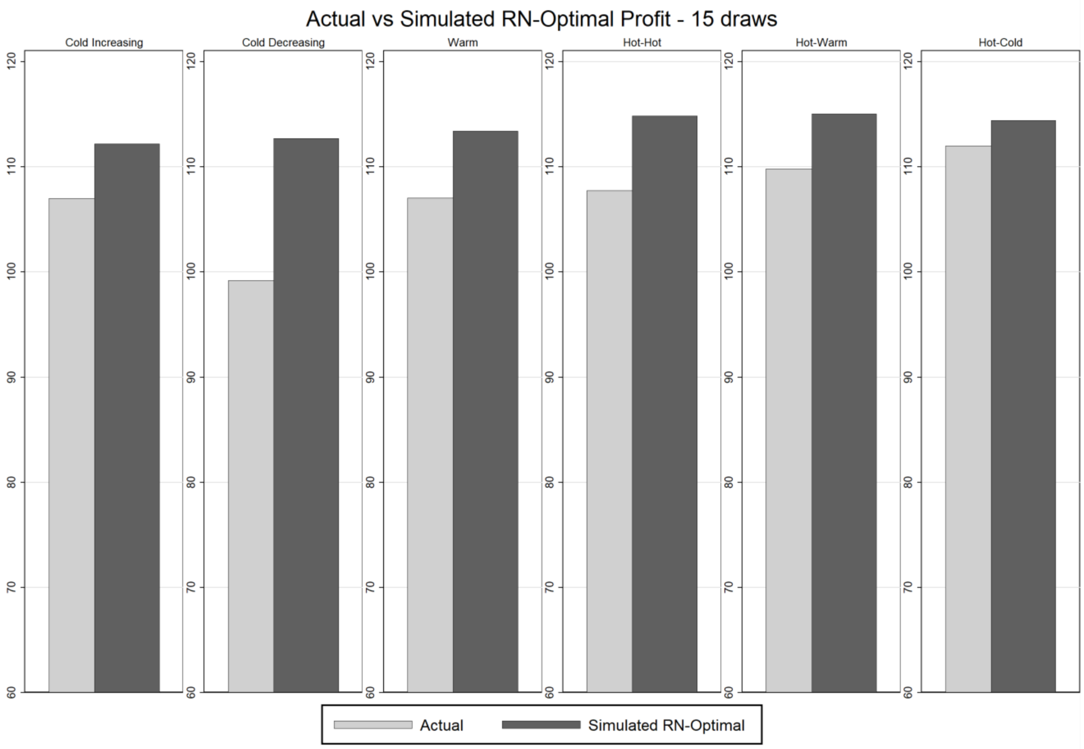

Figure 13: Actual vs. RN-optimal profits ($n = 15$) - all participants

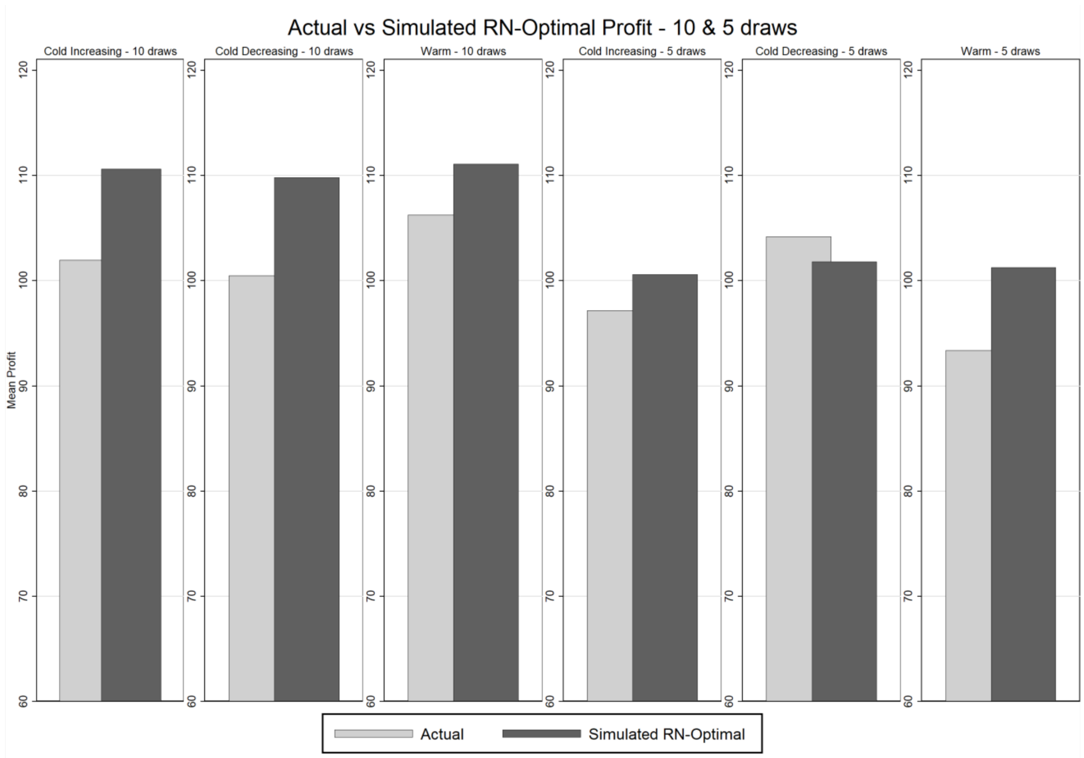

Figure 14: Actual vs. RN-optimal profits ($n = 10$ & $n = 5$) - all participants

Table 10: Acceptance thresholds in round 6 (when $n = 15$) and RN-optimal ones

Number of Remaining Trials		Cold-Increasing	Hot	RN-Optimal
14	Mean	99.854	104.404	112.784
	Std. Dev.	22.834	14.465	
13	Mean	100.313	104.192	112.139
	Std. Dev.	19.684	12.020	
12	Mean	101.208	103.077	111.408
	Std. Dev.	15.623	16.525	
11	Mean	101.188	104.038	110.573
	Std. Dev.	13.807	10.456	
10	Mean	98.333	102.865	109.608
	Std. Dev.	14.755	12.711	
9	Mean	97.417	102.115	108.480
	Std. Dev.	12.391	9.298	
8	Mean	95.521	100.538	107.143
	Std. Dev.	14.447	12.586	
7	Mean	96.104	101.135	105.528
	Std. Dev.	13.071	10.462	
6	Mean	93.375	100.000	103.535
	Std. Dev.	14.627	12.692	
5	Mean	91.188	100.288	101.007
	Std. Dev.	17.523	10.615	
4	Mean	90.854	97.904	97.671
	Std. Dev.	16.461	14.209	
3	Mean	84.438	97.231	93.030
	Std. Dev.	18.014	12.200	
2	Mean	83.167	97.096	86.000
	Std. Dev.	19.243	14.159	
1	Mean	75.000	87.808	73.500
	Std. Dev.	24.466	23.787	